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AEROPLANE STRUCTURES

APPLIED AERO-DYNAMICS

By L. BAIRSTOW, F.R.S.

With Illustrations and Diagrams. 8vo.

AEROPLANE STRUCTURES

By A. J. SUTTON PIPPARD, M.B.E., M.Sc., Assoc.M. Inst.C.E., Fellow of the Royal Aeronautical Society, and Captain J. LAURENCE PRITCHARD, late R.A.F., Associate Fellow of the Royal Aeronautical Society. With an Introduction by L. BAIRSTOW, F.R.S.

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NEW YORK, LONDON, BOMBAY, CALCUTTA, AND MADRAS

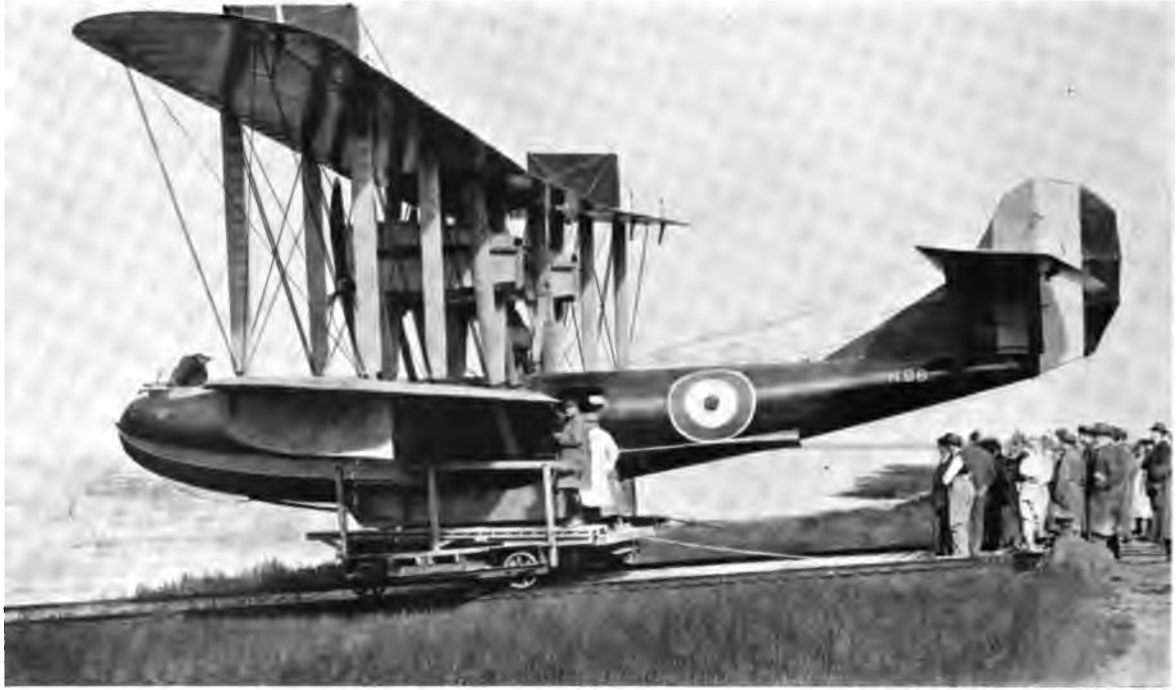

PLATE I.—Phœnix "P. 5." Boat Seaplane.

Frontispiece.

AEROPLANE STRUCTURES

BY

A. J. SUTTON PIPPARD, M.B.E., M.Sc.,
ASSOCIATE MEMBER OF THE INSTITUTION OF CIVIL ENGINEERS; FELLOW OF THE ROYAL AERONAUTICAL SOCIETY,

AND

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WITH ILLUSTRATIONS AND DIAGRAMS

WITH AN INTRODUCTION BY
L. BAIRSTOW, F.R.S.

NEW YORK:
LONGMANS, GREEN AND CO.
FOURTH AVENUE AND 30TH STREET
39 PATERNOSTER ROW, LONDON
BOMBAY, CALCUTTA, AND MADRAS
1919

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INTRODUCTION

BY

L. BAIRSTOW, C.B.E., F.R.S.

MESSRS. PIPPARD AND PRITCHARD'S book deals in an authoritative way with the applications of well-established engineering principles to a new science. The necessity for economy in structure weight has an importance in aviation which is unrepresented in the more common engineering problems of bridge building and construction of machinery. As a consequence of the greater stringency of the requirements, it has been found necessary to make common application of theorems of comparatively rare occurrence in other branches of engineering. An example of this may be mentioned in the universally adopted system by which the spars and wing structure of the aeroplanes are considered as elastic parts. Stresses in wing spars are only calculable with sufficient accuracy after consideration of the strains and the usual assumption that a redundant structure can be reduced to an equivalent rigid structure by the elimination of certain members is abandoned in aeroplane calculations. In applying the new methods, the authors of this book have taken a leading part, and the relative immunity from structure failure which has characterised British design is in no small measure due to them.

The most accurate method of calculation of spar stresses yet known is due to Mr. Arthur Berry, of King's College, Cambridge, and by the help of a number of tables which he compiled the work is not unduly laborious. The well-known theorem of three moments is by this means reduced to a simple form very suitable for drawing-office use.

The unique nature of the problems of aviation has made it necessary to abandon the older ideas of factors of safety, and to pay far greater attention to the possible loadings which may come on the internal structure. No other vehicle, certainly not the locomotive or steamship, is subjected to such a variety of changes of motion, the difference being immediately obvious if one tries to imagine looping or rolling over in the latter cases. On account of the freedom of motion it happens that loads occur in flight manœuvres which may be four or five times those in steady straight flying. The increase of weight required to secure a factor of safety of five or ten on the extreme loads has never been contemplated, and a figure less than two has proved to be adequate in conjunction with great care in the selection of materials and inspection of workmanship to meet the new drastic requirements. There has grown up an intimate connection between the strength factors required of the structure and the uses to which an aeroplane may be put. A heavy bombing aeroplane has nominally half the load factor of a fighter scout, but has probably a greater real factor. It can be made more stable, just because it is not required to undertake violent manœuvres for fighting, and unexpected emergencies are not likely to occur in an aeroplane with a reasonable degree of stability. For civil flying the emergencies should be still less rare, and there is no apparent difficulty in producing strength which is ample to secure safety in civil uses. Perhaps most change may be expected in the undercarriages of the future.

The War period has produced a unique opportunity for observing the factors of safety required, for manœuvres of all descriptions have been performed either by intention or accident. It is difficult to think of any new kind of aeroplane motion. With these operations Messrs. Pippard and Pritchard have been in close contact, and have developed their methods and extended them to meet the requirements of the moment. The science is still new and is progressive, and it is to the advantage of aviation that a work of this character is available, which summarises in working form the best known methods of ensuring that an aeroplane when built will have that degree of safety which experience shows to be necessary.

L. BAIRSTOW.

July 18, 1919.

PREFACE

THIS book, intended primarily for the use of aeroplane designers and students of aeronautical engineering, is the result of the Authors' experience during the war whilst engaged in the work of supervising experimental designs from the standpoint of structural strength.

To the designer, the results will appeal more than the mathematical proofs, but no apology is needed for inserting these latter wherever possible, since it is essential that the student should understand how the formulæ are obtained.

It is recognized by the Authors that this branch of engineering is an entirely new one, and much criticism of the first book devoted to aeroplane structures is to be expected. Such criticism will be welcomed, as it is only by the mutual exchange of ideas that progress in a new science can be made.

The Authors would like to point out that the chapters dealing with the more general structural and stress problems are only intended to be a resumé. It is assumed that the reader is already acquainted with a good deal of this general work, or has at least access to standard works on the subject.

Thanks are due to the Department of Aircraft Production for permission to use information and photographs; to Messrs. Aircraft Manufacturing Co. for permission to reproduce drawings; and to Mr. C. R. Fairey, of the Fairey Aviation Co., for permission to include his method of spar testing in Chapter XX. Other acknowledgments are given in the text, but if in any case they have been inadvertently omitted, the authors would like to take this opportunity of offering their apologies.

Further, the Authors wish especially to acknowledge their great indebtedness to their late colleagues in the Air Ministry, Dr. Hilda Hudson, O.B.E., and Captain T. M. Wilson, R.A.F., for their great help in the arrangement of the book and their valuable criticism of the manuscript.

A. J. SUTTON PIPPARD,
J. LAURENCE PRITCHARD.

LONDON,
1919.

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CHAPTER I

INTRODUCTORY

1. The General Problem.—To the engineer, aeroplane design presents questions differing in many ways from those he has to deal with in other branches of his work. Essentially an engineering problem, its satisfactory solution must on that account demand consideration of questions which are common to all structural work. At the same time other important factors which are peculiar to it, enter into and complicate it.

The American definition of an engineer as " A man who can do for one dollar what any fool can do for two " may well be amended in the case of the aeronautical engineer. The latter must build for one pound avoirdupois what any one could build for two, and in this importance of the weight question lies the primary difference between aeroplane and other structural design.

Not only is the saving of weight of primary importance in the structural members of the aeroplane, but also in the power unit. The greatest difficulty confronting the pioneers of flying was to obtain an engine which would give the necessary power for flight for a reasonable weight, and it is safe to say that the lack of this held back the science for many years. With the advent of the petrol engine, however, the problem was solved, and flight became a practical proposition, purely on this question of pounds per horse-power.

Given the engine it was now the turn of the structural designer, and his aim has always been to produce a framework giving the maximum of safety for the minimum of weight. He has been so successful in this endeavour that one may safely prophesy that an aeroplane can be designed for any engine which the future may develop, for at the present time further progress is limited by engine considerations alone.

While weight saving must be carried out as far as possible, it is of course essential that the necessary strength be obtained, and the most successful designer is the one who can obtain the greatest value of the ratio of strength to weight, with due regard to the work which the aeroplane has to do.

2. The Factor of Safety.—In engineering work of all classes the necessary working strength is ensured by the introduction of a " factor of safety.' This factor of safety is obtained by designing, not to the ultimate strength or elastic limit of the material employed, but to a figure much below it. This implies that under the worst loads which can occur in the structure the material is only stressed to a fraction of the figure at which it would fail.

This is the universal engineering practice, but in aeronautical engineering a radical change has to be made. It is impossible here to give a factor of safety in the ordinary sense, as this would mean such an increase of weight that the resulting structure would probably not fly at all, or at the best would put up a very poor performance. The best that can be done, therefore, is to estimate the maximum loads which can possibly occur in all the structural members and so design those members that a small margin of strength is in hand under the worst conditions.

Even this small margin of safety is not always obtainable, and especially in the smaller and more easily manœuvrable aeroplanes there are certain evolutions which would cause structural failure if attempted.

It becomes a critical question and a good test of the engineer's judgment, therefore, to estimate these forces and design members to take them correctly, since a small error may cause not only the loss of the aeroplane, but also loss of life.

This introduces another difference between aeroplane design and other branches of engineering. A mistake in any design may, of course, cause loss of life, as, for example, in the collapse of a bridge, but more often it entails financial loss only. Mistakes in the design of aeroplane structures, however, nearly always mean accidents which are fatal or result in serious bodily injury to at least one or two men. There is, therefore, no room for error in this branch of engineering, and a sound knowledge of theoretical structures becomes imperative, if successful designs are to be produced.

3. The Resistance Factor.—A further peculiar question arising is that of resistance. This is important as affecting the performance of the aeroplane, since the efficient use of the engine horse-power available depends on the reduction of the head resistance of the aeroplane to a minimum.

As an example of the importance of this reduction of head resistance, may be quoted the actual case of a scouting aeroplane which gained seven miles an hour in top speed by the substitution of wires of a streamline shape for circular cables in all exposed positions.

While this problem is not primarily a structural one, it has a great influence on the framework of the machine, since such members as struts and wires have to be designed with less freedom than in other engineering structures in order to ensure a good streamline section.

4. Limitation of Material.—The aeroplane designer is at present handicapped in a manner which is unknown in other branches of engineering by the limitation of materials. While for the bridge builder, steel, masonry, timber, and reinforced concrete are all available, the aeroplane designer is practically dependent upon two of these, timber and steel, and even with these his choice is limited to very few varieties. This restriction introduces further problems into the design question, and until more research has been made into light alloys and more experience in their use obtained, this handicap cannot be removed.

5. Resumé.—The successful aeroplane is the one which has the requisite margin of safety with the least weight of structure, combined with the requirements as to performance and load-carrying capacity. In most cases the expense is of secondary importance if these essential conditions are satisfied, at least where aeroplanes for war purposes are concerned, but there is no reason why the points above should not be combined with economical design.

To obtain the best results all round it is essential that scientific methods of calculation should be adopted in the structural design, and the present book aims at giving these methods in such a form as will render them of most value both to the student and the practical designer.

CHAPTER II

THE MECHANICS OF FLIGHT

6. Air Reaction on an Inclined Plate.—If a jet of fluid is deflected from its path there is a reaction between the jet and the deflecting object. The latter may be a curved tube as shown in Fig. 1; a shaped vane or a plate; the effect will be similar.

Suppose this jet to be of cross-sectional area a, and to have a velocity V feet per second. On impact with the plate let it be deflected through an angle β to its original line of flow, without loss of speed, *i.e.* assume there is no loss in the kinetic energy of the stream.

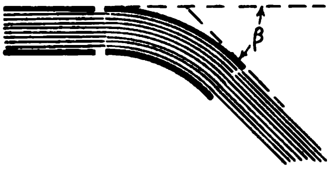


FIG. 1.

Then if ρ be the density of air, the mass of air deflected per second is $aV\rho$.

The momentum of air per second previous to impact $= aV^2\rho$.

The momentum of air per second in its original direction after impact $= aV^2\rho \cos \beta$.

Therefore the change of momentum per second due to impact in the direction of the original jet is $aV^2\rho(1-\cos \beta)$ and this is the component reaction in the same direction.

In the perpendicular direction there is created in unit time a quantity of momentum $aV^2\rho \sin \beta$, and this is the reaction perpendicular to the jet.

The total reaction is the resultant of these two components, and is $2aV^2\rho \sin \beta/2$, and its direction is along a line bisecting the angle between the original and final direction of the jet, *i.e.* it is at $(90 - \beta/2)°$ to the original line.

This force can be resolved into two components, one tending to lift the plate in a vertical direction and the other tending to move it horizontally. These resolved components are very important, and have a great bearing upon the problem of flight.

Suppose, instead of the jet impinging upon the plate, the latter is moved through still air at a velocity V. Exactly the same forces are brought into action and the plate tends to lift. Further, the force required to impel the plate at the given velocity is that necessary to overcome the horizontal force mentioned in the previous paragraph.

This is the method which is employed to render flight possible. The aeroplane is provided with plates or wings which are forced through the air by an engine powerful enough to overcome the horizontal force—

known as drag; the vertical force—known as lift—then causing the aeroplane to rise.

7. The Aerofoil Section.—In the case just considered of a jet impinging on a plate, the whole stream was supposed deflected; but when the plate is moved through the air it is clear that not only is the air below the plate affected, but also that above, and results of actual experiments show clearly that by far the greater portion of the lift force is caused by the nature of the disturbance above the plate.

The plate impinging on the air causes a flow somewhat as indicated in Fig. 2, and the effect of this flow is to cause an increased pressure on the underside of the plate and a partial vacuum on the upper side. It is the creation of this partial vacuum which is responsible for a portion of the lifting force.

So far it has been tacitly assumed that the plate is a flat one, but if the nature of the flow of air above it is of the importance suggested, it is evident that the shape of the plate in section as affecting the flow must have some bearing on the results obtained.

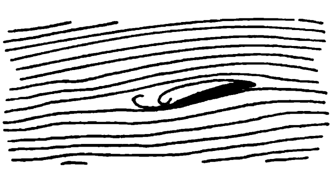


FIG. 2.

Experiment proves this to be the case, and very varying results are obtained by the use of plates curved in different ways in section. These plates are called aerofoils, and the properties of these aerofoils, and the effects of changes in their shape, are the subjects of continuous experimental research.

8. The Wind Tunnel.—In order to conduct these investigations into the properties of aerofoils it is necessary to have a steady air stream, the velocity of which can be measured accurately, instruments sufficiently delicate to measure small changes of force and accurate models of the aerofoils to be tested.

The last two requirements are a matter of mechanical skill, and the first is obtained by means of the apparatus known as the wind tunnel.

Essentially this is a long wooden box varying in section in different tunnels from three to seven feet square. An air screw sucks air through this tunnel, and by a proper regard to proportion of the apparatus a steady flow is maintained, which ensures a practically constant velocity.

In this air stream a model of the aerofoil under test is mounted upon a balance by means of a small spindle passing through it. The balance is so designed that the angle of the aerofoil to the stream can be varied, and the vertical and horizontal forces measured for all positions of the aerofoil.

A further measurement is made of the moment of the forces causing the model to turn about the supporting spindle. The lift and drag forces and the moment of the forces about the spindle are measured for all angles of the aerofoil to the line of air flow, and from these figures the value of the section for any particular purpose can be determined.

9. General Characteristics of an Aerofoil.—The results obtained by testing a large number of aerofoil sections show that while the different shapes vary in degree they are all similar in their general properties.

Fig. 3 shows a series of curves plotted for a typical aerofoil. The abscissæ denote the angle at which the aerofoil is inclined to the air stream. This angle is known as the angle of incidence of the aerofoil, and it is measured from the chord line (see paragraph 11).

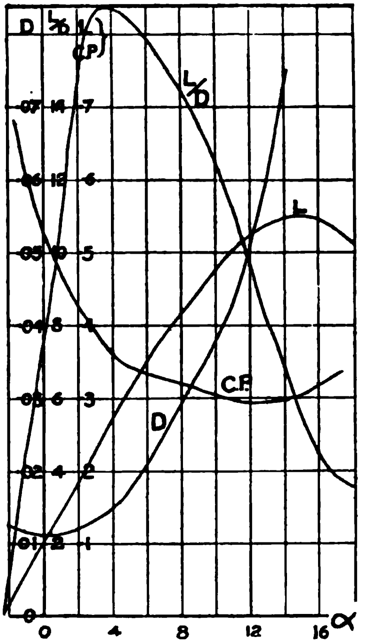


Fig. 3.

The first curve shows the variation of lift with the angle of incidence. It will be seen that this curve is a straight line for all practical purposes from the value $a = -2°$ to about 12° at which point there is a disturbance in the flow. This is usually known as the " burble point," and corresponds to the stalling point in an aeroplane (see paragraph 29). It is at this angle that the streamline flow of the air current past the aerofoil breaks down.

It will be seen that this curve does not pass through the origin, but that the angle at which there is no lift force is a small negative one. This is due to the fact that the upper and lower surfaces of the aerofoil are not symmetrical. The angle at which L, the lift force, vanishes is called the " angle of no lift."

The second curve is that for the drag force, and here again there is a great similarity for all aerofoil sections in the general shape of the curve. Having a minimum value in the neighbourhood of 1°, it rises rapidly each side of this point in a smooth curve and does not exhibit the peculiarity of a " burble point " shown in the lift curve.

The third curve shows the ratio of the lift to the drag forces plotted against a. This ratio is extremely important from the standpoint of flight, as it gives a direct measure of the efficiency of an aerofoil. This will be dealt with in detail at a later stage.

The fourth curve shows the position of the centre of pressure for the various angles of incidence. The centre of pressure for any particular angle of incidence is arrived at as follows. The values of the vertical and horizontal forces having been determined by direct measurement in the tunnel, the value of the resultant force can be calculated. In addition, the moment of this force about the spindle is measured, and so it is a simple matter of division to determine the distance from the spindle at which it may be considered concentrated. At the angle of no lift, the moment does not vanish, and the centre of pressure is at a very great distance from the spindle. The centre of pressure varies its position for most aerofoils in a very similar manner to that shown in Fig. 3.

10. Expression of Results.—The results for an aerofoil having been obtained in the wind tunnel, it is necessary for them to be reduced to a form which is convenient for use, and in which they can be applied to the design of an aeroplane.

If a plate be moved through still air it is clear from a consideration of the equations of the change of momentum quoted at the commencement of this chapter that the lift and drag forces are proportional to the area of the plate, the density of the air, and the square of the velocity at which the plate is moved through the air,

i.e. $L \propto \rho S V^2$

or $L = k_L \rho S V^2$

where k_L is a constant to be determined by experiment, and S is the area of the plate. k_L is known as the absolute lift coefficient, and is non-dimensional.

Similarly $D = k_D \rho S V^2$

where k_D is the absolute drag coefficient.

The use of these coefficients is very convenient, and a similar method is adopted for reducing the moment

$$M = k_M \rho S V^2 c$$

where k_M is the absolute moment coefficient, and c is the chord length of the aerofoil. k_M is quoted so that the expression above gives the value of the moment about the nose of the aerofoil.

The centre of pressure is defined by a coefficient which denotes the distance of the centre of pressure from the leading edge of the aerofoil in terms of the chord length.

11. The Shape of an Aerofoil and its Effect.—An aerofoil has a cambered top surface. The lower surface may be flat, or curved, and may be either convex or concave upwards. The front of the section or the nose A is known as the leading edge, and the rear end B as the trailing edge.

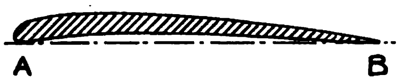


FIG. 4.

The chord line is an arbitrary datum line from which all dimensions of the aerofoil are measured, and it is from this line that the angle of incidence is reckoned. This datum line is usually the common tangent to the curve of the under surface near the leading and trailing edges, or in some cases is the line joining the leading and trailing edges. The intercept on this line of perpendiculars from the leading and trailing edges is called the chord length, or simply the chord.

The maximum camber is usually at a point about one-third of the chord from the leading edge, and the depth of the aerofoil at this section varies from about ·06 to rather over ·1 of the chord in the commonest aerofoils.

The selection of a wing for any special aeroplane requires considerable judgment. Some sections give a high lift coefficient at moderate angles of incidence, and these are useful for weight-carrying machines. The disadvantage of this type of section is, however, the greater resistance offered, and so if a wing is needed for speed a section is chosen which while not giving such a high value of the lift coefficient shows a better ratio of lift to drag.

In making a comparison of two wings it is necessary to reduce them to a common datum, and for this purpose the angle of no lift as determined by tests is the best starting point. This angle may be considered as the true zero for the wing.

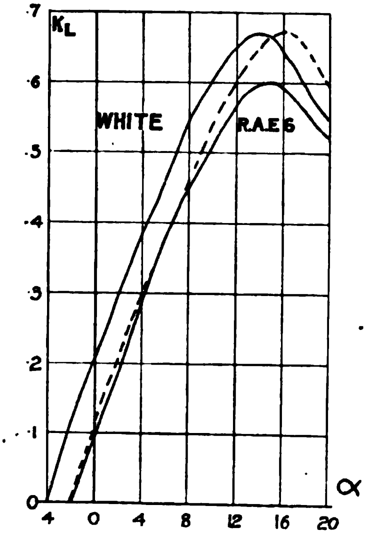


Fig. 5.

This point is best illustrated by an example.

Fig. 5 shows plotted the curves of k_L against angle of incidence for the two wing sections R.A.E. 6 and the "White." Taking any value of a it is seen that k_L for the White section is higher than for R.A.E. 6.

The White curve, however, cuts the axis of a at the point $-4°$, whereas the R.A.E. 6 curve cuts it at $-2°$. These angles may be considered as the true zeros for the two sections respectively, and in order to make a fair comparison it is necessary to make these points coincide. If, then, we move one curve along parallel to the axis of a until the two origins coincide, it will be seen that whereas before one wing section appeared much better than the other, there is now very little to choose between them. Indeed, a selection would now be made not on the k_L curves, but on the L/D curves.

Bearing the method of comparison in mind it is now possible to make some general statements as to the effect of proportions in the aerofoil.

An alteration in the maximum camber of the wing affects both L/D and L. The L/D ratio is highest when the maximum camber is ·05 of the chord, and the lift coefficient is a maximum when this value is ·075 of the chord. The best form would appear to be therefore one in which the maximum camber falls between these values. In the German Fokker aeroplanes the section is much deeper than this, but this appears to be principally in order to get a very deep spar.

An alteration in bottom camber has very little effect upon the characteristics of an aerofoil.

The best position of the maximum camber is at one-third of the chord from the leading edge both for values of L and L/D. Both these quantities are affected adversely by any shift of this position, but a certain instability of flow renders it advisable to arrange the maximum camber at three-eighths of the chord from the leading edge, and sacrifice a little in the value of L/D.

An alteration in the leading edge shape or "entry" only affects the drag, and hence, of course, the L/D values. A fine entry is better from this point of view than a blunt one.

The trailing edge does not affect L or L/D very appreciably, but by giving it a reversed curvature the centre of pressure movement can be greatly reduced. The angle which this reversed portion makes with the chord should not exceed 15°.

It must be remembered that the above notes are generalizations, and are only true within limits. The best possible information on any particular point will usually be found in the Reports of the Advisory Committee for Aeronautics in the form of research work carried out at the National Physical Laboratory.

The characteristics of a number of aerofoils are given at the end of the book.

12. The Characteristics of an Aerofoil as affecting Flight.—So far we have only considered the properties of aerofoils as regards their behaviour under test in the wind tunnel. It now remains to be seen how these characteristics, as they are termed, can be employed in the design of the aeroplane. In the first place, it has already been shown that the primary object to be obtained is that of getting sufficient lift to raise the aeroplane and its load off the ground. The lift may thus be considered as the useful force brought into play.

In order to obtain this lift, however, the wing must be moved forward through the air at a certain speed. This entails some energy being employed, and the amount of this energy depends upon the resistance which the wing offers to movement at the speed required. This resistance we know as the drag, and it may be considered as the opposing force.

The most efficient wing section is obviously the one which gives the required lift for the smallest drag at the necessary speed. The efficiency of an aerofoil is thus measured by the value of the ratio L/D.

Suppose it is required to compare two aerofoil sections, the following method may be adopted.

It has already been shown that the lift $L = k_L \rho S V^2$, so that for equal lifts at speed V equal values of k_L are required from the two aerofoils. When given the value of L required, the section which shows the better value of L/D is the more efficient. If, then, for the two sections curves of k_L/k_D are plotted upon a base of k_L, it can be seen at a glance which section is the better at any given point, and the selection can then be made. Such curves are shown plotted in Fig. 6 for the same two aerofoils represented in Fig. 5.

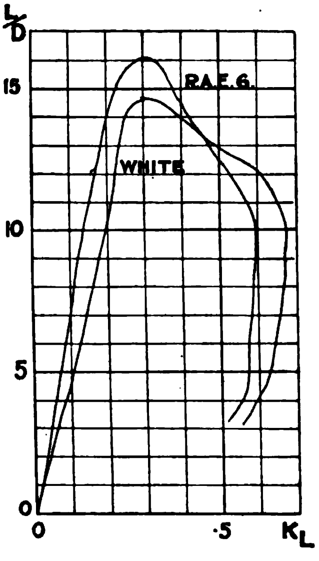


Fig. 6.

In a biplane or a multiplane the forces and moments upon the various planes must be compounded to find the total action of the air on the wings. It is usual to state the results, in theoretical work at least, in terms of an imaginary "equivalent plane," whose position, chord and area are so chosen that correct equations can be written down as if the craft were a monoplane with this wing.

This is possible only if the values of k_L and k_D for any one plane bear to their respective values for another plane ratios which are independent of α. This is approximately true over the greater part of the range, but is not so accurate near the angle of no lift.

13. The Aeroplane as a Whole.—So far the wings only have been considered acting as a separate unit, but a complete aeroplane consists of many other parts all of which must be taken into consideration.

From the standpoint of the mechanics of flight the only points which concern us in the remainder of the various units of the aeroplane are their weights and resistances.

When any body is propelled through the air it offers resistance to such propulsion, this resistance being approximately proportional to the square of the velocity with which the body is moving. While the wings, therefore, have a resistance following a law determined only by experiment and shown on the curve of k_D for any given aerofoil, the body, wheels, and other portions of the aeroplane follow a square law within limits. The total resistance of an aeroplane may therefore be split up into two parts—

(1) The resistance of the wings found by model experiments $= D_W$.

(2) The resistance of the body, wheels, skids, and so on, represented by D_B, where $D_B \propto V^2$.

For any speed, therefore, the total resistance D of the aeroplane is given by

$$D = D_W + D_B$$

It has already been shown that for any speed V the value of D_W can be expressed in the form

$$D_W = k_D \rho S V^2$$

where k_D is the absolute drag coefficient. The value of D_B can be similarly expressed by $D_B = k_{D'} \rho S V^2$ where ρ, S, and V have their usual significance, and $k_{D'}$ is a coefficient of resistance for the aeroplane without the wings. This coefficient, unlike k_D, is practically independent of the angle of incidence.

Then

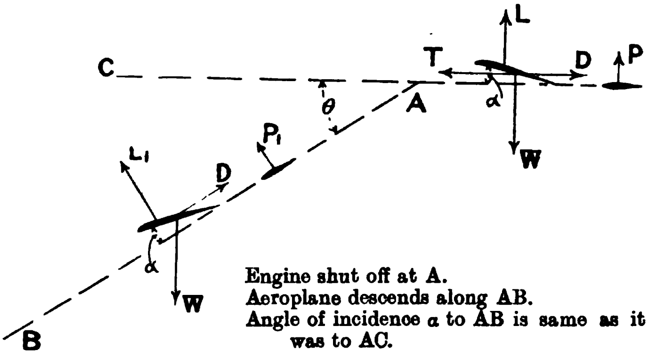
$$D = \rho S V^2 (k_D + k_{D'})$$

If the aeroplane be assumed flying level at a velocity V, the total lift forces on the main planes and other lifting surfaces are just equal to its total weight, and in order to fly steadily at this speed the horse-power developed must be just sufficient to overcome the value of D.

The smaller the horse-power required, *i.e.* the smaller the value of D for a given speed, the better is the aeroplane, and so the ratio W/D may be considered as a measure of the aerodynamic efficiency of the aeroplane. When W is the weight of the aeroplane W has already been shown to be equal to $k_L \rho S V^2$ for horizontal flight, and $D = \rho S V^2 (k_D + k_{D'})$, so that W/D becomes $\frac{k_L}{k_D + k_{D'}}$.

14. Gliding Angle.—Suppose an aeroplane to be flying horizontally at a speed V. Let the angle of incidence of the main planes be α. The forces acting upon the craft are—

(1) The lift acting vertically through the centre of pressure;
(2) The weight acting vertically through the centre of gravity;
(3) The drag acting horizontally;
(4) The thrust of the engine acting horizontally;
(5) A tail load acting vertically.



Engine shut off at A.
Aeroplane descends along AB.
Angle of incidence α to AB is same as it was to AC.

FIG. 7.

If it is assumed that the thrust and the drag act through the centre of gravity, and that the propeller slip stream may be neglected, no couple is brought into play when the engine is cut off. Since the aeroplane is in steady flight the forces are in equilibrium. Suppose the engine is now cut off. The velocity decreases and the lift is no longer equal to the weight, so the aeroplane commences to descend. The effect of the addition of a downward component of motion is to increase the angle of incidence of the main planes and of the tail plane, and if the aeroplane is stable restoring couples are brought into play which bring the craft into the same attitude relative to the new flight path which it had relative to the horizontal path.

Let the angle of the new flight path to the horizon be θ. Then the lift on the planes normal to the inclined flight path is L. W still acts vertically, and since the forces must be in equilibrium

$$L = W \cos \theta$$

But $L \propto V^2$

Therefore the velocity on the new flight path is

$$V\sqrt{\cos \theta}$$

Further, since the component along the inclined flight path is D

$$\sin \theta = D/W$$

θ is called the gliding angle, and is generally expressed by the value of its cosecant W/D.

Since k_L and k_D vary for every angle of incidence it is clear that the value of the expression $\frac{k_L}{k_D + k_{D'}}$ will not be constant. It follows, therefore, that there is a different gliding angle for every flight speed of the aeroplane. The highest value of the ratio occurs usually in the neighbourhood of from 2° to 4° angle of incidence, and for a modern aeroplane is from 8 to 10.

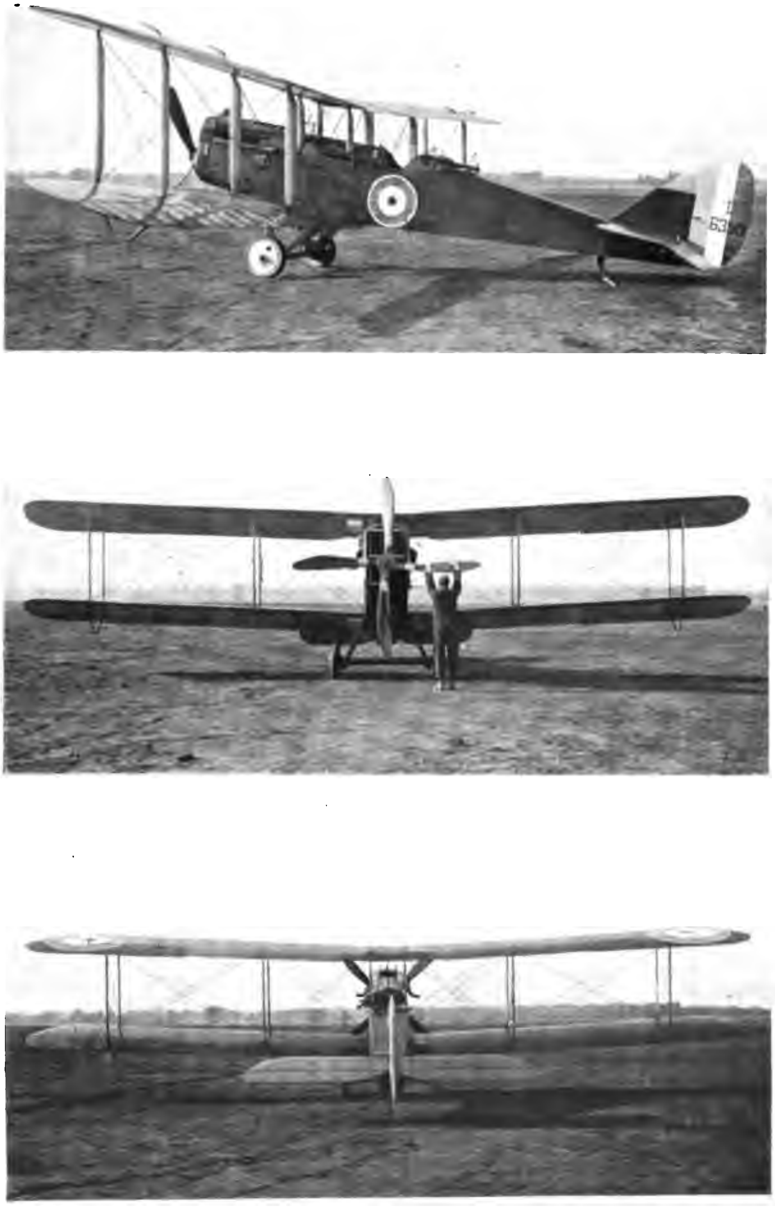


PLATE II.—Airco, 9A Aeroplane.

CHAPTER III

THE STRUCTURE OF AN AEROPLANE

15. Main Structural Units.—The two succeeding chapters will be devoted to a consideration of the various parts of an aeroplane, their functions and relations one to the other. The present chapter will deal with the main units composing the complete structure, and the one following will consider these units in detail.

The complete aeroplane structure always consists of four main sections or units.

(1) **The main planes, wings or glider unit.**
(2) **The body unit.**
(3) **The tail unit.**
(4) **The undercarriage or chassis.**

The forms which these units take differ, of course, in different types of aeroplane, but their functions remain unaltered in spite of differences in design.

16. The Main Planes.—The main planes, wings or glider unit consist of fabric-covered surfaces which give, when forced through the air, the lift reaction necessary to support the aeroplane and its loads (Plates II. to VII.). They are cambered in section for the sake of efficiency, as described in Chapter II. The exact type of camber which is used depends on the functions of the aeroplane.

The main planes must be placed upon the aeroplane in such a position that the centre of gravity of the latter as a whole lies as closely as possible in the same vertical as the centre of pressure on the planes at cruising speed. As a general rule, it may be taken that the planes should be so arranged that the centre of gravity of the complete aeroplane lies between ·28 and ·40 of the mean chord of the wings measured from the leading edge. Unless this is done there may be a considerable distance between the lines of action of the weight carried and the supporting force, bringing a large couple into play which is difficult properly to counteract. In addition the stability of the aeroplane is adversely affected.

The glider unit may consist of a single plane or a number of planes, and aeroplanes are commonly classified in the first instance by the number of the planes.

The **monoplane** has **one** supporting surface.
The **biplane** has **two** superposed supporting surfaces.
The **triplane** has **three** superposed supporting surfaces.
The **quadruplane** has **four** superposed supporting surfaces.

Up to the present no serious attempt has been made to extend the number of planes beyond four, and indeed the number of quadruplanes built is not large, as no great success has attended this type. There is, however, no reason why, as it becomes necessary to increase the lifting surface and maintain reasonable overall dimensions, the multiplane should not become as great a success as the biplane and some triplanes have proved themselves.

Attached to the main planes and forming part of them are the lateral control surfaces. These consist of movable flaps known as wing flaps or ailerons, usually placed at the wing tip and forming part of the surface of the wing itself. The function of these flaps is to control the aeroplane laterally or enable it to bank correctly. This is done from the pilot's seat by depressing the flaps on one side of the aeroplane and raising those on the other side simultaneously. The effect is to increase the lifting force on one side of the aeroplane and decrease it on the other, thus applying a moment to the aeroplane which causes it to bank or take up an angle to the horizon (see Chapter V.).

The control of these surfaces is exercised by the pilot either by sideways movement of a control column or the rotation of a wheel control.

17. The Body.—The function of this unit is to contain the engine, fuel, pilot, passengers and useful load.

Aeroplanes are not limited to one body. In the case of a multiple engine aeroplane there may be one or two main bodies and also subsidiary bodies for the purpose of containing one or more of the engines.

The engines may be placed to drive propellers either in front or in the rear of the main planes. When the former arrangement is adopted the aeroplane is usually known as a tractor type (Plate II.). When the propeller is behind the main planes, however, the aeroplane is known as a pusher (Plates III. and IV.).

In multiple engine aeroplanes a combination of these types is common, that is to say, some of the engines drive pusher and some tractor airscrews.

If the aeroplane is a tractor the propeller is placed at the very front of the body, which can then be extended right aft to support the tail plane without any trouble. This body is then called the fuselage. With the pusher type, however, this method of construction must be modified, and the fuselage gives way to a smaller body or nacelle for the pilot, engine, etc. The tail unit is then supported by means of a braced structure, extending from the main planes, known as the tail booms or outriggers.

In a multiple engine aeroplane it is a common form of design to have either a central fuselage carrying one tractor engine and the greater part or even the whole of the useful load, with wing nacelles carrying the other engines, or else to have two fuselages in the wings with a central nacelle. In either case the tail unit is supported on the fuselages, no outriggers being necessary.

18. The Tail Unit.—The function of the tail unit is to stabilize the aeroplane and render it controllable by the pilot. The tail unit is attached by suitable bracing to the rear of the fuselage or the tail booms. It is

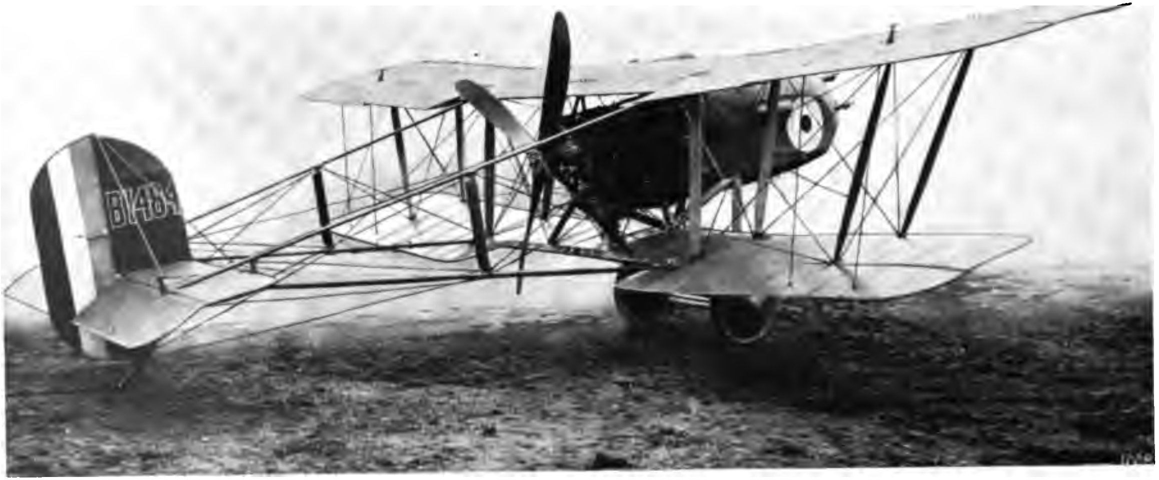


PLATE III.—Vickers' Pusher Scout.

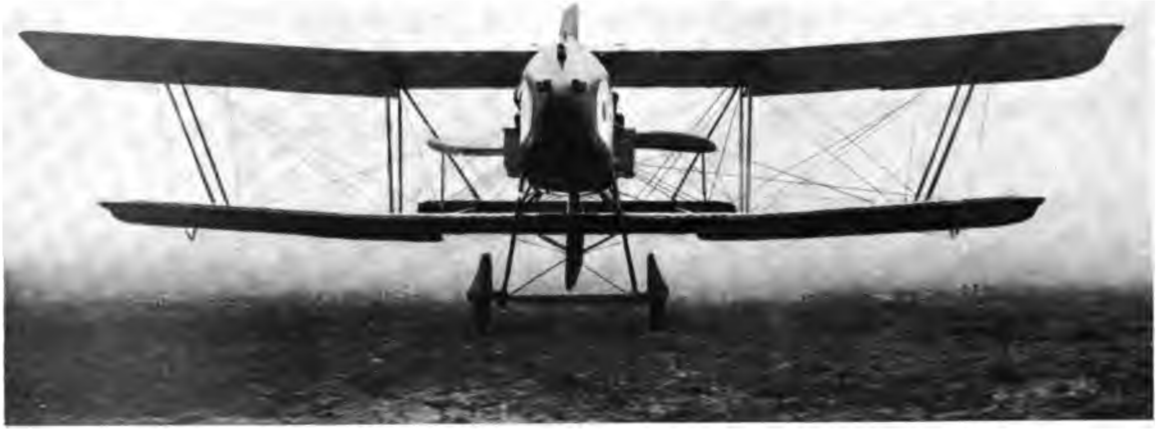


Plate IV.—Vickers' Pusher Scout.

divided into two sections, one for fore and aft and one for directional control and stability.

Fore and aft control and stability are obtained by a horizontal surface known as the tail plane or empennage, which is provided with adjustable flaps known as the elevators, controlled by the pilot by a fore and aft movement of the control column.

These flaps enable the tail control load to be governed, and thus regulate the angle of incidence of the main planes, and consequently the speed of flight of the aeroplane (in conjunction with the engine).

Directional control and stability are obtained by vertical surfaces corresponding to the horizontal ones just mentioned. The fixed surface is known as the fin and the adjustable portion as the rudder. The rudder is controlled either by a rudder bar or by pedals operated by the pilot's feet.

19. Chassis.—The chassis or undercarriage is the structure upon which the aeroplane makes its run in order to acquire flying speed preparatory to leaving the ground and upon which it alights at the termination of the flight.

In aircraft operating from the land the undercarriage consists of wheels or skids supported upon a suitable framework.

In seaplanes the wheels or skids are replaced by floats or hulls of suitable shape and of such buoyancy that they will easily keep the craft afloat in water.

In float seaplanes (Plate VII.) there are two main floats provided, the buoyancy of each being about equal to the weight of the fully loaded seaplane. In addition there is a tail float which takes the place of a tail skid in an aeroplane, and possibly wing tip floats to keep the wings from entering the water should the seaplane roll badly, or while taxying.

In the type of aircraft known as the boat seaplane the two floats are replaced by a hull like a boat, and in this type of seaplane tail floats are unnecessary. The hull takes the place of the floats and also serves the purpose of a fuselage (Plate VI.).

In most boat seaplanes the engines are mounted separately in the wings to give the necessary propeller clearance from the water.

CHAPTER IV

THE DETAIL STRUCTURE OF AN AEROPLANE

20. Main Planes.—The previous chapter has dealt with the main units into which the aeroplane structure can be resolved, and it now remains to analyse the construction of these units in detail, before proceeding to determine the strength considerations which govern their design.

Fig. 8 shows in outline the main planes of a monoplane. This is braced to the fuselage DEFG in the following manner. Struts DH, *d*H, etc., are attached to the fuselage by means of suitable fittings. These struts meet at the point H, forming a rectangular pyramid. The whole of the centre structure is called a cabane. The struts HD, etc., are pylon struts. Wires brace the points B, C, *b*, *c* on the wings to the point H, and these same points are wired to the fuselage at F and *f*.

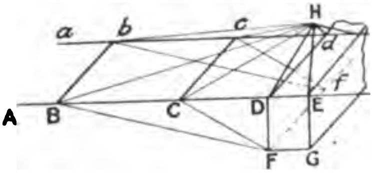


FIG. 8.

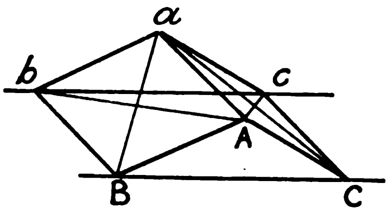


FIG. 9

The lower wires FB, FC, etc., which are operative in normal flight, are known as the lift, load, or flying wires. The upper wires HC, HB, etc., which carry the weight of the wing structure when the aeroplane is standing on the ground, are known as the weight or the antiflying wires.

The cabane instead of being constructed as just explained may be made as shown in Fig. 9.

In this case wires *a*B, A*b*, etc., are needed to complete the bracing, these being known as cabane incidence wires. The advantage of this type of construction over the pyramidal cabane lies in its greater rigidity against torsional movements.

Fig. 10 shows a biplane structure in which the two planes or decks are of equal span. These are braced together by a system of wires and struts as indicated. AG, BH, *ag*, *bh* are interplane or gap struts braced by means of the flying, lift or load wires AH, BJ, etc., and the antiflying or weight

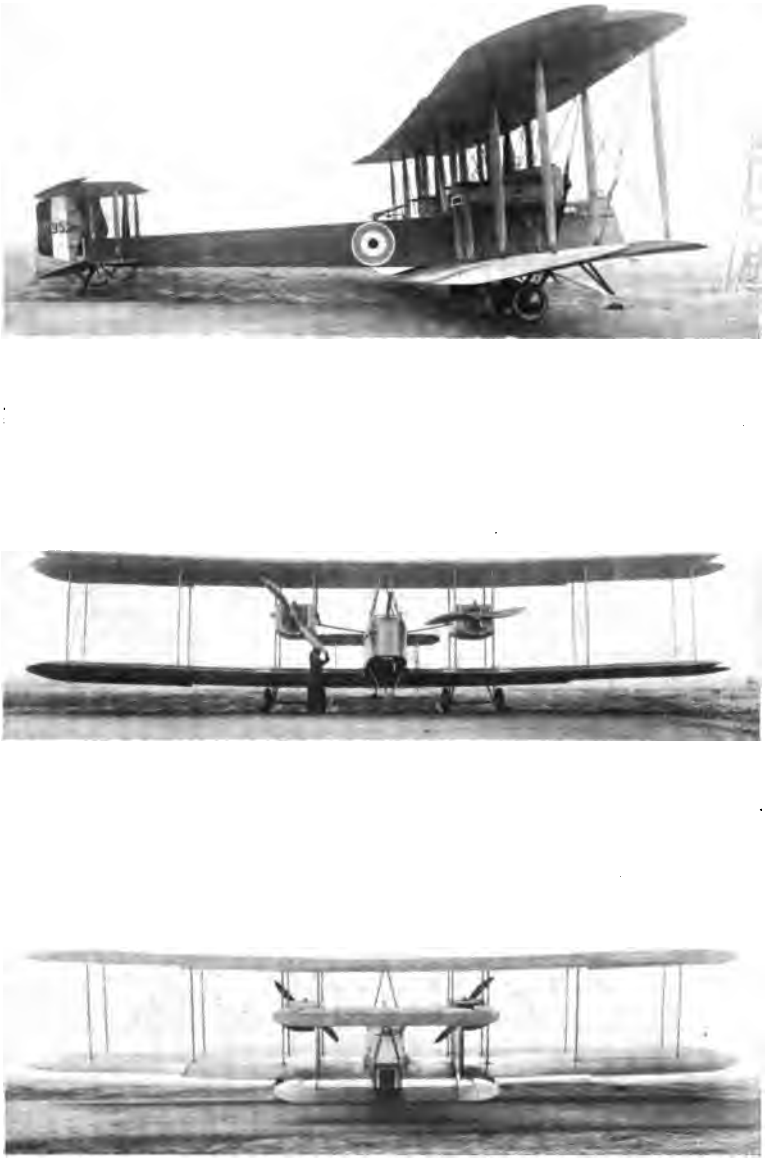


Plate V.—Vickers' Vimy Aeroplane.

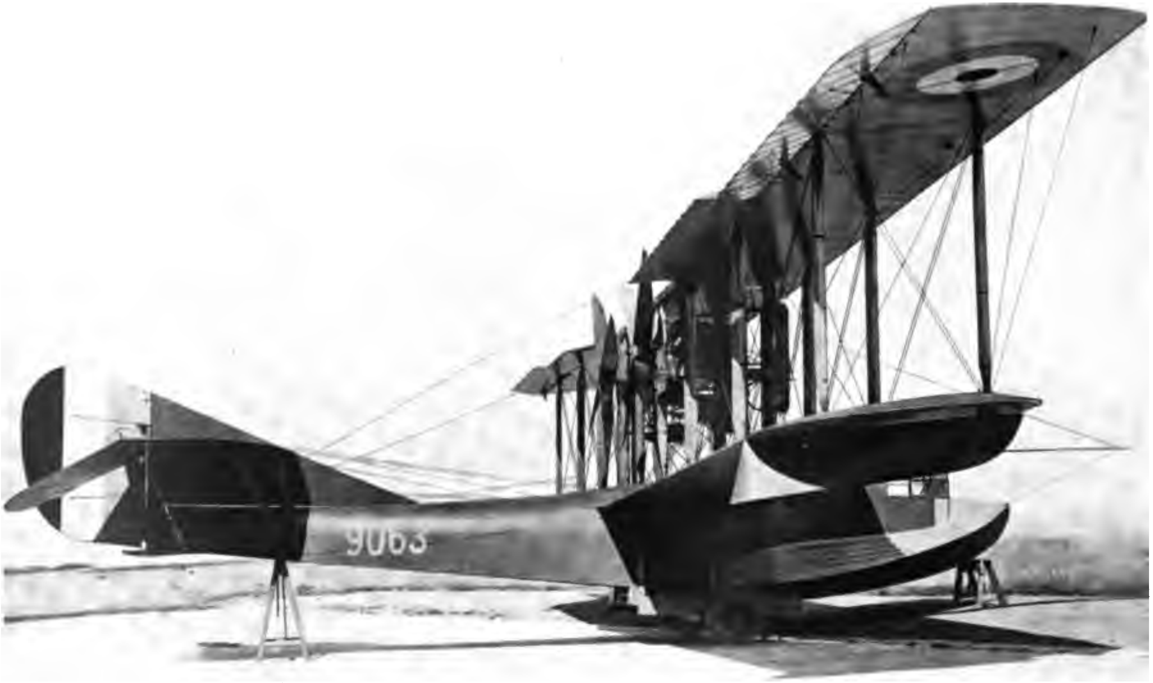


PLATE VI.—Norman Thompson Boat Seaplane.

wires BG, EH, etc. The spars or main horizontal members of the planes form, together with these struts and wires, two complete frames or trusses ABEJHG and *abejhg*, distinguished as the front and rear trusses respectively.

Connecting these trusses in the plane of the front and rear interplane struts are bracing wires A*g* and *a*G, etc., called incidence wires, which, though strictly redundant members in the structural sense, serve a valuable purpose in duplicating the lift wires. The small struts EL, *el*, are centre section or body struts. The panel EL*le* should be provided with incidence bracing.

Fig. 11 shows a biplane structure in which the top plane has a greater span than the lower one. Two methods may be used for bracing this extension CD. The first consists in adopting the pylon system like a monoplane, the cabane being either a pyramid or a prism, the latter being preferable as already explained. The second method consists in making the members ED and *ed* struts instead of wires, so that they may carry either an upward or downward load on the wings. In this type of structure the portion of the top plane extending beyond the outermost interplane strut is called the extension plane or the overhang, and the struts ED, *ed* are extension struts. The top wires AD A*d*, etc., are pylon wires, and the struts AC, A*c* are pylon struts.

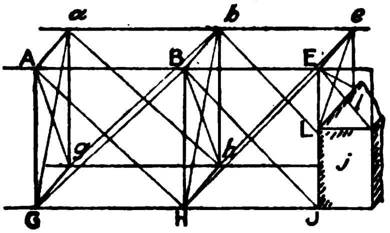


FIG. 10.

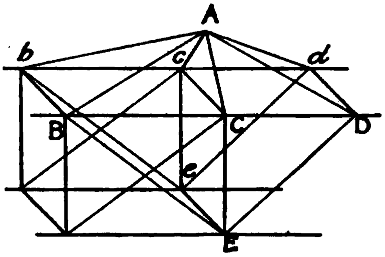


FIG. 11.

Having described the external bracing between the planes, the actual construction of the planes themselves will now be examined in detail.

Fig. 12 shows the internal structure of an aeroplane wing. The main members are the longitudinal beams S and *s*, known as the front and rear spars respectively. These members form the booms of the front and rear trusses, and the interplane bracing struts and wires are attached to them.

The front and rear spars are braced together by drag bracing, which consists of a system of struts and wires contained in the plane of the wing. The struts may take either of two forms, one the compression rib (CR), which is a stiff member shaped to the section of the wing, and the other a drag strut, which is simply a wooden or metal strut not conforming to the wing section.

These drag struts or compression ribs are braced by wires or cables called the internal drag wires (DW). There are usually two drag bays subdividing each complete interplane bay. In addition to the internal drag wires, external drag wires are sometimes taken from the rear truss

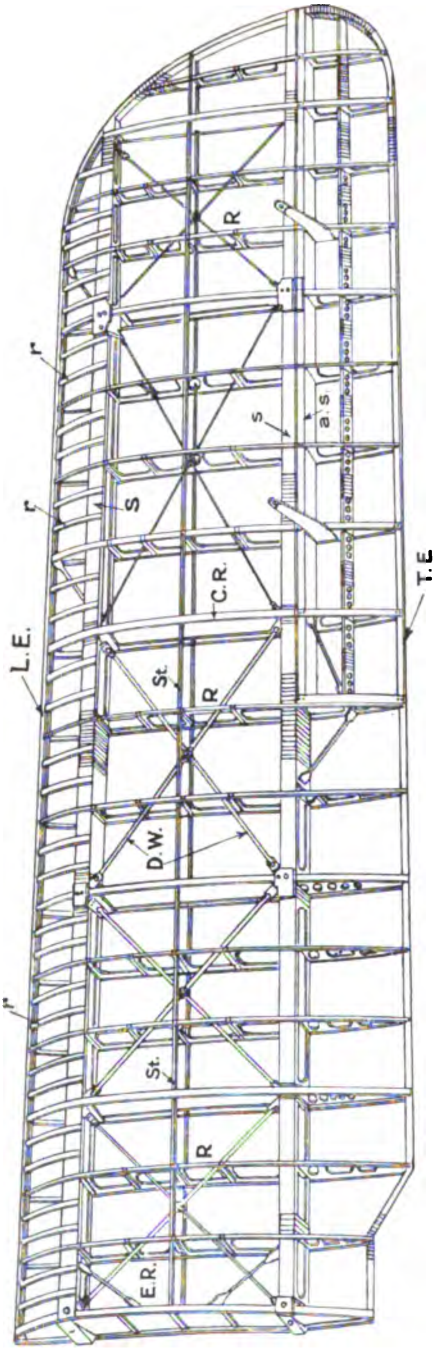


Fig. 12.

inner interplane strut attachment points to the forward part of the fuselage.

The shape of the wing is kept by means of ribs or formers (R), which consist of light wooden structures made to the contour of the wing section and rigidly attached at intervals of about 12 inches to the main spars. These ribs transmit the load from the wing covering to the main spars. To the front edge of these ribs a metal or wooden member called the leading edge (LE) is attached. This runs parallel to the spars throughout the wing and forms the nose piece of the structure. This member must be very stiff as it has to carry heavy pressures. To the aft portion of the ribs the trailing edge (TE) is attached, forming the tail of the wing. This member need not be of special strength, and balloon cord or wire stretched tightly is very commonly used for it instead of wood or tube.

The leading edge and the nose of the wing (*i.e.* the part between the leading edge and the front spar) must be of rigid construction for two reasons: first, the heaviest concentration of load occurs there, and secondly, it is the most important part of the wing from the standpoint of aerodynamic efficiency.

On this account while formers or ribs at intervals of 12 inches or thereabouts are sufficient for the structure of the wing behind the front spar, it is necessary to have a larger number of members between the leading edge and the front spar. This point is met by attaching small members known as riblets or nose ribs (*r*) to the front spar, these being spaced between the ribs proper. The riblets may consist simply of the nosepiece of a rib or of a small curved strip of spruce or metal attached to the front spars and to the leading edge (Fig. 13).

The ribs are of very light construction, and often require support to prevent lateral failure under the loads upon them in flight. This support is provided by very small section laths running along the tops and bottoms of the ribs and attached to them. These laths are called stringers (St), and are attached to the rigid drag struts or compression ribs, thus stabilizing the whole series of ribs attached to them.

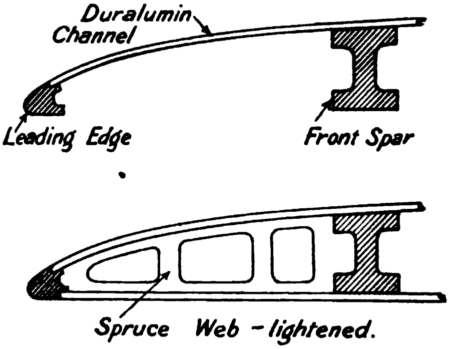


FIG. 13.

Another method of stabilizing is by means of tapes arranged like the herring-bone strutting of the ordinary joist floor, *i.e.* two tapes are attached to the tops and bottoms of alternate ribs, as shown in Fig. 14.

The fabric covering of the wing is tightened up by the application of dope, and in contracting it exerts a pull upon all the boundaries of the planes. The leading and trailing edges are sufficiently strong to withstand this pull, and all other edges are provided with special members to take the load. These members usually take the form of specially stiff ribs called end ribs. One of these is shown in Fig. 12 at the end of the plane which attaches to the centre section (ER). Sometimes the ordinary rib is used, and in this case it must be strutted as shown in Fig. 12, where the plane is cut back for the aileron.

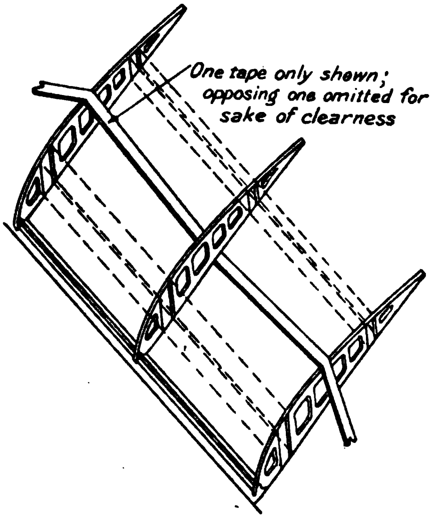

One tape only shown; opposing one omitted for sake of clearness

FIG. 14.

The aileron is constructed in a very similar manner to the remainder of the plane. Running parallel to the rear spar of the main plane and attached to it by the aileron hinges is the aileron spar (*as*). The ribs at this section of the plane are replaced by short ribs in the fixed part of the wing. These consist of the leading edge portion and the portion between the spars. The trailing edge part is separate, and is attached to the aileron spar. An end rib is required at the inner boundary of the aileron to take the fabric pull, or the ordinary rib must be specially strutted.

The aileron is controlled by means of a lever attached to the aileron spar (Fig. 15). From the ends of this lever, wires are taken to the control column in the pilot's cockpit, and through these the lever is manipulated. This lever is called the aileron control lever. The control forces may be transmitted to the aileron structure in various ways, which will be dealt with when considering the design of control surfaces.

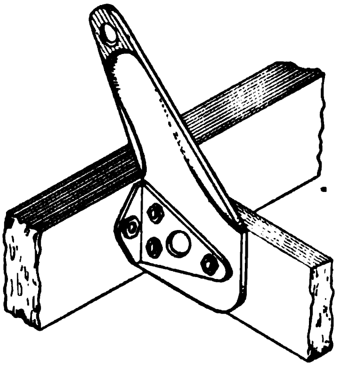


FIG. 15.

The whole wing is covered with fabric of the best quality linen. This is tightened and rendered waterproof by the application of dope.

21. The Fuselage.—Fig. 16 is a diagram of a fuselage of the usual design. It consists of a built-up girder of the N type. The longitudinal members forming the booms are called longerons or fuselage rails, and are generally four in number.

These, usually made of spruce or ash, are curved to give a streamline shape to the fuselage, and are braced by two systems of panels, one vertical and the other horizontal. These panels are formed by struts braced up by cables, wires or tie rods. At each bulkhead formed by two vertical and two horizontal struts cross bracing is provided, by means of bulkhead wires. At each joint there is a fitting to take the strut ends and the wires. These fittings are known as fuselage clips.

In the diagram the pilot's cockpit is shown in the centre section of the wings; here the horizontal bracing wires are replaced by stiff ash hoops which are rigidly screwed and lashed to the longerons. The seat in the cockpit is supported on bearers which are attached to the two vertical struts as shown.

The body struts are shown attached to the fuselage fore and aft of the pilot's cockpit, and the bottom plane main spars are attached to the fuselage immediately below.

Considerable tension has to be transmitted across the fuselage from these spars and the flying wires, and a tension rod should for this reason be carried through the body and rigidly attached to the spars.

The longerons usually taper to a knife edge at the rear portion, and this is almost always in a vertical plane. The "knife edge" is formed by a specially strong member known as the stern post, and to it the rudder is hinged. The tail plane as well as the fin and rudder is attached to the rear portion of the fuselage.

The forward portion of the fuselage usually carries the engine, the type of engine of course determining the form of mounting used.

In the rotary engine the mounting consists of a steel plate through which the crankshaft passes and to which the engine is bolted (Fig. 17).

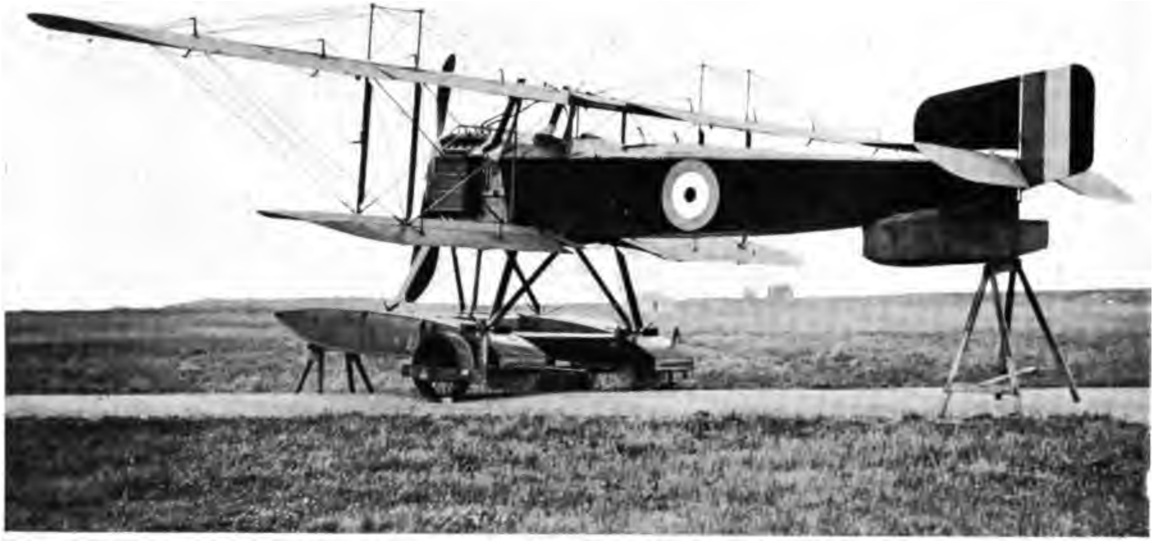


Plate VII.—Fairey Float Seaplane.

The crankshaft then passes through a second plate in order to give a support against the moment induced by the overhung weight of the engine.

The plate is usually lightened by holes drilled in it, and any portions which may be thrown into compression are flanged in order to increase the stiffness.

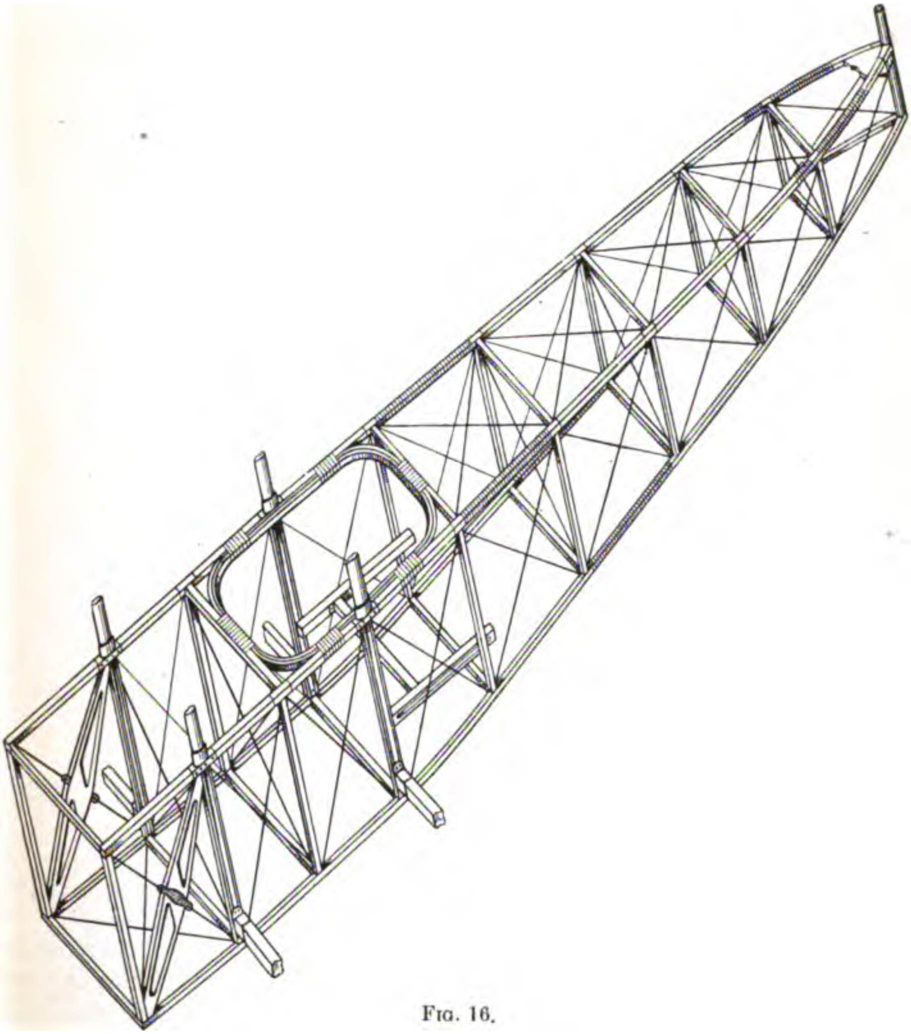


FIG. 16.

This form of mounting can only be used for rotary and radial engines. In the case of a stationary engine, which is always very much longer than those mentioned, the mounting must take a different form.

The method usually adopted is shown in Fig. 18. The engine mountings (EM) are frames of steel or multi-ply wood carried as bulkheads across the fore part of the fuselage or the engine nacelle. Beams or engine bearers

(EB) are carried between these, the feet of the engine being bolted to the bearers.

Mountings of a cantilever type are sometimes constructed in steel tube. This construction makes a very good and neat job.

In order to cut down the resistance of the engine in flight and improve the performance of the aeroplane it is always streamlined off by a light structure known as the cowling (Fig. 19). This is not a main structural member and carries no loads, its function being merely to reduce resistance.

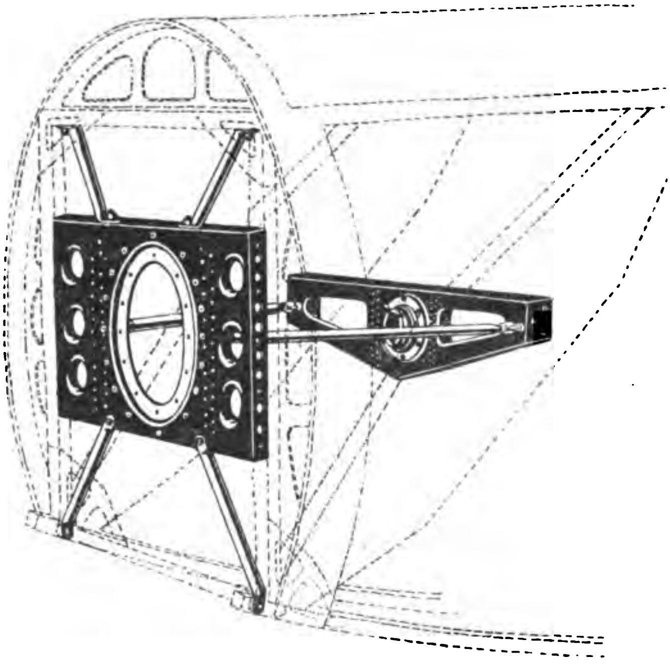


Fig. 17.—Engine Mounting on Avro 504 K.

In an air-cooled engine it is important to ensure that a proper flow of air is allowed through the cowling, or else the engine will overheat.

The remainder of the fuselage also requires to be streamlined to reduce head resistance in flight, and this is done by means of fairing. This consists of a light structure attached to the main members, and its sole function is to give the correct streamline shape to the unit.

One common type of fairing is shown in Fig. 20. *ff* are light formers in spruce or plywood attached to the horizontal fuselage struts by means of screws and glue. These formers are cut to the required shape and are notched at intervals round the circumference to take stringers (*ss*) which in turn carry either fabric or a very light veneer. In the neighbourhood of the cockpits the fairing is neatly finished off and a padded roll

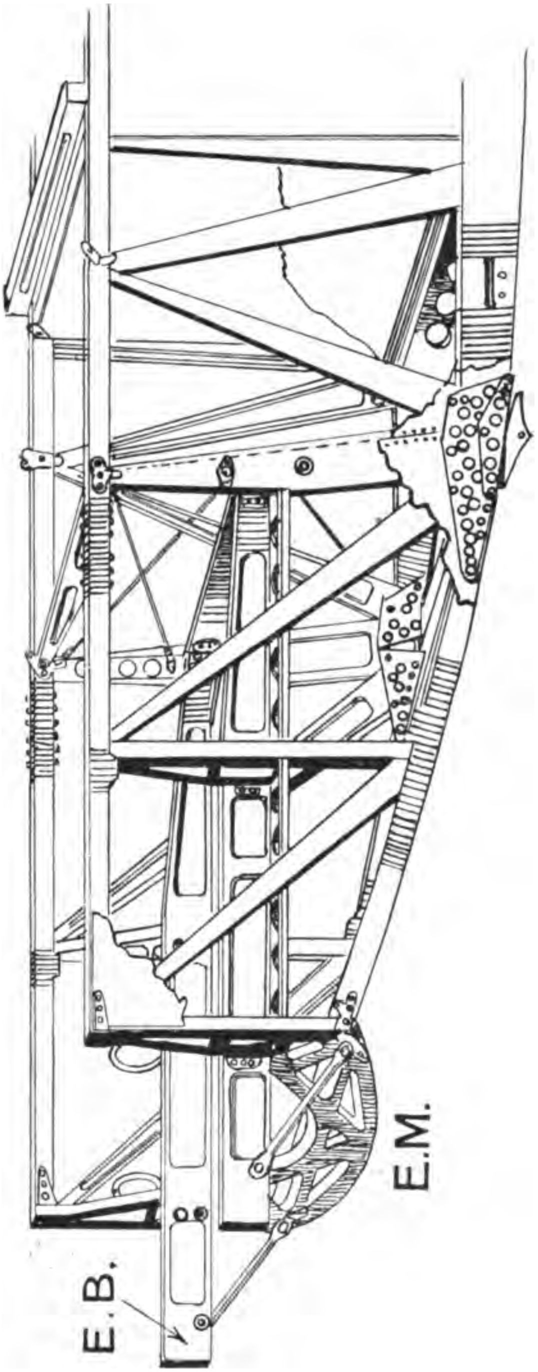


FIG. 18.—Airco Engine Mounting.

attached to it all the way round (Fig. 21). Even if the main fairing is fabric it should be replaced by veneer in this region to give the extra stiffness required.

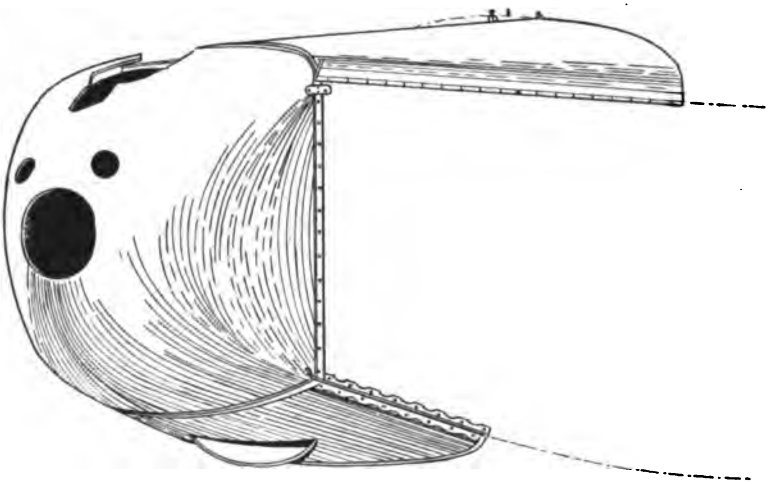


FIG. 19.

The whole fuselage is covered with fabric, which need not necessarily be of the fine quality demanded for the main planes and control surfaces, although, of course, it must be up to a good standard.

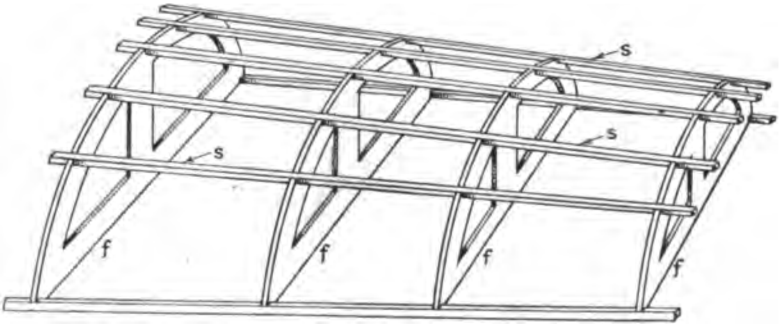


FIG. 20.

The fuselage described so far is a common type, but there are one or two other methods employed in building this part of an aeroplane which should be noticed.

In the first of these the bracing wires are dispensed with, and instead of covering the structure with fabric, three-ply of a good quality is used, as shown in Fig. 22. The three-ply is simply glued and screwed to the longerons and struts, and being able to withstand tension takes the place of both the fabric and bracing wires. At heavily loaded sections the three-

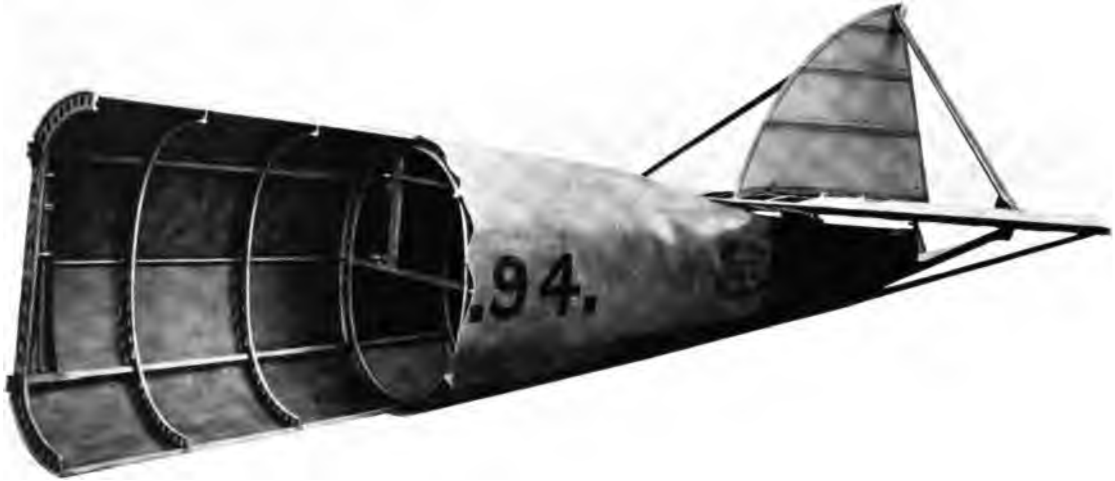


Plate VIII.—Monocoque Fuselage for "Panther" Aeroplane.
(Designed by Messrs. Parnall and Sons, Bristol.)

ply is stiffened by bracing struts. This type of construction has proved itself very satisfactory.

Another method which has also proved very successful is that known as the monocoque type of construction. The principle adopted is as follows: formers are cut to the shape of the fuselage and these are spaced at intervals of from two to three feet along the length of the structure. A thin veneer or plywood shell or skin is attached by screws and glue to these formers (Fig. 23).

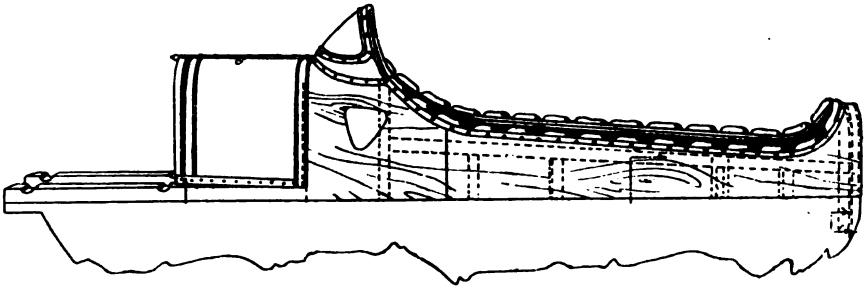


FIG. 21.

There are several varieties of this type of construction. In some there are longitudinal stringers between the formers, and the skin is laid on longitudinally, and may be either in small width planks or in broad strips even up to a complete quadrant of the circumference.

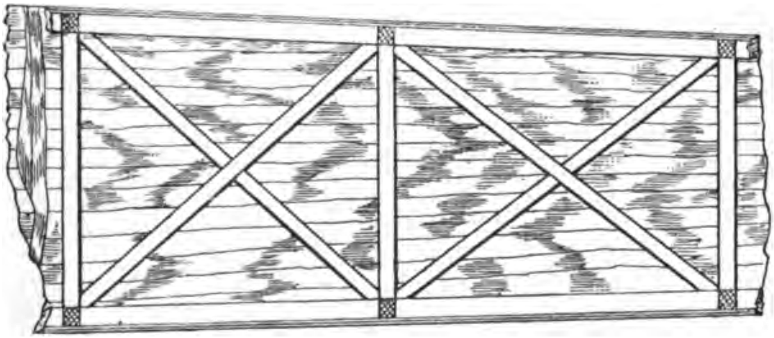


FIG. 22.

This construction is light, cheap and strong. It gives a very roomy fuselage and does away with all bracing wires, fittings, and fairings (Plate VIII.). It was originated in France, but has since been used in all countries. One advantage of it is that it cannot get out of truth and hence no trueing up is rendered necessary. A disadvantage lies in the fact that a small local damage may mean the loss of the whole structure, as repairs cannot easily be made. Also, of course, there is some danger of warping, but this is not a serious contingency.

22. Undercarriage.—The design of the undercarriage unit is of considerable importance in a successful aeroplane. While there are many types in use it will be sufficient to analyse one in detail as all of them have the same components, the differences in design being chiefly those of arrangement.

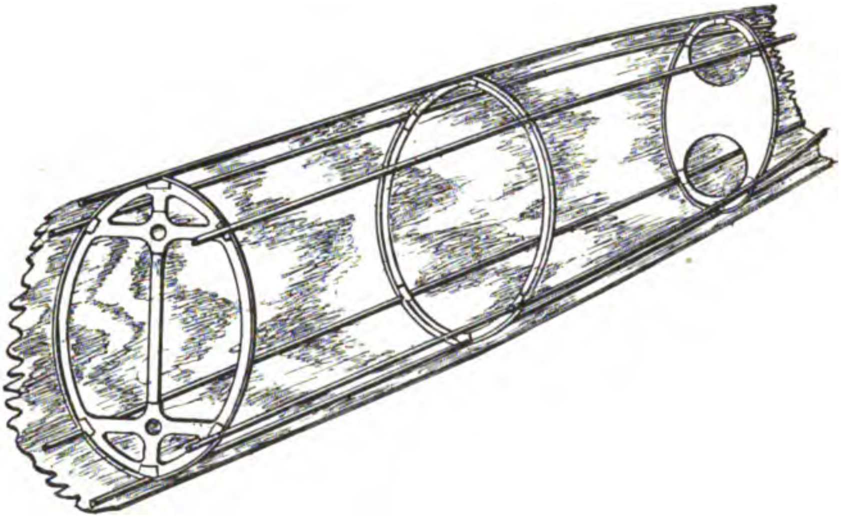


Fig. 23.

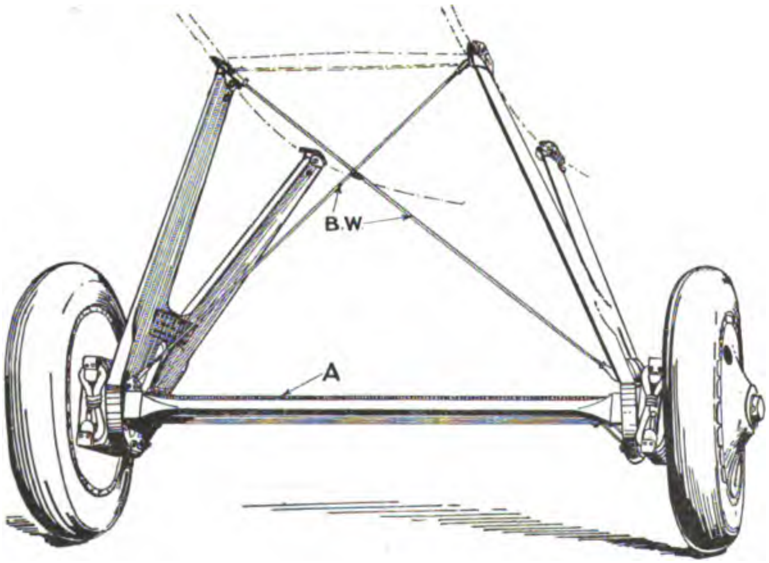


Fig. 24.

Fig. 24 shows a Vee chassis, a common and very effective type in smaller aeroplanes. It consists primarily of the main chassis structure of two V frames built up either of wooden streamline struts or of steel tubes properly faired. These V frames, whose function is to form a support for the axle A, which carries the whole weight, are attached to the fuselage by means of strong fittings. In the front view the V frames are braced together by cables or wires (BW).

The axle is not attached rigidly to the V frames, but through a shock absorber, so that on landing the aeroplane shall not receive too severe an instantaneous load.

In the simplest form this shock absorber consists of corded rubber cable which is used for lashing the axle to the frames. This leaves the wheels and axle free to move vertically under loads, but in so doing the rubber is stretched, and the landing shock dissipated (Fig. 25).

Another method is to use a helical spring contained in the steel tubular struts forming the chassis frames, the compression of these on landing giving the necessary absorption.

Still another method, as used on Avro aeroplanes, is to make the main struts in two portions, connecting them through the medium of a shock absorber, made up either of coiled rubber cord or a spring (Fig. 26).

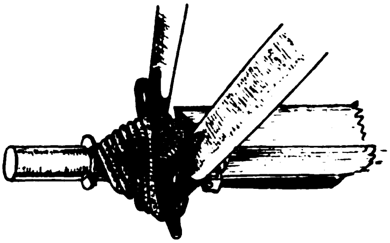


FIG. 25.

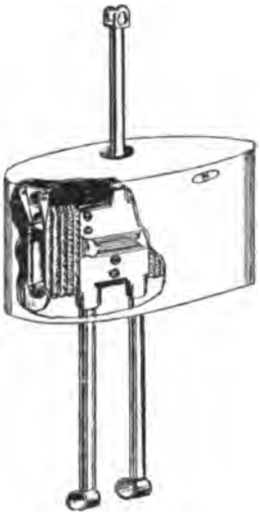


FIG. 26.

The oleo landing gear is an alternative type which is based on the oil dashpot principle. The shock of landing is transmitted to an oil chamber, and oil is forced through a small aperture, the energy being thus dissipated.

Many other devices have been tried, but these are the main forms in use. In all types the points to be borne in mind are lightness, reduction of head resistance, and efficiency in doing the job for which they are intended.

It is of great importance that a certain amount of side play be permitted on the chassis, to allow for taxying over rough ground.

The aeroplane wheel is similar to that used on a motor car, but is of lighter construction, being of the tension spoke type and carrying a rubber tyre. The whole wheel is faired off by a detachable canvas fairing. The sizes of wheels are specified by their diameter and the diameter of the tyre. The table at the end of Chapter XIV. gives a list of standard aeroplane tyres and wheels and their weights. Plate IX. shows a number of standard types of aeroplane and seaplane chassis.

In some cases the wheels are dispensed with and skids are provided for running and landing. These are specially useful for aeroplanes designed for work from the decks of ships.

23. Tail Plane and Elevators.—Fig. 27 shows a typical arrangement of a tail plane and elevators. It is very similar in construction to the main planes, and consists of leading edge, spars, ribs, and bracing wires and struts.

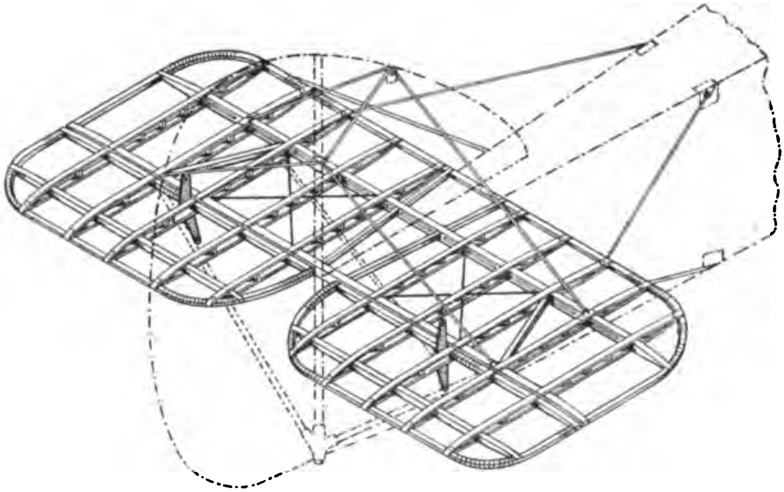


FIG. 27.

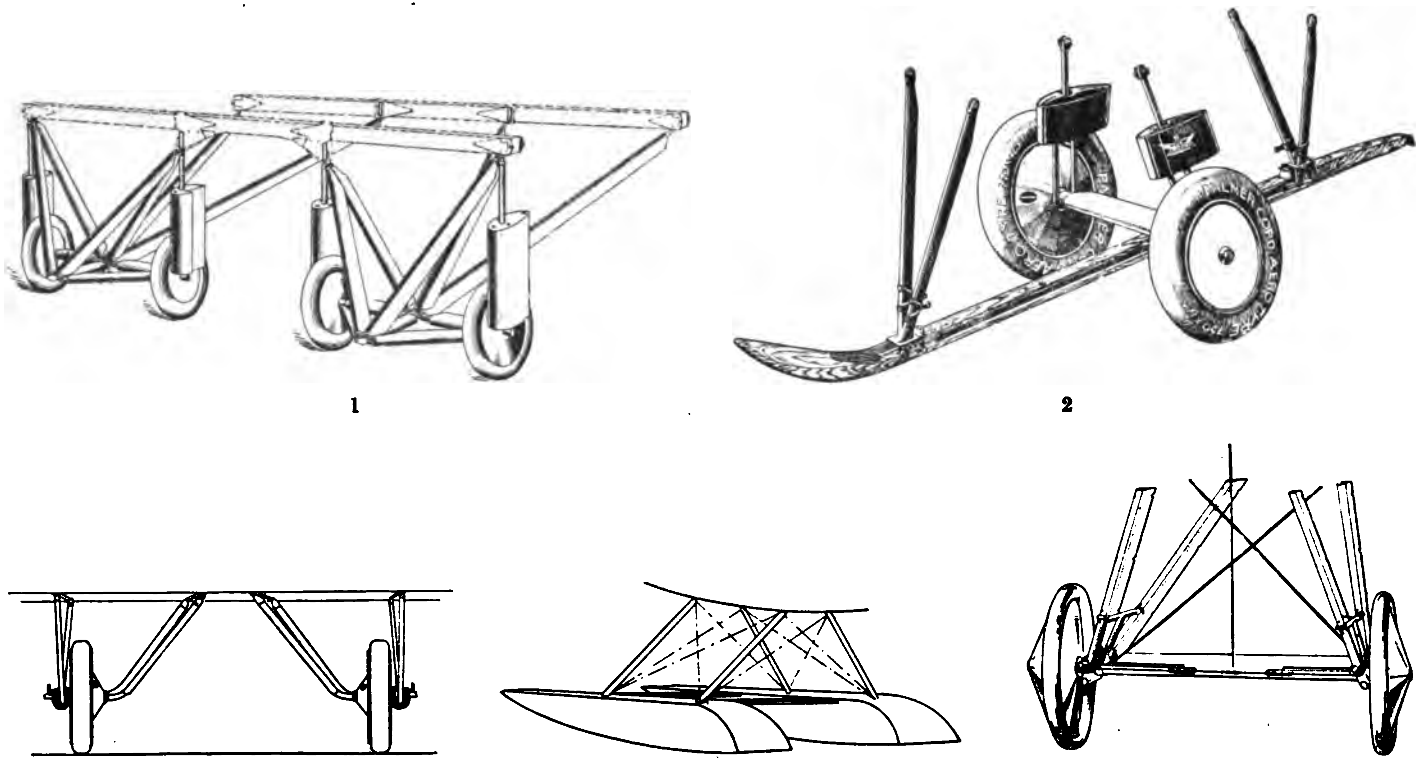
The actual bracing may consist either of wires or tubes from the fuselage and rudder post, which must be made of sufficient strength to carry the loads arising from the tail. The bracing must be such as can withstand loads either upwards or downwards upon the tail plane.

The elevator is similar in construction to the wing flaps, and consists of an elevator spar which carries the ribs, and is hinged to the rear spar of the tail plane. To this spar the elevator king posts which carry the control wires from the pilot's cockpit are attached. These king posts are usually metal plate or a wood and steel composite structure.

The elevators may be made to operate either through the torsion of the elevator spar caused by pulling on the top of the king post by the control cable, or by transmitting the pull to other parts of the structure and so eliminating most of the torsional loads.

In some designs wires are led from the top of the king post to a point either on the trailing edge or to a spar running through the elevator. These wires are called back wires. An internal arrangement of struts is needed to take the components of this back wire load to the elevator spars. This method relieves the control king post of all bending, and gives it a pure compressive load.

Various forms of the internal structure of tail planes and elevators are given in Plate X.



1

2

3

4

5

PLATE IX.—Typical Chassis.

1. Handley Page. 2. Avro. 3. Sopwith. 4. Typical float chassis. 5. Sopwith.

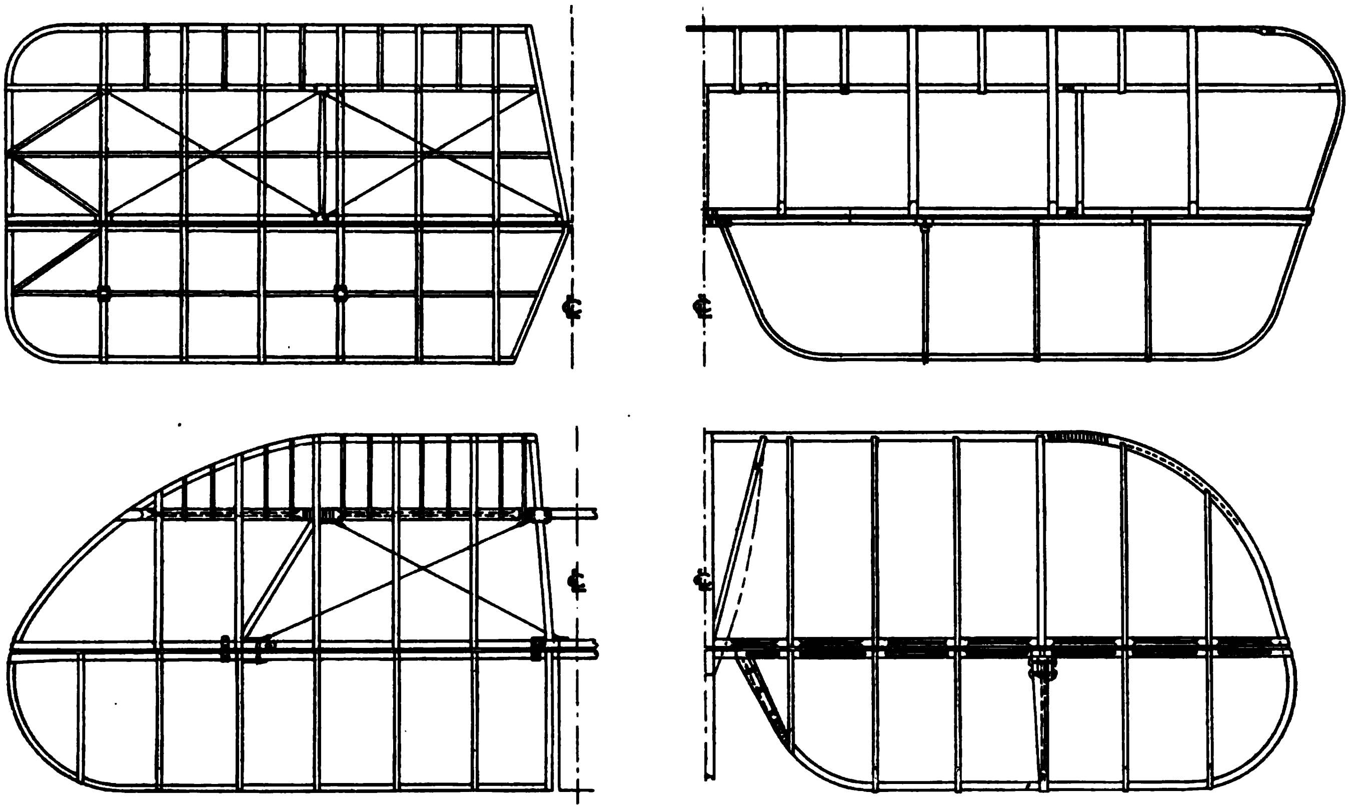


PLATE X.—Internal Construction of Tail Planes and Elevators.

24. Fin and Rudder.—A common form of construction for the fin and rudder is shown in Fig. 28. The rudder is of the type known as unbalanced, and is constructed as follows. The front edge is formed by the rudder post (RP) which is hinged to the fin post or stern post (FP). The rudder post carries the control lever or king post (KP), which is manipulated in an exactly similar manner to the elevator king post previously described. The trailing edge (TE) is usually formed of steel, aluminium or duralumin streamline tube of a small section bent to the shape of the rudder. A vertical rib (*vr*) is often placed as a stiffening member, and horizontal ribs or formers (*r*) connect the rudder post, vertical rib and trailing edge. The whole is covered with linen fabric of the best quality. The fin is constructed in a similar manner, the trailing edge being the fin post or stern post (FP). Horizontal and vertical ribs complete the structure except for the fabric.

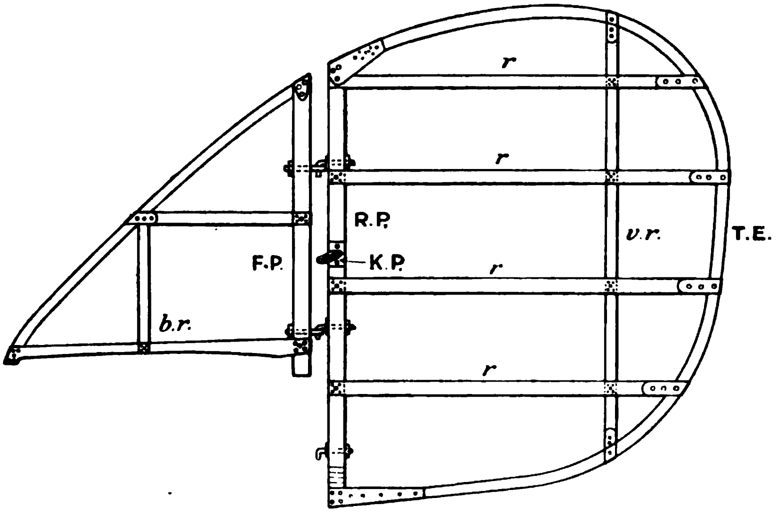


Fig. 28.

The lowest rib (*br*) is a specially stout one, and is attached to the fuselage by suitable fittings.

In the balanced type of rudder a portion of the movable surface is arranged forward of the rudder post in order to bring the centre of pressure of the whole nearer to the axis of rotation. This relieves the load which has to be exerted by the pilot in controlling the aeroplane.

Plate XI. shows the construction of various types of fins and rudders, including both the balanced and unbalanced patterns.

The rudder and fin may be of cantilever construction from the fuselage, but as a general rule they are braced to the tail plane spars by wires.

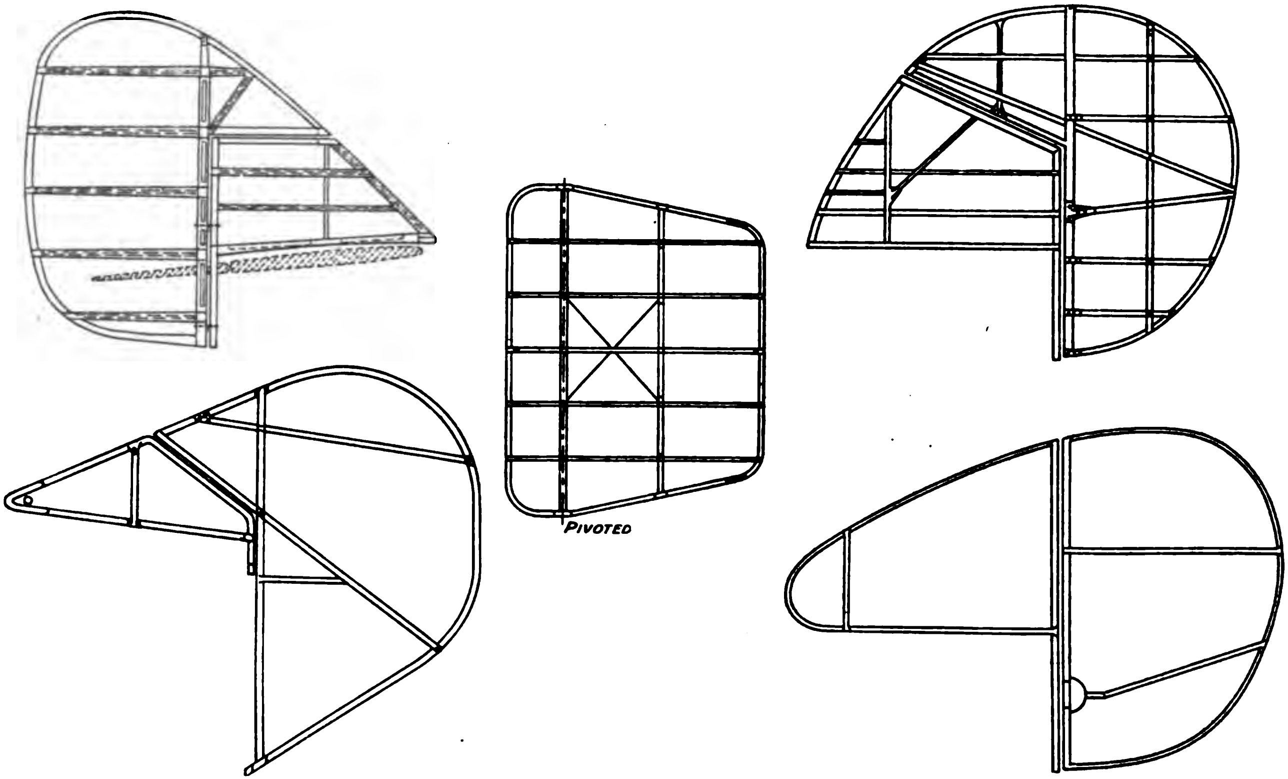


PLATE XI.—Internal Construction of Fins and Rudders.

CHAPTER V

THE EVOLUTIONS OF AN AEROPLANE IN FLIGHT

25. Controlling Units.—Before the evolutions which an aeroplane can be made to perform can be understood, it is necessary to grasp clearly the various agents entering into the control of the aeroplane, and the functions fulfilled by them.

There are four units employed in the control of an aeroplane—

(1) The engine.
(2) The elevators.
(3) The rudder.
(4) The wing flaps or ailerons.

26. The Engine as a Controlling Unit.—The engine is the primary factor controlling the height. This statement should be clear from a consideration of the conditions of flight discussed in Chapter II. It was shown there that for any given horizontal flight speed there is a definite angle of incidence of the main planes, and a definite horse-power required from the engine to overcome the head resistance of the aeroplane.

Supposing an aeroplane to be flying level with the engine throttled down so that it is giving the exact horse-power required for this condition. If the aeroplane is stable in all respects it will continue to fly at the original speed and attitude without any attention from its pilot. Suppose now that the pilot, without touching any more of the controls, opens up his engine, then the aeroplane is receiving a greater power than it requires to maintain its horizontal flight, and it immediately commences to climb, maintaining its original attitude along its new flight path. Conversely, if instead of being opened out the engine is throttled down still more than originally, the aeroplane will lose height and will glide downwards, still keeping the same attitude relative to the new flight path (para. 14). From this it is evident that the principal controlling unit for height is the engine.

Suppose now that our aeroplane is again flying steadily in a horizontal direction, and it is desired to alter the speed of its flight. In order to do this it is necessary to set the main planes at the appropriate angle of incidence for the desired speed. A pitching couple is applied to the aeroplane by manipulating the elevators, and this swings it about its centre of gravity into the new attitude desired. The aeroplane then takes up the speed corresponding to the new angle of incidence. The elevators then are the organs controlling the speed of the aeroplane. There is one point here, however, which must not be lost sight of, and that is as follows. When the speed of the aeroplane is altered the horse-power required to overcome the head resistance is changed, and so if it is still desired to fly on a horizontal path the engine must be controlled so that the horse-power developed is just correct to meet the altered conditions of flight. Unless this is done the aeroplane will either climb or fall as explained before.

27. Directional and Lateral Control.—So far we have only considered straight flight. It is however, essential to have good control for turning an aeroplane, and this is provided by the rudder.

It will be wise to examine how this surface fulfils its function in some detail, as it is bound up with other points in control and manœuvrability.

Fig. 29 (*a*) shows the plan view of an aeroplane flying in the direction of the arrow at a velocity V. In order to turn in a counter clock-wise direction the rudder is set as indicated.

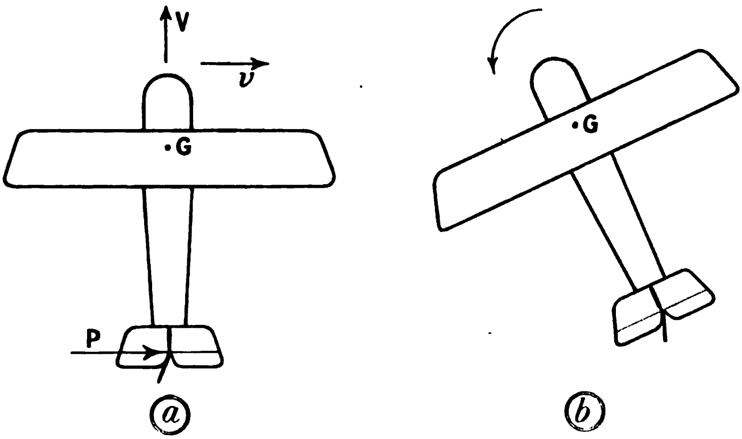


Fig. 29.

The rudder now forms a plane set at an angle to the direction of motion, and in consequence there is a lateral force P on the surface as shown in the figure.

The effect of this force is to cause the aeroplane to slip bodily sideways with a velocity v, and there is in consequence a lateral force acting through the centre of side pressure of the aeroplane. This together with P forms a couple, turning the aeroplane as shown at Fig. 29 (*b*).

When the aeroplane commences to turn a centrifugal force F acting through the centre of gravity is brought into play, and unless the aeroplane is banked, the forces acting through this point will not be in equilibrium. The craft will then continue to side slip. Fig. 30 (*a*) shows the aeroplane on an even keel. The forces acting on it are the lift L, the weight W, and the centrifugal force F, all through the centre of gravity.

It is obvious that these forces cannot balance, and side slip occurs as stated.

If the aeroplane is correctly banked, however, as shown at Fig. 30(*b*), the weight and lift no longer balance each other, and the three forces L, W, and F form a system in equilibrium so that side slip is eliminated.

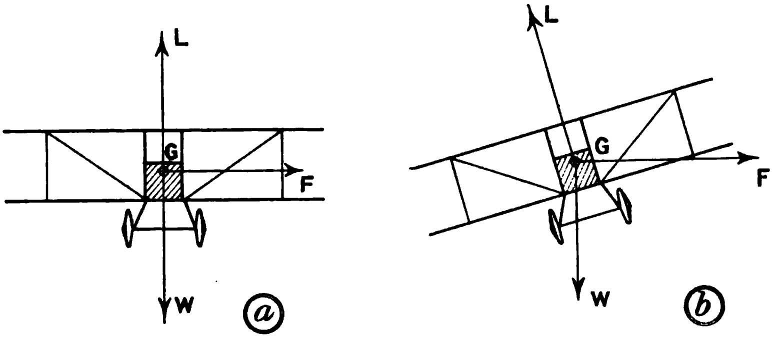


Fig. 30.

The function of the rudder is therefore to turn the aeroplane, and of the wing flaps to give the necessary bank to ensure the elimination of side slip, *i.e.* the rudder gives directional and the wing flaps lateral control.

28. Control System.—It has been tacitly assumed so far that the pilot can control the engine, elevators, rudder and wing flaps, but it is necessary to enter somewhat into the mechanism of his control.

The engine controls are not important for the purposes of this book. The control of the surfaces is, however, of great importance in understanding the manœuvres through which an aeroplane is put, and also for appreciating the loads to be dealt with in design.

The control levers in the pilot's cockpit consist of a control column or " joy stick " and a rudder bar. The control column is usually vertical when the controls are central, and by it the wing flaps and elevators are governed, the former by lateral movements and the latter by fore and aft. A forward motion of the stick tends to make the aeroplane dive, that is to say, the elevators are depressed, while a backward motion raises the elevators and tends to stall the aeroplane.

A lateral movement of the stick to the pilot's right makes the aeroplane roll to the right, *i.e.* it is correctly banked for a right-hand turn, and conversely for movement to the left. A right lateral movement causes the wing flap on the port or left-hand side to be depressed, and the wing flap on the starboard or right-hand side to be elevated, so increasing the port and decreasing the starboard lift forces, and providing the required banking couple.

The rudder bar is a horizontal lever pivoted in the centre and controls the rudder only. It is manipulated by the feet, and by pushing the right foot forward the aeroplane is turned to the right, and conversely. The controls are therefore quite simple in themselves, the aeroplane following the motion of the levers, *i.e.* right rudder causes right turn, right stick causes right bank, and forward stick causes nose downward and vice versa.

The connections between the control column and rudder and control surfaces are made by means of cables running over pulleys or through fairleads, and Plate XII. shows a diagrammatic view of a control system for a modern aeroplane.

In place of a lateral movement of the joystick for controlling the ailerons it is common practice to provide a wheel similar to a motor car steering wheel, attached to the head of the control column. The fore and aft motion still controls the elevators, but the ailerons are controlled by rotation of the wheel. In some aeroplanes, too, the rudder bar is replaced by pedals and directional control is obtained by pressing the appropriate pedal.

29. Evolutions in Flight.—Having now briefly described the controls and their functions, we may proceed to a description of some of the evolutions which an aeroplane may perform in the ordinary course of flying.

The first operation of the pilot after starting up his engine is to get his aeroplane into the air. In order to do this he runs the aeroplane across the aerodrome under its own power against the wind until the speed is sufficient to sustain him in the air. The start from rest is made with the tail skid of the aeroplane resting on the ground, but as speed is gained, the tail rises and the aeroplane is taxying with the top rail of the fuselage practically horizontal, the elevators being depressed during this stage. When flying speed is reached the pilot pulls his stick in towards him, the tail of the aeroplane drops, giving a large angle of incidence to the main planes, and the lift is then sufficient to raise the aeroplane into the air. Once in the air the aeroplane is levelled, and the flight can proceed.

Climbing.—To climb, the attitude of the aeroplane is controlled so that its forward speed does not utilize the whole of the available horse-power, the excess being used for the climb.

Gliding.—In gliding, the engine is either cut out or throttled down so that the horse-power developed is not sufficient to maintain horizontal flight. The aeroplane then commences to descend along a sloped path, the steepness varying with the horse-power being developed, and the design of the aeroplane.

Banking.—This consists in throwing the aeroplane on to one side, and is done on a turn. Rudder appropriate to the direction of the turn is put on, and the control column put in the same direction. The wing flaps are then brought into play, and introduce a couple on the aeroplane which throws it on to the bank. The bank may be any angle from a degree or two to nearly 90°. Any angle over 45° is commonly known as a vertical bank (Fig. 31). An important point to be remembered in connection with this latter manœuvre is that the controls

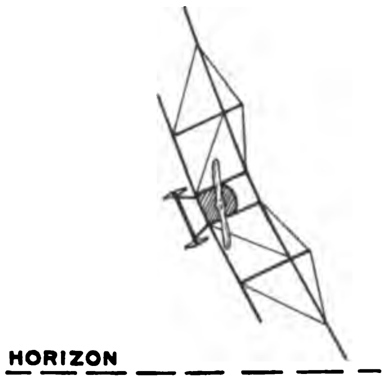


Fig. 31.

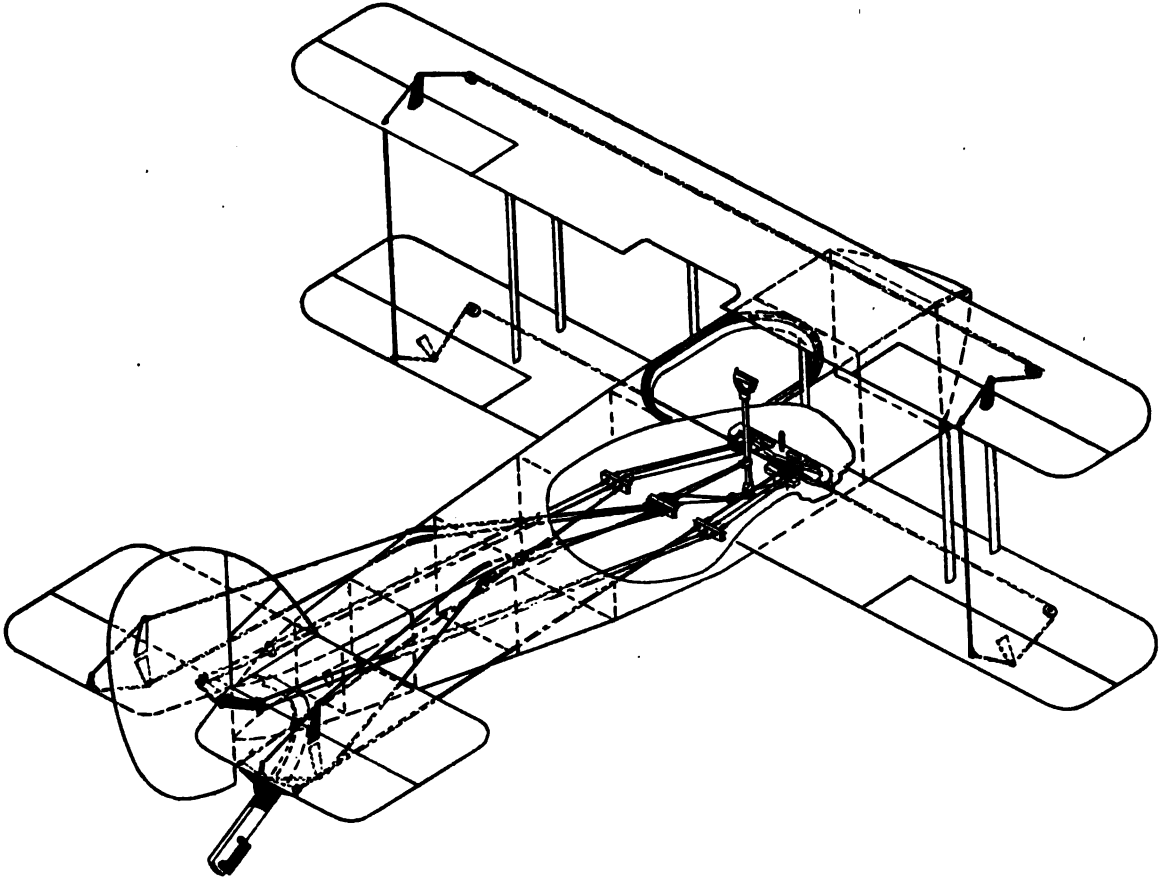


PLATE XII.—Control System of an Aeroplane.

(elevator and rudder) become intermixed, that is to say, the rudder is used partly to keep the nose of the aeroplane up or down, and the elevators to control the direction of flight in a horizontal plane. In the extreme case of an absolutely vertical bank the functions of the two surfaces have become quite interchanged.

Turning.—This is closely bound up with banking, and the operation has already been described. There is one point, however, on the turn which must be noted. The propeller is revolving at such a speed that there is a very considerable gyroscopic effect arising from it, which effect is greatly increased in the case of a rotary engine by the revolving parts of the engine itself. The effect of this gyroscopic action is to elevate the nose of the aeroplane during a turn in one direction and to depress it on the opposite turn. This is a very well marked phenomenon, and one which must be counteracted by appropriate elevator control. (See para. 36.)

Spiral.—In the spiral the aeroplane is steeply banked in the ordinary manner and is kept on the bank. The aeroplane then descends on a spiral path, keeping a perfectly steady and stable motion. This manœuvre may be done with the engine on or off, and is very useful in landing on an aerodrome from a height immediately above.

Spin.—In a spin the aeroplane descends nose down, and rotating more or less rapidly about a vertical axis (Fig. 32). The first essential in executing a spin is that the aeroplane shall be at an angle of incidence greater than the stalling angle. Many types will automatically spin when this condition is attained, but in others it is necessary to start a banked turn at the same moment. Since the wings are at such a large angle during this evolution the drag on them is very large, and in consequence the speed of the aeroplane through the air is comparatively small. The spin may be either a stable or an unstable motion, in the former case the speed through the air and the rotation being uniform. In the case of an unstable spin, however, the aeroplane gathers speed, and though a safe manœuvre under certain conditions, it may be a source of grave danger.

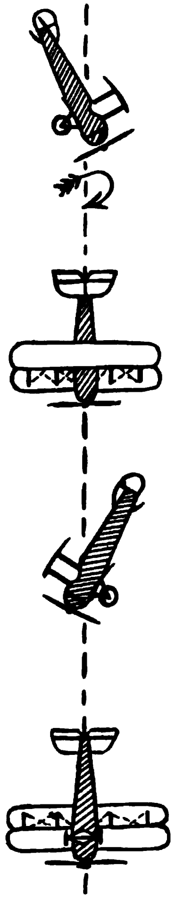


Fig. 32.

Zooming.—If an aeroplane is flying at a high speed and the elevators are suddenly pulled up there is a large increase in the lifting force on the wings, which causes the craft to climb very rapidly (Fig. 33).

This manœuvre, called zooming, is dangerous from the structure standpoint, and many types of aeroplane would be broken if it were attempted on them. (See para. 37.)

Nose Diving.—This is the limiting case of a glide. The nose of the aeroplane is forced down by pushing the control column forward, and it finally comes down in a vertical path at a very high speed (Fig. 34). High speed is always a possible cause of danger to the structure, and care must be taken in pulling out of a nose dive on this account. (See para. 40.)

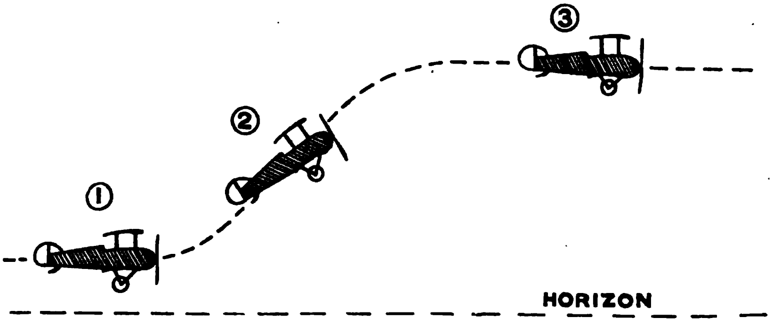


Fig. 33.

Stalling.—An examination of the curves of lift coefficients shown in Chapter II. will show that at a certain angle of incidence the value reaches

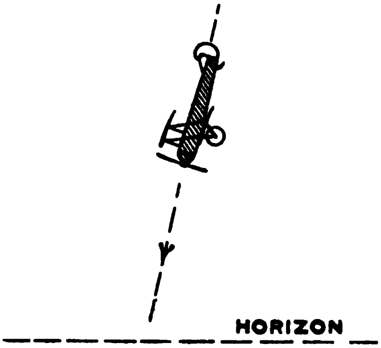


Fig. 34.

a maximum, after which it falls off again. If an aeroplane is flying horizontally at the angle corresponding to this maximum lift coefficient, any increase in this angle will be accompanied by a breakdown of stable airflow. This speed is known as the minimum flying speed, and if the control column is moved back the slightest amount in an attempt to diminish the speed, control of the aeroplane is partly lost, and it is said to have stalled.

Tail Slide.—If the aeroplane is stalled it may descend some distance tail first. This manœuvre is known as a tail slide, and the aeroplane comes out of it into a nose dive until flying speed is regained.

Pancaking.—If, instead of tail sliding, an aeroplane on being stalled drops bodily, it is said to have " pancaked " (Fig. 35).

Rolling.—This manœuvre is executed from normal horizontal flight. The aeroplane when flying level is banked and rudder is applied in the same sense. It is thus made to revolve completely about its longitudinal axis.

Looping.—The aeroplane is dived at a fairly high speed, and the control column then brought back. The effect is a vertical turn known as a loop, and the aeroplane emerges in the same direction as it entered (Fig. 36).

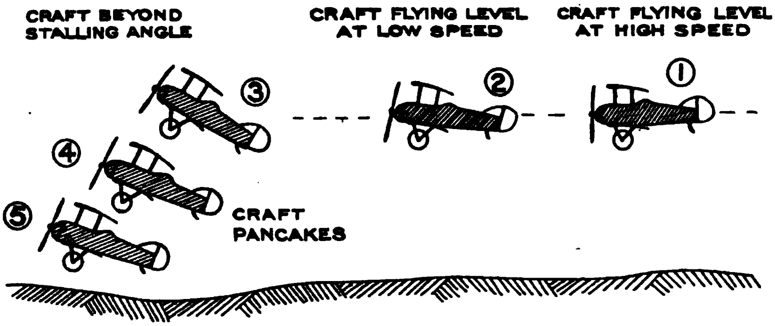

CRAFT BEYOND STALLING ANGLE
CRAFT FLYING LEVEL AT LOW SPEED
CRAFT FLYING LEVEL AT HIGH SPEED
①
②
③
④
⑤
CRAFT PANCAKES

FIG. 35.

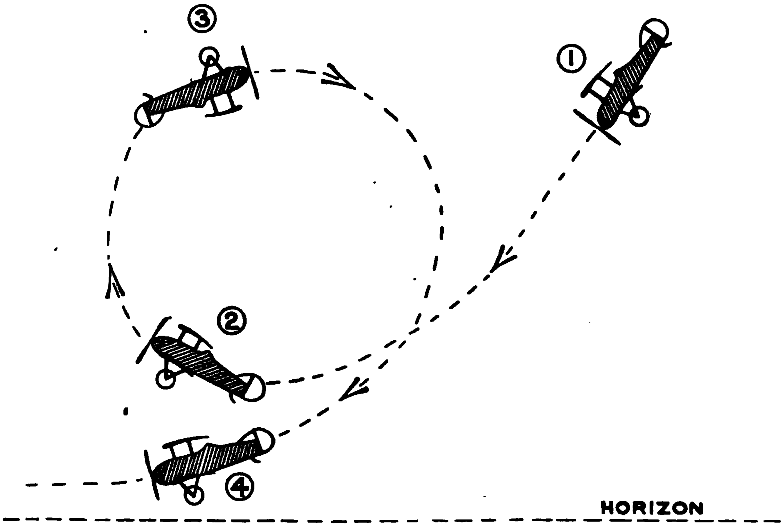


FIG. 36.

Landing.—In landing, the aeroplane is brought down on the glide to a short distance from the ground, and is then flattened out, *i.e.* the angle of incidence is made as large as possible without actually stalling the aeroplane. The chassis wheels touch and the aeroplane rolls along the ground, losing speed until it finally puts its tail down and comes to rest.

CHAPTER VI

LOADS ON AN AEROPLANE IN FLIGHT

30. General Considerations.—Some of the manœuvres which may be executed upon an aeroplane have been dealt with in the previous chapter, but no mention has been made so far of the loads which are thrown upon the structure during these evolutions.

This aspect of the case must now be considered, since the designer must meet the loads by the provision of a sufficiently strong and trustworthy structure to prevent failure during flight. The present chapter deals only with the magnitude of the loads without any attempt to distribute them among the various parts of the structure. The latter problem is dealt with later.

Two methods are available for obtaining the magnitude of the loads occurring during flight, and recourse must be taken to both. The first is by calculation and the second by experiment. Of the two the latter is probably the more reliable, but experiments are, of course, from their very nature, limited in extent. It is obviously impossible, for example, to obtain the worst loads upon an aeroplane when it is known that these loads will cause failure of the structure.

At some later date it will be possible to build an aeroplane of quick manœuvrability and at the same time make it unbreakable in flight, so that the maximum loads really obtainable will be capable of measurement directly. Until that time arrives, however, there are certain aspects of this problem which must be arrived at by calculation.

When an aeroplane is flying level at a constant speed it is obvious that the supporting force upon the wings and other lifting surfaces must be exactly equal to the weight of the aeroplane. This is unit load upon the wings.

Suppose now that the aeroplane, keeping its flying attitude, suddenly has extra power put into it, *e.g.* by opening the engine throttle. The first result is an increase in the horizontal velocity with an acceleration upwards, followed by a steady condition of climbing. This acceleration throws extra loads upon the wings and more than unit weight is carried. All manœuvres involve accelerations, and it is these accelerations, which throw the increased loads upon the structure, which have to be met by the designer.

31. Factor of Safety and Load Factor.—Suppose in any manœuvre the acceleration given to the aeroplane is Ng, where g is the acceleration due to gravity. Then the force acting upon the aeroplane producing this acceleration is $P = W/g \times Ng = WN$, where W is the total weight of the aeroplane. This force WN is the resultant of the weight and the air forces upon the various surfaces, and the lift lies between $N \pm 1$ times that carried under level-flight conditions.

If an aeroplane has under any conditions of flight to carry N times its normal steady flight load, the structure must be designed accordingly, and is said to have a load factor N.

The load factor is often spoken of as the factor of safety of the aeroplane, but this is an error which should be carefully avoided. Factor of safety is a term which has a perfectly definite meaning well understood in other branches of engineering, and it should be reserved for that purpose. When a structure such as a bridge is designed, the worst possible loads which can come upon it are estimated, and the bridge is then designed so that when this maximum load is carried the material is only stressed to about one-fifth of its ultimate. In other words, the bridge could carry five times the greatest possible load it ever has to bear before failure would occur. This figure of 5 is then the factor of safety of the structure.

With an aeroplane, however, the case is different. The least possible load to maintain flight in a straight line is estimated, and factors are based on this. A small scout aeroplane may well have a load factor of seven, but a factor of safety of under one; that is to say, it would be possible by clumsy flying to break the aeroplane in the air.

If the worst load occurring under any flight conditions could be estimated, it might be possible to estimate a true factor of safety for an aeroplane, but at the present moment this is not customary, at any rate for the wings.

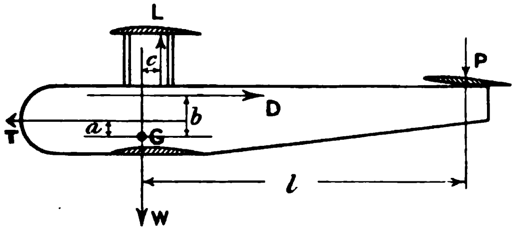


Fig. 37.

32. Analysis of Forces on an Aeroplane in Horizontal Flight.—Fig. 37 shows a line diagram of an aeroplane in steady horizontal flight. G is the centre of gravity of the aeroplane. Under this condition of flight the upward lift upon the supporting surfaces must be equal to the weight of the aeroplane, and further the sum of the moments about any point must be zero.

The forces acting are—

(1) The lift on the main planes L, acting vertically.

(2) The weight of the aeroplane W, acting vertically through the centre of gravity G.

(3) The thrust of the propeller T.

(4) The down load on the tail surface P.

(5) The drag of the aeroplane D.

The moment of the air forces due to drag and lift is given by $Db - Lc$ which is the moment of the resultant force about G. It should be noted that this has a definite value, though b and c may vary in any number of ways.

Then equating vertical forces—

$$L - P = W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Equating horizontal forces—

$$T = D \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Taking moments about G and calling nose-diving moments negative and stalling moments positive

$$Pl + Db - Lc - aT = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From these equations the values of P and L can be found.

It should be noticed particularly that the tail load P can be either upwards or downwards, depending primarily upon the position of the centre of pressure on the main planes. This varies for different angles of flight, being well forward for slow speeds and back for high speeds.

Thus an aeroplane carries a bigger down load on its tail plane for high speed flight than for low speed. A big down load on the tail, of course, signifies an increased up load on the main planes, so that in high speed flight the main planes are carrying more than the weight of the aeroplane, and at slow speeds are carrying less as a general rule.

In a modern aeroplane the moment term involving the thrust is relatively unimportant, since its line of action usually passes very close to the centre of gravity.

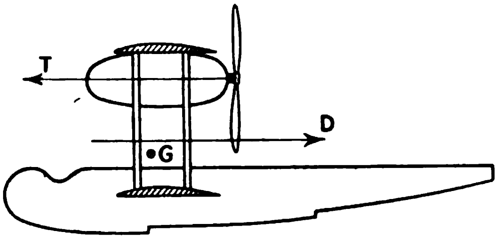


FIG. 38.

In some cases, however, this term is of great importance. In Fig. 38 is shown a boat seaplane with the engine carried between the wings. Since the main part of the weight is carried in the hull the centre of gravity of the seaplane is very low while the thrust line, on the contrary, is especially high.

Imagine such an aeroplane flying horizontally. The moment term due to the thrust is—

$$aT \text{ as before}$$

But whereas in the case of a normal aeroplane this term is usually negligible, it is in this case of considerable importance, and has the effect of increasing the control load very appreciably, so that even at slow speeds there is a down load on the tail plane.

33. Slip Stream.—The air stream from a propeller is called the slip stream and is travelling at a greater velocity than the entering stream, and this effect is felt for some distance behind the propeller. If, then, the control surfaces of an aeroplane are placed so that they are within the affected area, the loads upon them are greater than if they were outside such area. Suppose an aeroplane to be travelling at a speed V ft. per sec. and let the increased air speed in the slip stream of the propeller be v.

Then the air speed of surfaces in the slip stream is $(V + v)$. Forces and resistances are proportional to the squares of these speeds, so that the forces acting on surfaces in the slip stream are $\left(\frac{V + v}{V}\right)^2$ as great as if these surfaces were outside the slip stream.

This fact is made use of in some cases. Returning to a consideration of the boat seaplane shown in Fig. 38, suppose this seaplane is flying level and is in trim with the propeller on. If the engine be cut out, a nose diving couple aT is immediately lost, and the seaplane stalls until balance is automatically restored. Sometimes it would be impossible for this balance to be regained and the machine might stall completely.

If, now, the tail plane is placed in the slip stream of the propeller, a much better state of affairs can be obtained. When the engine is running, the loads on the tail plane are proportional to $(V + v)^2$ and are downwards, *i.e.* they produce a stalling couple on the seaplane. When the engine is cut out these loads are immediately reduced by the amount due to the velocity of the slip stream and are only proportional to V^2, *i.e.* the stalling moment is reduced in the ratio of $\left(\frac{V}{V + v}\right)^2$ at the same instant that the nose diving moment is reduced by the term Ta. By suitable design the moment terms can be balanced so that the seaplane continues without any change of attitude, *i.e.*—

$$P\left\{1 - \left(\frac{V}{V + v}\right)^2\right\}l = Ta$$

34. Torque of Engine.—The reaction of the propeller puts a torque upon the aeroplane which must be considered in connection with the loads upon it.

Let the speed of rotation of the propeller be n revs. per min., the horsepower developed be P, and the torque be Q ft.-lbs. Then the work done by torque is $2\pi nQ$ ft.-lbs. per min.

$$\text{H.P. exerted by torque} = \frac{2\pi nQ}{33,000}$$

$$\textit{i.e. } P = \frac{2\pi nQ}{33,000}$$

or

$$Q = \frac{33,000P}{2\pi n}$$

The torque has the effect of making the aeroplane fly with one wing down unless means are taken to prevent it happening. The tendency can, of course, be counteracted by a very slight aileron movement; but in most aeroplanes the angle of incidence of one wing is increased, and of the other decreased, just at the tips in order to give an unequal lift on the two tips. This creates a moment on the aeroplane to counteract the torque.

The increase of incidence is known as "wash in," and the decrease as "wash out." The same result may be obtained by erecting the aeroplane so that when the aileron control appears central to the pilot, there is in reality a slight effect just sufficient to balance the torque. This torque effect is not serious, except in very small span aeroplanes with fairly powerful engines.

It is a specially important point to be considered in the case of a twin or multiple engine aircraft where the engines are mounted in the wings, as the torque introduces loads upon the main trusses. If the engine is mounted on members a distance *d* apart, there is an up load on one and a down load on the other, each equal to Q/d lbs., and these loads must be considered in the structure.

If the engine is mounted on a plate, the bolt circle of which is of radius R, the total load on the bolts resisting the torque is Q/R. If there are N bolts, the load on each is Q/NR. The load is in turn transmitted to the fuselage structure through the attachments of the engine plate.

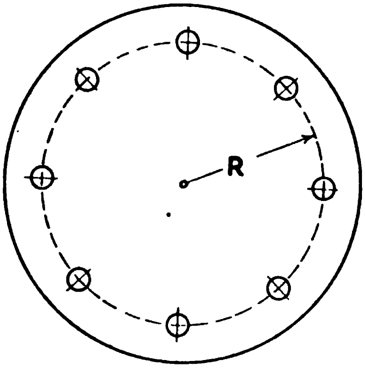


FIG. 39.

35. Propeller Thrust.—The thrust of the propeller must also be taken into account when designing the forepart of the fuselage and the main plane structure. It is usually transmitted direct to the longerons by means of bracing cables, and resolves itself into a tension in each of these members.

Let $V =$ the speed of the aeroplane in flight in ft.-sec.
$P =$ the horse-power of the engine.
$\eta =$ the efficiency of the propeller at this speed.
$T =$ the thrust of the propeller.

Then
$$\frac{TV}{550} = \eta P$$

or
$$T = \frac{550\eta P}{V}$$

giving the thrust at any speed of flight.

When the engine is at rest the thrust of the propeller is a maximum, and is about 7 lbs. per H.P.

36. Gyroscopic Effect.—With a large propeller in use, and especially in connection with a rotary engine, there is a serious gyroscopic effect produced, which is often found troublesome on quick turns in any plane. An aeroplane with the above combination will be found on a quick turn to put its nose either up or down, depending on the direction of the turn.

Suppose an aeroplane is flying at a speed V ft.-sec. Let the angular velocity of the airscrew be Ω radians per sec. Let the moment of inertia of the rotating parts be I. If the aeroplane banks to an angle ϕ with an angular velocity of turning of ψ radians per sec., a gyroscopic couple tending to turn the nose of the machine up or down is introduced. The magnitude of the couple is given by—

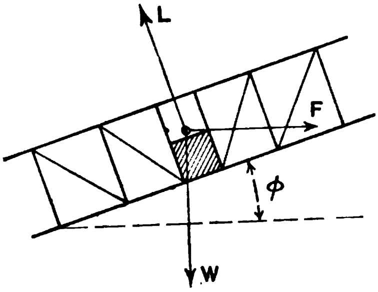


FIG. 40.

$$C = \frac{I\Omega\psi}{g} \quad . \quad . \quad . \quad (1)$$

When the aeroplane is banking without side slip its equilibrium is maintained by the three forces, weight, lift, and centrifugal force acting through the C.G., the first vertically downwards, the last horizontal, and the lift normal to the wings (Fig. 40).

Hence $$F = W \tan \phi$$

But $F = \frac{W}{g}\psi^2 r$, where r is the radius of the circle on which the aeroplane is turning.

Also $$\psi r = V$$

therefore $$\frac{W}{g}\psi V = W \tan \phi$$

or $$\psi = \frac{g \tan \phi}{V} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The gyroscopic couple is transmitted to the aeroplane through the engine mounting, and is resisted by control forces on the rudder and elevators.

If C_1 is the couple resisted by the rudder
C_2 „ „ „ elevators
Then $C_1 = C \sin \phi$
$C_2 = C \cos \phi$.

This couple becomes of importance in small scout aeroplanes with large rotary engines and large propellers.

This result is an approximate one only; but even if the whole of the factors are taken into account strictly, there is not a very marked departure from the figures arrived at by its use.

We must now turn to a consideration of the effect of some of the more violent manœuvres upon the loads which come upon the aeroplane. Most of these cases cannot be dealt with mathematically, although a treatment can be given in some of them. In a fair number of cases, however, experimental work has been done which throws considerable light on the intensity of the loading produced.

37. Effect of Sudden Change of Attitude.—Suppose an aeroplane to be flying horizontally at a speed V miles an hour. Let the angle of incidence of the main planes at this speed be α. Let the lift coefficient corresponding to α be k_L. By a violent movement of the controls the pilot can suddenly change the attitude of the aeroplane so that the wings have a greatly increased angle of incidence α' with a greatly increased lift coefficient $k_{L'}$. At the same time the speed is appreciably constant. That is to say

$$\text{Lift on wings} = L = k_{L'}\rho SV^2 \quad . \quad . \quad . \quad . \quad (1)$$

Let the horizontal flying speed of the aeroplane corresponding to an angle of incidence α' be v.

Then if the weight of the aeroplane be W

$$W = k_{L'}\rho Sv^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\frac{L}{W} = \frac{V^2}{v^2} \text{ from (1) and (2)}$$

Now $$\frac{L}{W} = N \text{ where N is the load factor}$$

therefore $$N = \frac{V^2}{v^2}$$

The extreme case of this occurs when an aeroplane which is being nose-dived at its terminal velocity is suddenly pulled out. It may reach an angle of incidence equal to the stalling angle.

As an example, suppose an aeroplane diving at a speed of 160 m.p.h. is suddenly pulled out. If it reaches stalling angle, as it easily can do before much speed is lost, v becomes stalling speed. Suppose this to be 50 m.p.h.

The loading upon the wings then becomes $(160)^2/(50)^2$ times normal, *i.e.* a load factor of over 10 is required to prevent failure of the structure. The case taken is not an extreme one for a scout aeroplane, and as this class is usually designed to a load factor of not more than 7, it is clear that most aeroplanes of this type can be broken in the air by such a manœuvre.

38. Forces on Fuselage and Tail Plane in Horizontal Flight.—When an aeroplane is in flight on a steady path it has already been shown that the centre of pressure on the main planes has a definite position. This position does not, except for one speed, coincide with the centre of gravity of the aeroplane, and hence a tail load is required to balance the aeroplane.

When the centre of pressure is forward of the centre of gravity the load upon the tail plane to balance the aeroplane must be in an upward direction; and conversely when the centre of pressure falls aft of the centre of gravity a down load is required.

The statement above may be modified in the case of an aeroplane with a high centre of thrust, where the moment introduced by the thrust about the centre of gravity has an important part in determining the load upon the tail (see para. 32).

However, in order to find the load which comes upon the tail in normal flight, it is only necessary to take moments of the lift and drag forces and the propeller thrust about the centre of gravity of the aeroplane and balance the net moment by a force on the tail plane. The equation for this purpose has already been quoted (para. 32).

It is clear, then, that there will be a different load on the tail plane for each angle of incidence of the main planes, and the strength factor for the fuselage must cover all such cases.

The main planes are generally designed in normal flight for two positions of the centre of pressure, and the strength of the fuselage should be calculated so that the latter structure has at least the same factor as the main planes for the centre of pressure considered.

If N_1 is the load factor required on the aeroplane for C.P. forward, N_2 is the load factor required on the aeroplane for C.P. back; P_1 is the up load on the tail plane with the C.P. of main planes forward; P_2 is the down load on the tail plane with C.P. of main planes back. Then the fuselage should stand up to a load at least equal to N_1P_1 upwards on the tail, and at least equal to N_2P_2 downwards.

It should be noticed that, except in very rare cases, this will not be the criterion for the fuselage strength, since heavier loads will be brought into play under other conditions of flight, but these must always be watched.

39. Loads on Fuselage and Tail Plane in a Nose Dive.—As the speed of an aeroplane increases, so the centre of pressure shifts aft and the tail load required to maintain balance increases.

An investigation was made in the Air Department of the Admiralty to determine what the limiting conditions were, and the results were published as a Confidential Information Memorandum by Miss Cave-Browne-Cave. The result showed that the tail load increased steadily until the aeroplane reached its terminal velocity, which occurred at an angle slightly greater than that of "no-lift." The results can be expressed in graphical form, which enables the tail load to be determined for any aeroplane under these flight conditions, a different set of curves being required for every wing section.

The method of treatment is as follows :—

Fig. 41 shows an aeroplane upon a diving course θ to the horizon at a speed V.

The forces keeping it in equilibrium are—

W = the weight acting vertically downwards.
D_W = the drag of the wings concentrated at the equivalent plane.
L = the lift of the planes perpendicular to the flight path supposed acting at the leading edge of the equivalent plane.
M = the moment of the lift force about the leading edge of the equivalent plane.
D_B = the resistance of the aeroplane apart from the wings, known for convenience as body resistance.
P = the control load on the tail plane.

The notation adopted is as follows :—

ρ = density of the air.
k_D = coefficient of resistance of the main planes.
k_L = lift coefficient of main planes.
k_M = coefficient of moment about the nose of the aerofoil.
$k_{D'}$ = coefficient of body resistance.
a = angle of incidence of main planes relative to the wind.
c = chord of the equivalent plane.
d = distance of line of action of body resistance below the centre of gravity of the aeroplane.
ϕc = perpendicular distance from C.G. of aeroplane to the chord of the equivalent plane.
κc = distance from the foot of this perpendicular to the leading edge of the equivalent plane.
l = distance from C.G. of aeroplane to line of action of tail load.
S = area of main planes.
S′ = area of tail plane and elevators.
Δ = loading on the main planes.
δ = loading on the tail plane and elevators.
U = the volume of the main plane = S × c.
u = the volume of the tail plane = S′ × l.
Ω = ratio $\frac{\delta u}{\Delta U}$.

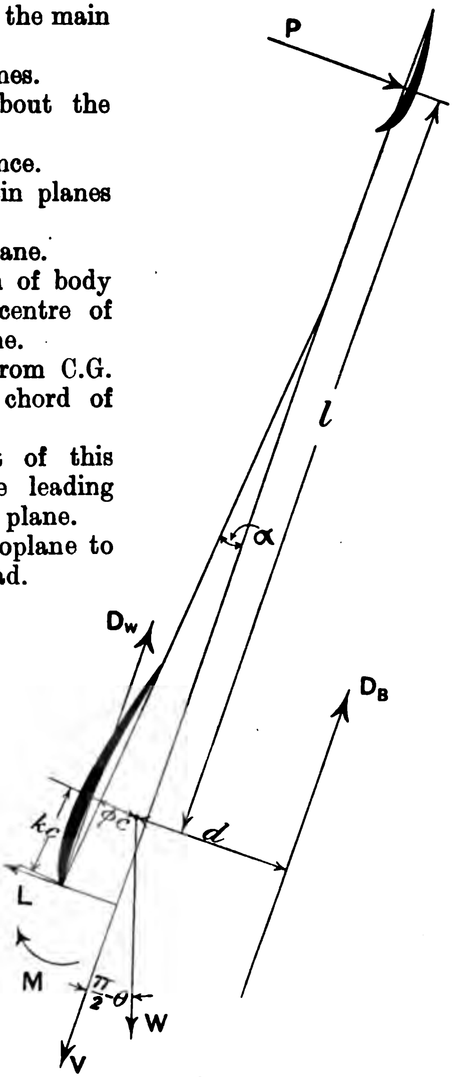


FIG. 41.

Resolving along and perpendicular to the trajectory and taking moments about the C.G. we get—

$$W \sin \theta = D_W + D_B \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$W \cos \theta + P = L \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$M - D_B d + Pl + Lc(\kappa \cos a - \phi \sin a) + D_W c(\kappa \sin a + \phi \cos a) = 0 \quad (3)$$

Also we have the equations—

$$L = k_L \rho S V^2; \quad D_W = k_D \rho S V^2; \quad M = k_M \rho c S V^2; \quad D_B = k_{D'} \rho S V^2 \quad . \quad (4)$$

Equations (1) and (2) give—

$$W^2 = (D_W + D_B)^2 + (L - P)^2 \quad . \quad . \quad . \quad . \quad (5)$$

Substitute in (5) and (3) for L, D_W, M and D_B from (4)—

$$W^2 = (k_D + k_{D'})^2\rho SV^2 + (k_L\rho SV^2 - P)^2 \quad . \quad . \quad . \quad (6)$$

$$k_M\rho SV^2 - k_{D'}\rho SV^2\frac{d}{c} + P\frac{l}{c} + k_L\rho SV^2(\kappa\cos a - \phi\sin a)$$

$$+ k_D\rho SV^2(\kappa\sin a + \phi\cos a) = 0 \quad . \quad . \quad . \quad . \quad . \quad (7)$$

From (7) we get—

$$P\frac{l}{c} = \rho SV^2\left[k_{D'}\frac{d}{c} - k_M - k_L(\kappa\cos a - \phi\sin a) - k_D(\kappa\sin a + \phi\cos a)\right]$$

Put—

$$\gamma = k_{D'}\frac{d}{c} - k_M - k_L(\kappa\cos a - \phi\sin a) - k_D(\kappa\sin a + \phi\cos a) \quad . \quad (8)$$

Then $$Pl/c = \rho SV^2\gamma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Substituting this value of P in (6) we have—

$$W^2 = (k_D + k_{D'})^2(\rho SV^2)^2 + \left(k_L - \frac{c}{l}\gamma\right)^2(\rho SV^2)^2 \quad . \quad . \quad (10)$$

Now $$\frac{1}{\Omega} = \frac{\Delta U}{\delta u} = \frac{\Delta Sc}{\delta S'l} = \frac{Wc}{Pl} = \frac{W}{\rho SV^2\gamma}$$

therefore $$\left(\frac{\gamma}{\Omega}\right)^2 = \left(\frac{W}{\rho SV^2}\right)^2 \quad . \quad . \quad . \quad . \quad (11)$$

Using (10) and (11) we get—

$$\left(\frac{\gamma}{\Omega}\right)^2 = (k_D + k_{D'})^2 + \left(\frac{c}{l}\gamma - k_L\right)^2 \quad . \quad . \quad . \quad (12)$$

The next step is to simplify this expression. In the original work a definite aeroplane was considered, and the wing section was R.A.E. 14.

The values of k_D, $k_{D'}$, etc., were determined for this aeroplane, and a curve showing the variation of Ω with a was plotted. This curve showed that the maximum value of Ω occurred at a value of $a = a_0$ very nearly that of "no-lift."

On the same figure were next plotted a series of curves, varying one of the constants κ, $k_{D'}$, d/c, ϕ, and l/c at a time, *i.e.* each of the new curves had all the constants equal to those in the original curve with one exception. These curves showed the effects of the constants on the result and these effects were as follows :—

(1) The maximum value of Ω always occurred at about the same angle a_0.

(2) The variation of l/c or κ had little effect upon the maximum value of Ω.

(3) The variation of $k_{D'}$, d/c or ϕ affected the maximum value of Ω considerably.

The result of (12) was then simplified by taking the value of Ω as a maximum, when $a = a_0$, assuming l/c and κ were constants, and thus making the value of $\Omega_{\max.}$ a function only of $k_{D'}$, d/c, ϕ and the wing section. Equation (12) then becomes—

$$\left(\frac{\gamma}{\Omega}\right)^2 = (k_{D'} + A)^2 + \left(\frac{\gamma}{B} - C\right) \quad . \; . \; . \; . \; . \; (13)$$

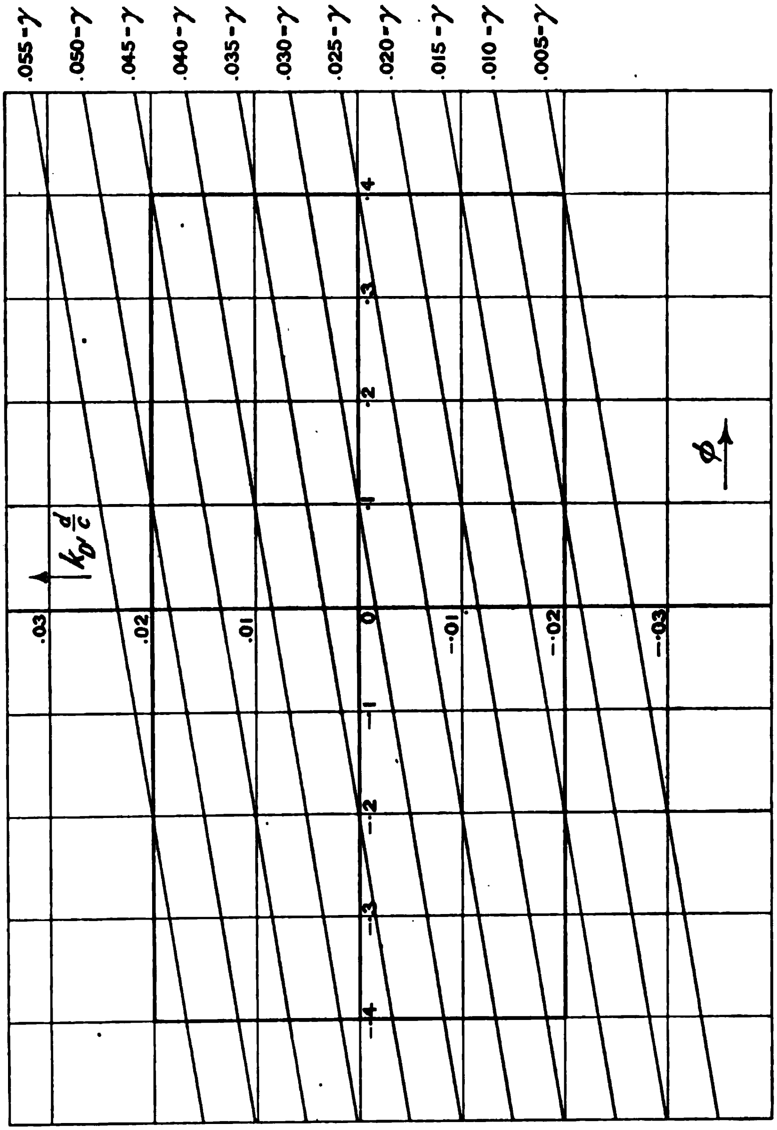


Fig. 42.—R.A.E. 14.

and equation (8) becomes—

$$\gamma = k_{D'}\frac{d}{c} + D - E\phi \quad . \; . \; . \; . \; . \; . \; . \; (14)$$

where A, B, C, D, and E are constants whose values are determined from a knowledge of the wing section and by giving any reasonable values to l/c and κ.

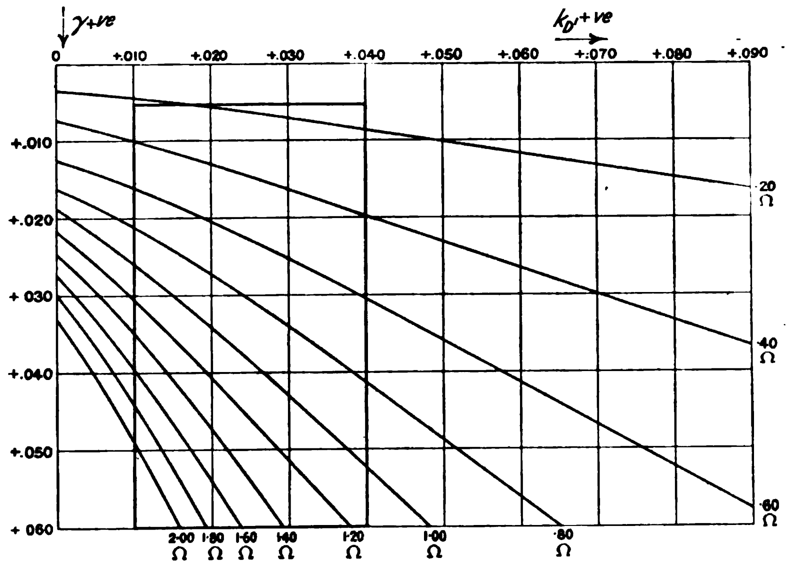


FIG. 43.—R.A.E. 14.

Having found A, B, C, D, and E a family of curves of ϕ against $k_{D'} . d/c$ for varying values of γ may be plotted, and another family of γ against $k_{D'}$ for varying values of Ω.

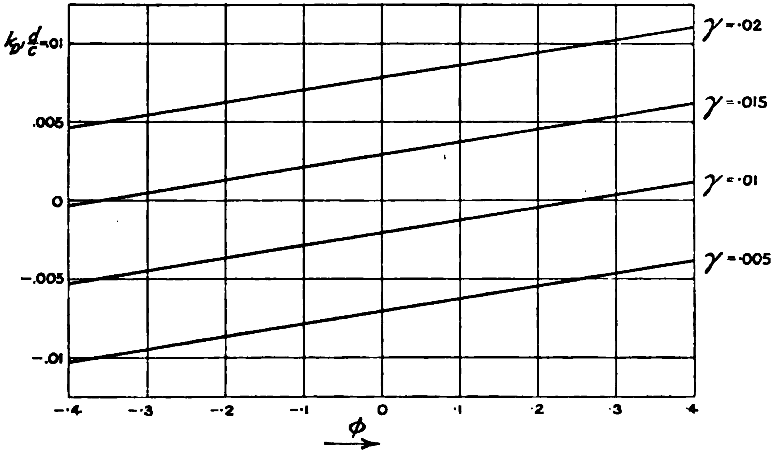


FIG. 44.—R.A.E. 15.

Such a series of curves is given in Figs. 42, 43 for R.A.E. 14 aerofoil, and in Figs. 44, 45 for R.A.E. 15 aerofoil.

To use these curves, first calculate the values of $k_{D'}$, ϕ, and d/c for the aeroplane under consideration. From the γ curves find the value of γ given at the intersection of ϕ and $k_{D'}d/c$. Then from the Ω curves find the value of Ω given at the intersection of this value of γ and $k_{D'}$.

Ω having been found in this way, the tail loading can be determined from the definition of Ω

$$\Omega = \frac{\delta u}{\Delta U}$$

This method gives a good means of obtaining the maximum steady load on the fuselage in a terminal velocity nose dive, but it is not capable of very rapid application.

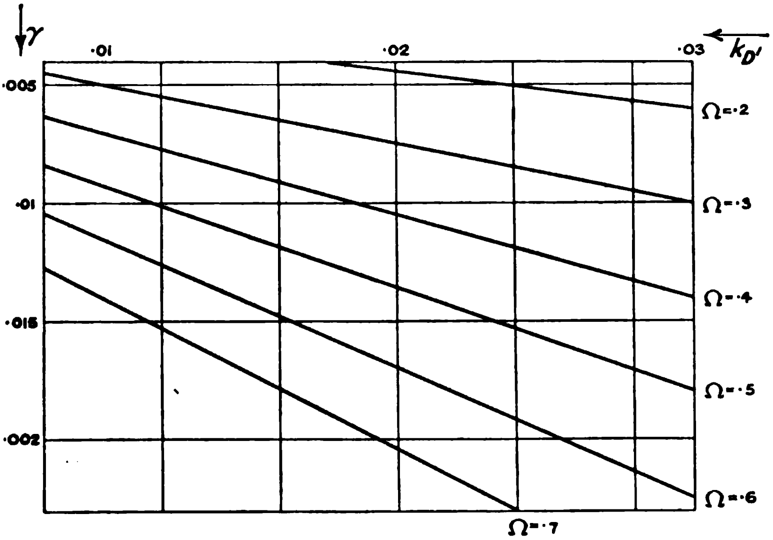


FIG. 45.—R.A.E. 15.

A series of aeroplanes was calculated both on R.A.E. 14 and R.A.E. 15 aerofoils, and the values of Ω obtained. It was found that Ω was very nearly constant and could be taken as 1 for R.A.E. 14 and ·6 for R.A.E. 15, so that the general result may be approximated still further by putting Ω equal to a constant K.

We then have $$\delta = \frac{\mathrm{K}\Delta\mathrm{U}}{u}$$

or $$\delta\mathrm{S}' = \frac{\mathrm{K}\Delta\mathrm{S}c}{l}$$

but $$\Delta\mathrm{S} = \mathrm{W} \quad \text{and} \quad \delta\mathrm{S}' = \mathrm{P}$$

therefore $$\mathrm{P} = \mathrm{K}\frac{\mathrm{W}c}{l}$$

for R.A.E. 14, $K = 1$, and for R.A.E. 15, $K = {\cdot}6$, so that the result is obtained

for R.A.E. 14 $$P = \frac{Wc}{l}$$

for R.A.E. 15 $$P = \frac{{\cdot}6Wc}{l}$$

A factor of safety should in all cases be allowed on these loads to cover faulty material or workmanship, and a figure of 1¼ to 1½ has worked well in the authors' experience.

40. Pulling out of a Nose Dive.—The foregoing work relates only to the load upon the tail plane and fuselage during a steady dive at the terminal velocity of the aeroplane. During the period of pulling out from such a dive a heavier load is momentarily required, and the magnitude of such load is a function of the rate at which this transition into horizontal flight is made.

The combination of terminal velocity with a quick pull out is one which would break practically any aeroplane, and, as a general rule, the pull out from a dive of anything like a high speed is done very slowly, so that these excessive loads on the tail do not occur.

The exact mathematical investigation of this case is very complicated and laborious, but interesting papers on the subject by G. P. Thomson, S. B. Gates, T. W. K. Clarke and others have been published by the Advisory Committee for Aeronautics.

For practical purposes, however, the problem may be tackled as follows :—

Let V_0 = stalling speed of the aeroplane.

Let V = maximum speed from which it is just safe to pull up suddenly.

Let N = factor of loading allowed on the front truss of the aeroplane.

Then it has already been shown in para. 37, that—

$$N = \left(\frac{V}{V_0}\right)^2$$

so that $$V = V_0\sqrt{N}$$

N and V_0 being known V is thus determinate.

Since the load factor of the main planes is N there is no point in making the fuselage structure of greater strength than this.

Suppose, then, the aeroplane is diving at the velocity V ft. per sec. A sudden pull out will load the main planes to N times normal, *i.e.* to breaking load.

Let the tail plane incidence (making allowance for down wash from the main planes) at this speed be α_T, and imagine the elevators are pulled suddenly to such an angle as to give the maximum k_L possible on the tail. This k_L may be found from model tests and will be in the neighbourhood of ·6.

Then the loading on the tail plane is $k_L\rho V^2$ lbs. per square foot. This loading will correspond to the breaking load on the main planes, and the fuselage may be designed to it.

41. Loads on Main Planes in a Nose Dive.—The limiting case of a dive occurs when the aeroplane travels in a nearly vertical path at its terminal velocity. Under this condition of flight there is a moment upon the wings tending to nose dive the aeroplane still further, and a small lift force. The aeroplane is kept in equilibrium by a load acting downwards upon the tail.

The full treatment of this case has been considered in para. 39, where the method of obtaining the tail load is fully given, and the reader is referred to that paragraph for details.

In the meantime it will be considered that this control load is known and is P.

The moment on the main planes gives rise to a downward load on the front truss and an upward load on the rear truss.

Let the total load on the front truss $= F_1$.

Let the total load on the rear truss $= F_2$.

Let the distance between the two trusses be a.

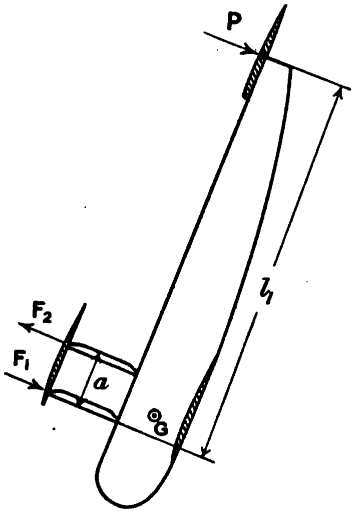


FIG. 46.

Let the distance from the front truss to the line of action of the tail load $P = l_1$.

Then by moments about the front truss—

$$l_1 P = a F_2 \quad \dots \dots \dots \quad (1)$$

or

$$F_2 = \frac{l_1}{a} P \quad \dots \dots \dots \quad (2)$$

Also

$$F_2 = F_1 + P \quad \dots \dots \dots \quad (3)$$

therefore

$$F_1 = F_2 - P$$

or

$$F_1 = \left(\frac{l_1}{a} - 1\right) P \quad \dots \dots \dots \quad (4)$$

From equations (2) and (4) the values of F_1 and F_2 are found, and hence the loads per foot run on the front and rear trusses respectively.

The values of these being known, the two trusses are treated in exactly the same manner as for normal flight conditions, the only difference being in the fact that the loading on the front truss is reversed in direction. This method is only approximate, since the forces F_1 and F_2 are each much larger than their difference P.

No account has so far been taken of the drag forces for this condition of flight, but since the equilibrium of the aeroplane is maintained, it is clear that for a steady velocity the weight of the aeroplane must be exactly balanced by the drag upon the various parts.

Analysis of the drag forces upon an aeroplane for the terminal velocity may be made approximately in the following manner.

Let k_D = the drag coefficient of the wings at an angle α.
Let k_D' = ,, ,, ,, remainder of the structure at the angle α.
Let k_L = the lift ,, ,, wings at the angle α.

Then

$$W = k_L \rho S V^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

$$R_W = k_D \rho S V^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

$$R_B = k_{D'} \rho S V^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

where

W = the weight of the aeroplane
R_W = the wing resistance
R_B = the residual resistance

From equation (1) the speed of the aeroplane for various angles of incidence can be obtained. These angles should be taken from the angle of no lift to the neighbourhood of about 2°.

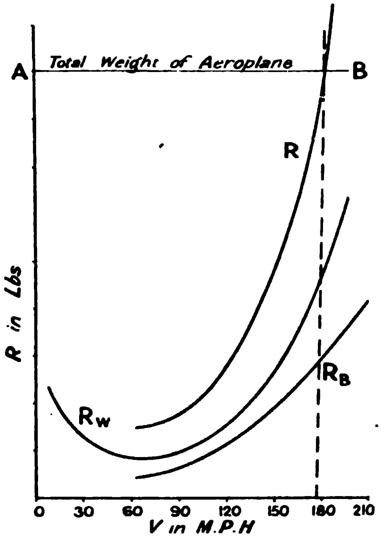


FIG. 47.

The values of V for the range of α having been found by this method, the equations (2) and (3) are used to determine the wing and body resistance for the same series of speeds.

Plotting these values of R_W and R_B against the speed of the aeroplane two curves are obtained, as shown in Fig. 47. These curves are added to give the value of the total resistance R of the aeroplane at any speed.

The line AB is drawn at the value of R equal to the total weight of the aeroplane, and the point at which this cuts the curve of R determines the terminal velocity of the aeroplane.

These curves should be plotted for any aeroplane under consideration, and the values of the various drag forces can then be found for any speed.

Examples worked out by the authors show that at the terminal velocity the drag upon the wings is approximately 50 per cent. of the total weight of the aeroplane, and this figure appears to hold fairly consistently for quite a wide range of types.

As an approximate rule, therefore, for preliminary design purposes it may be assumed that at the terminal velocity the wings carry 50 per cent. of the weight of the aeroplane as drag forces, the remainder being split up between the body, chassis, tail, etc.

In the case of a twin-engine aeroplane it may be assumed that 12½ per cent. of the weight is carried as drag by each engine nacelle, 25 per cent. by the central fuselage, tail, etc., and 50 per cent. by the wings.

In the nose diving case, therefore, the following forces upon the wings must be considered—

(1) An up load on the rear truss.

(2) A down load on the front truss.

(3) A drag upon the wings.

(4) The weights of the various units of the structure acting vertically, *i.e.* roughly parallel to the chord.

These having been determined, the forces in the main plane structure are determined in the manner given in Chapter XI.

As an example consider the case of a twin-engine, high-performance aeroplane as shown in Fig. 48.

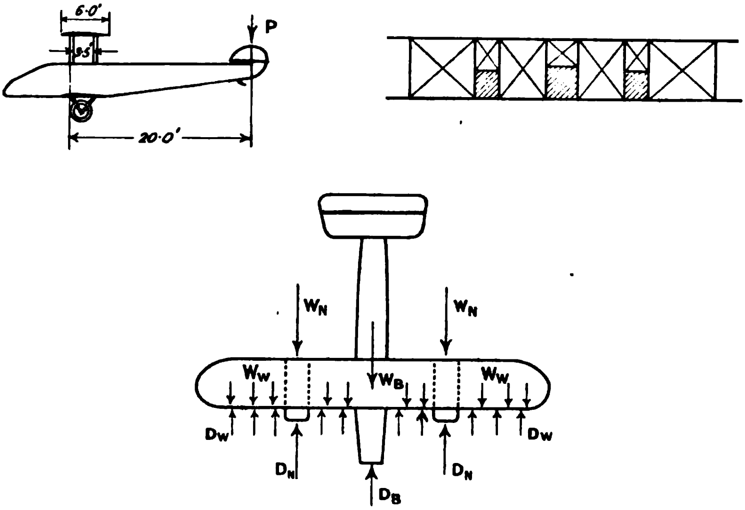


FIG. 48.

Let the weight be 6000 lbs., say.

Let the tail load be 1700 lbs.

Then taking moments about the front truss—

$$\text{The load on the rear truss} = \frac{20 \times 1700}{3 \cdot 5} = 9720 \text{ lbs. up}$$

$$\text{And on the front truss} = 9720 - 1700 = 8020 \text{ lbs. down}$$

These loads are distributed over the span of the aeroplane in the ordinary way.

Consider next the drag forces on the structure.

Total drag	= 6000 lbs.	
Drag on wings	= 50 per cent. of 6000 lbs.	= 3000 lbs.
,, nacelles	= 12½ per cent. of 6000 on each	= 1500 lbs.
,, body, etc.	= 25 per cent. of 6000	= 1500 lbs.
	Total	6000 lbs.

These drag forces act upward, as shown on Fig. 48, plan view. Balancing these and acting downward are

(1) The weight of the wings W_W—evenly distributed along the span.

(2) The concentrated weights of nacelles, bodies, etc., acting at their appropriate points.

These forces are in equilibrium, and the structure can therefore be properly examined.

42. Loads during a Turn.—In the earlier part of this chapter we have only considered forces acting perpendicular to the tail plane, but there are other conditions to be examined.

When an aeroplane turns it does so in virtue of air forces on the fin and rudder which are transmitted through the fuselage structure to the centre section.

The fact that the fin and rudder are not usually symmetrically placed about the centre line of the fuselage introduces in addition to the direct load a torsion of the fuselage which requires consideration. The determination of the load which can occur on a rudder is a rather difficult question to settle, but a rough rule is to assume that when the aeroplane is flying at its normal top speed V the rudder is put hard over. The k_L value for this condition is about ·4, the low figure being due to the bad aspect ratio and shape of the rudder surface.

The loading per square foot is then given by $k_L\rho V^2$, where V is in feet per second.

The centre of pressure of the fin and rudder must be estimated or judged. Suppose this to be l_1 feet above the centre line of the fuselage, the torque on the structure is $k_L\rho S_1V^2l_1$ foot-lbs., where S_1 is the area of the rudder.

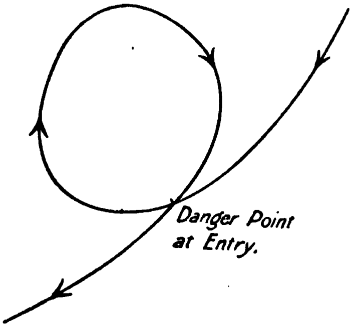


FIG. 49.

43. Loads in a Loop.—In looping, the aeroplane is dived or flown at a high speed and then pulled out into the loop. From what has been said previously with regard to a sudden change of attitude, it should be clear that the most dangerous part of the loop is during the first quadrant, and the entry should on this account be as easy as possible.

The loading upon the wings when in the loop can be obtained approximately if the radius of curvature is known.

Let W = weight of aeroplane.
,, V = velocity of aeroplane at any moment.
,, R = radius of curvature of flight path at same moment.

$$\text{Then the centrifugal force} = \frac{WV^2}{gR}$$

The resultant of this and the weight is the air load at the moment, and given V and R it can be calculated.

Figures are quoted (para. 49) for actual measured accelerations during this manœuvre, and these may be used as a basis of design.

The loads arising from this manœuvre are covered by the ordinary calculation of nose diving, and for the centre of pressure in its most forward position, and there is therefore no need to make special calculations in this case.

44. Loads in a Spin.—The spinning of an aeroplane may be either stable or unstable. The latter condition usually arises from too small a fin and rudder, and the spin becomes faster and faster, and finally ends in a spinning nose dive which often terminates in an accident.

With this condition of flight we are not concerned, as it is one which only arises from a faulty design of the aeroplane, and should therefore never occur.

There is, however, another spin of a different nature. In this the aeroplane spins whilst dropping vertically. This spin is perfectly stable and is a steady uniform rotation with a steady vertical speed downwards.

It has been possible by means of a camera obscura to analyse this motion, and the following points were brought to light.

(1) The downward speed of the aeroplane is slow—of the order of 70 miles per hour.

(2) The angle of incidence of the main planes was well above stalling angle—in some cases it was as high as 45°.

From these facts it is clear that the equilibrium of the aeroplane must be maintained almost entirely by the drag upon the wings, since body drag, etc., at such a slow speed is not very great.

It would appear desirable, therefore, so to design the wings of an aeroplane of the smaller types that the drag bracing is sufficiently strong to transmit the total weight of the aeroplane when this is distributed along them. A factor of safety is required on this, and 2 seems a reasonable figure to adopt.

As will be seen later, the strength of the drag bracing is usually determined by the condition when one of the flying wires has been broken, and so the spinning case now under consideration is not of extreme practical importance from the structural standpoint.

45. Loads on a Bank.—When an aeroplane turns correctly there is no side-slip, and the centrifugal force, weight of craft, and lift on the planes are in equilibrium, as has been already shown in para. 27.

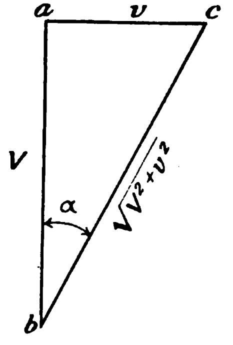


FIG. 50.

The question of incorrect banking is not an important one, viewed from the standpoint of the structure, since the extra loads brought into play are not very great.

In an extreme case, however, there is one point which should be noticed, and that is the effect of the side wind on the aeroplane struts.

Suppose an aeroplane is travelling with a forward velocity V in the direction of *ba*, and is side-slipping with velocity v in the direction *ac*, as shown in Fig. 50.

The resultant motion is in the direction of *bc*, but since the aeroplane is heading along *ba* it is clear that the wind strikes the struts at an angle α (Fig. 51), such that

$$\alpha = \tan^{-1} v/V$$

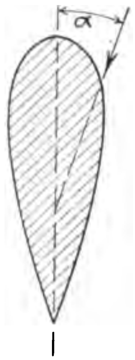


FIG. 51.

The struts act in exactly the same way as an aerofoil, and the result is therefore to give a distributed lateral load upon these members, which should be considered in conjunction with the normal end load arising from the lift forces on the wings.

The value of k_L per degree for a streamline section is approximately ·033, so that the force per square foot of strut side area is

$$F = \cdot033\alpha\rho(V^2 + v^2)$$

but $\quad v = V \tan \alpha$

therefore $\quad F = \cdot033\alpha\rho V^2 \sec^2 \alpha$

The critical flow point for a streamline section occurs at about 16°, so that in design α should be taken at this figure, V being taken as the top speed of the aeroplane.

The equation above then reduces to

$$F = \cdot00135\ V^2$$

This point assumes most importance in aeroplanes with very large struts, but it is also advisable to take it into account when designing interplane struts for any type.

46. Loads in Upside-Down Flight.—In the great majority of aeroplanes the case of upside-down flight cannot or does not occur, but in those of the fighting scout class it frequently happens that it is essential for the aeroplane to travel some little distance in the inverted position.

The case as regards loads in the trusses is exactly similar to the normal flight condition except, of course, that the lift forces are now in the opposite direction relative to the aeroplane. In most cases it will be found that reversed flight is only possible with a large angle of incidence, so that the centre of pressure is well forward. Thus the front truss will get the heaviest loads under these conditions, and, as a rule, these loads are not as heavy as the down load developed in a terminal velocity nose dive. It may, however, give the worst case for the bottom rear spar.

47. Landing Loads on Main Planes.—When an aeroplane lands the shock is absorbed in the shock absorbers in the under-carriage, and loads are thrown upon the chassis structure, which will be dealt with in a later chapter.

The inertia of the wings, however, causes loads in the main plane structure itself, and this requires some consideration.

If the chassis is designed to a given factor the wings should be made to carry the same number of times their own weight, acting downward, since the stoppage of their mass will stress them to the same degree as the structure of the undercarriage.

If engines or concentrated weights of any kind are carried in the wing structure, they may cause serious loads to be developed in the members of the main planes, since they will have the same retardation as the structure of the undercarriage, and the wings should be carefully considered from this point of view.

If, however, there are no concentrated loads in the wings, the loading induced is not of a serious nature and usually needs no detailed consideration.

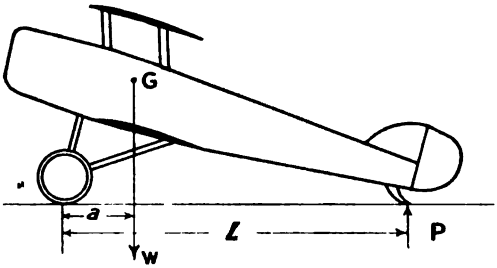


Fig. 52.

48. Landing Loads on Tail Skid and Fuselage.—Fig. 52 shows an aeroplane standing upon the ground with its tail skid down, also resting on the ground. The distance between the tail skid and the wheel contact is L feet.

The weight of the aeroplane acts vertically downward through the centre of gravity, and the line of action cuts the ground line at a distance a feet from the wheel contact line. The load on the tail skid is then given by

$$P = \frac{a}{L}W$$

and the reaction of the ground may be considered as an upward force on the end of the fuselage.

The weight of the whole tail unit W_T relieves this load as far as the fuselage is concerned, and may be deducted, so that the net upward load at this point is

$$\frac{a}{L}W - W_T$$

In landing, the tail skid bumps along the ground, and so a factor of loading is required on this figure. It is an impossibility to calculate what this should be, but a figure varying from 7 to 4 may be adopted, depending on the class of aeroplane which is under consideration.

49. Experimental Results.—By the use of an accelerometer attached to the aeroplane the following actual accelerations were measured in flight. Since the load on the aeroplane is given by

$$P = Wf/g$$

where f is the acceleration, the numerical figure is an index to the number of times normal load thrown on the wings during the particular manœuvre considered.

During a loop at 95 m.p.h. the	maximum	acceleration was	$3{\cdot}2g$
,, ,, 87 ,,	,,	,,	$2{\cdot}8g$
Rolling	,,	,,	$2{\cdot}8g$
Stalling suddenly at 85 m.p.h.	,,	,,	$2{\cdot}8g$
Spinning nose dive at 55 m.p.h.	,,	,,	$2{\cdot}0g$
Ordinary bumpy weather	,,	,,	$\cdot 5g$ to $1{\cdot}5g$

In a mock fight between an S.E. 5A and an R.E. 8, three times normal load was of common occurrence, while on one or two occasions four times and over was measured. In a spinning nose dive the conditions of flight were analysed through the medium of a camera obscura, and it was found that the angle of incidence of the wings was from 25° to 40°, while the downward speed was low, being approximately 70 m.p.h.

CHAPTER VII

THE ELASTIC PROPERTIES OF MATERIALS

50. Stress and Strain.—This and the succeeding chapters include only those general principles of engineering structures and materials which are necessary for the examination of the strength of aeroplanes.

Stress is the equal and opposite action and reaction which take place between two bodies, or two parts of the same body, transmitting force. In the first case, it is an external stress, and in the second, an internal stress. The same stress may be external from one point of view, and internal from another. The external forces acting on any portion of a body in equilibrium are balanced by the internal stresses. The air reaction on the wings of an aeroplane, for example, is balanced by the stresses in the spars, drag struts, wires, fabric, and so on.

Strain. A body is altered in shape or dimensions by the stresses acting on it, and is said to be strained.

There are three principal kinds of stresses: (1) tensile, (2) compressive, (3) shear, with three resulting strains: (1) an extension, (2) a contraction, (3) a slide or shear. These three kinds of stresses and strains are best illustrated by examples.

The acting wires of an aeroplane are in a state of tensile stress. The external air forces on the aeroplane tend to elongate these wires.

The interplane struts and drag struts are in a state of compressive stress. They have loads in them which tend to shorten them.

A bolt carrying a wiring lug at an angle to its axis is in shear combined with tension.

All three stresses may exist at the same time in a member. The main spars under normal flight conditions are in a state of shear, tensile, and compressive stress.

Every stress produces a corresponding strain, and within certain definite limits and for certain materials the strain is directly proportional to the stress causing it. Thus if the stress is doubled the strain is doubled. If, to take a concrete example, a pull of one hundred pounds on a wire increases its length from L to $L+dL$, then a pull of two hundred pounds will increase its length to $L + 2dL$. The same stress, however, will not produce the same strain in different materials.

The intensity of stress is the stress per unit area, sometimes called also unit stress. If, for example, the area of the wire above is A square inches, and the pull on it is T lbs., then the stress per unit area of the cross section of the wire is T/A. Unit stress is frequently referred to simply as stress.

Unit strain is the strain per unit length of the material. If a wire, for example, of length L is stretched by a pull T to $L+dL$, then the unit strain is dL/L. Unit strain is frequently referred to simply as strain.

It has been remarked that as the stress increases there is a corresponding increase in strain. This may be plotted as a curve. This curve is a straight line for most materials up to a limit known as the elastic limit, but beyond this limit the strain increases faster than the stress.

When a body has been strained beyond its elastic limit, and the stress inducing that strain removed, it will be found that it does not return completely to its original shape. This difference or amount of the change of shape is known as the permanent set. Within the elastic limit of the material a body will return to its original shape when the forces producing distortion have been removed. The proportionality of stress to strain was first enunciated by Hooke in 1676, and Hooke's law may be stated in the form

$$\frac{\text{unit stress}}{\text{unit strain}} = \text{constant}$$

This constant is known as Young's Modulus, or the coefficient of elasticity of the material. It is always denoted by E. Fig. 53 shows a typical stress-strain curve for steel. It will be noticed on this curve that the material fractures considerably beyond the elastic limit.

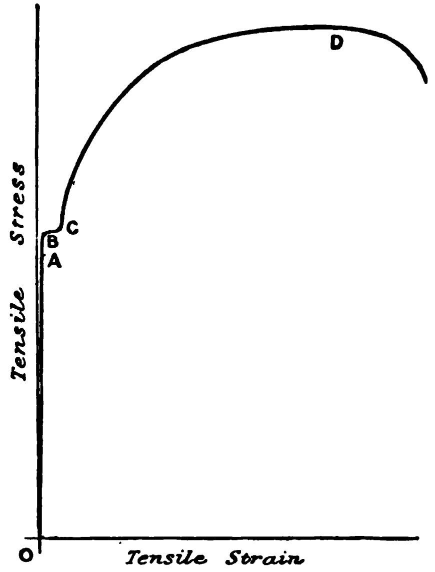


Fig. 53.

(From Morley's "Theory of Structures," Longmans, Green & Co.)

The elastic limit of the material should be taken in calculation and not the rupture point. Beyond the elastic limit the strain increases more rapidly than the stress, and there comes a point where there is a sudden increase in strain with no corresponding increase in stress. This is called the Yield Point. The stress at the yield point of the material is the stress to which a large number of designers in the past have worked. It is preferable, however, to work on the lower limit, the true elastic limit of the material, as any member loaded beyond this stress would show a permanent set

From the definition of Young's Modulus we get, if a member of length L and cross section A is acted on by a force P, and the extension or compression is e, the relation

$$E = \frac{\text{stress}}{\text{strain}} = \frac{P/A}{e/L} = \frac{PL}{eA}$$

or

$$\text{extension} = e = \frac{PL}{AE}$$

51. Oblique Stresses.—The stress across any section of a body may be neither normal nor tangential to the section. In this case it will be found convenient to resolve this stress into a compressive or tensile component normal to the section p_n, and a shear stress tangential to it, p_t.

Let a small prism of cross section a sq. ins. be under an intensity of tensile stress in the direction of its length of p, say. This is normal to the section A′B′.

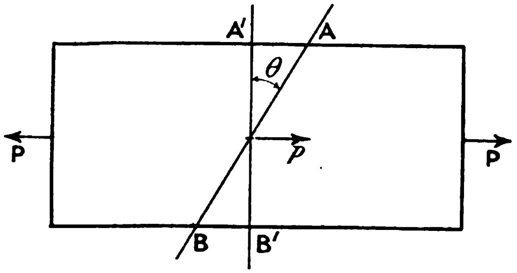


Fig. 54.

Now suppose it is required to find the stress intensity normal and tangential to AB, a section at an angle θ to A′B′.

By resolution, since the area of the surface AB is $a \sec \theta$,

we get
$$p_n = p\,\frac{\cos\theta}{\sec\theta} = p\cos^2\theta$$

and
$$p_t = p\,\frac{\sin\theta}{\sec\theta} = p\sin\theta\cos\theta$$

It is clear that p_t is a maximum when $\theta = 45°$, so that the maximum shear stress occurs in planes inclined at an angle of 45° to the plane A′B′.

52. Complementary Shear Stress.—For every shear stress there is one of equal intensity at right angles which is called the complementary shear stress. To demonstrate this, let ABCD be a small rectangular prism of material, with one pair of sides AB, CD parallel to the direction of the shearing stress. This stress, of intensity q, say, forms a couple tending to rotate the prism, and from statical considerations of the equilibrium, no system of forces can balance this couple unless they are such as to form an equal and opposite couple to the original one. Let the side AB of the prism be a, and AD be b, and let its length be l. Further, let q' be the intensity of the tangential stress at right angles to q as in Fig. 55.

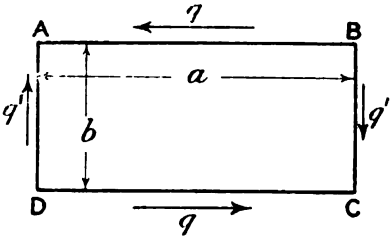


Fig. 55.

Then the forces on AB and CD are each alq, and the forces on AD and BC are each blq', and equating the moment of the two couples

$$alq \times b = blq' \times a$$

i.e.

$$q = q'$$

When as in the figure there are two simple shear stresses, of equal intensity, but opposite sign, the body is said to be in a state of simple shear.

A body that is subjected to only one kind of strain, *i.e.* extension, compression, or shear, is said to be in a state of simple strain; while if subjected to more than one, to be in a state of complex strain.

53. Principal Stresses.—If a body is under a complex system of stresses these stresses may be resolved into three simple tensile or compressive stresses in planes at right angles to each other. These simple stresses are called the principal stresses, and the planes are called the principal planes.

It is necessary to be able to find the magnitude and inclination of the principal stresses, in order to obtain the maximum intensity of stress.

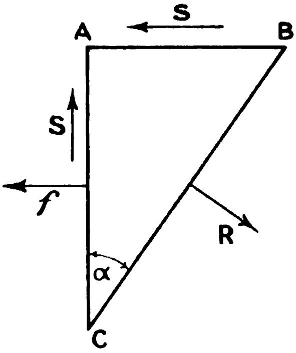


FIG. 56.

As an example, the principal stresses are found due to a normal stress f and a tangential stress s at right angles to one another.

Let these stresses act in a plane cutting the plane of the paper in AC (Fig. 56), and let BC represent similarly one of the planes of principal stress. Let the intensity of this principal stress be R.

Then, if AB is at right angles to AC, we have acting along AB, since for every shear stress there is an equal and opposite shear stress at right angles, a shear stress of intensity s.

Resolving along AB and AC

$$f.\mathrm{AC} + s.\mathrm{AB} = \mathrm{R}.\mathrm{BC}\cos a \quad \ldots \quad (1)$$

$$s.\mathrm{AC} = \mathrm{R}.\mathrm{BC}\sin a \quad \ldots \quad (2)$$

From (1)

$$f.\frac{\mathrm{AC}}{\mathrm{BC}} + s.\frac{\mathrm{AB}}{\mathrm{BC}} = \mathrm{R}\cos a$$

i.e.

$$f\cos a + s\sin a = \mathrm{R}\cos a \quad \ldots \quad (3)$$

From (2)

$$s.\frac{\mathrm{AC}}{\mathrm{BC}} = \mathrm{R}\sin a$$

or

$$s\cos a = \mathrm{R}\sin a \quad \ldots \quad (4)$$

From (3) and (4)

$$\mathrm{R}(\mathrm{R} - f) = s^2$$

i.e.

$$\mathrm{R}^2 - \mathrm{R}f - s^2 = 0$$

From which

$$\mathrm{R} = \frac{f}{2} \pm \frac{1}{2}\sqrt{f^2 + 4s^2}$$

$$= \frac{f}{2}\left(1 \pm \sqrt{1 + \frac{4s^2}{f^2}}\right)$$

The negative sign of the root corresponds to the second principal stress, and the positive sign to the maximum stress, which is the one we want.

The direction of the plane in which this stress acts is found as follows :—

From (3) $$R\cos\alpha - f\cos\alpha = s\sin\alpha$$

From (4) $$R\sin\alpha = s\cos\alpha$$

Therefore $$R = \frac{s\cos\alpha}{\sin\alpha}$$

therefore $$\frac{s\cos^2\alpha}{\sin\alpha} - f\cos\alpha = s\sin\alpha$$

or $$s = \frac{f\sin\alpha\cos\alpha}{\cos^2\alpha - \sin^2\alpha} = \frac{f\sin 2\alpha}{2\cos 2\alpha}$$

and $$\tan 2\alpha = \frac{2s}{f}$$

The planes of maximum shear stress are inclined at an angle of 45° to the principal planes, and it can easily be shown that the intensity of this stress is

$$\sqrt{\frac{f^2}{4} + s^2}$$

54. Two Perpendicular Stresses.—Let XX′Y′Y (Fig. 57) be a block of material subjected to two pulls P, Q, at right angles to one another, and let p, q be the stresses per unit area across sections normal to these forces. Let AB be any section inclined at an angle θ to the force P.

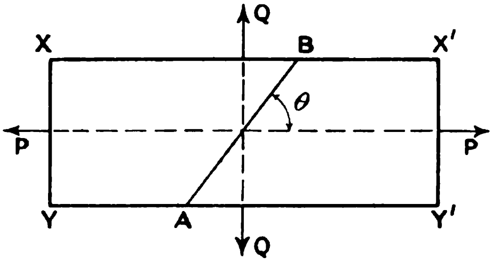


Fig. 57.

Then the normal component of p across AB $= p\sin\theta/\text{cosec}\,\theta$

$= p\sin^2\theta,$

and the tangential component of p across AB $= p\cos\theta/\text{cosec}\,\theta$

$= p\sin\theta\cos\theta$

So the normal component of q across AB

$$= q\sin(90^\circ - \theta)/\text{cosec}\,(90^\circ - \theta)$$
$$= q\cos^2\theta$$

and the tangential component of q across AB

$$= -q \cos (90^\circ - \theta)/\operatorname{cosec} (90^\circ - \theta)$$
$$= -q \cos \theta \sin \theta$$

since the tangential component here is in the opposite direction to that due to p.

The total normal component is $f_n = p \sin^2 \theta + q \cos^2 \theta$, and total tangential component $= f_t = (p - q) \sin \theta \cos \theta$.

The resultant of these components f_r

$$= \sqrt{f_n^2 + f_t^2} = \sqrt{(p \sin^2 \theta + q \cos^2 \theta)^2 + (p - q)^2 \sin^2 \theta \cos^2 \theta}$$

The quantity under the root reduces, on rearranging terms, to

$$p^2 \sin^2 \theta + q^2 \cos^2 \theta$$

therefore $$f_r = \sqrt{p^2 \sin^2 \theta + q^2 \cos^2 \theta} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If ϕ is the angle of this stress to AB it is given by

$$\tan \phi = \frac{f_n}{f_t} = \frac{p \sin^2 \theta + q \cos^2 \theta}{(p - q) \sin \theta \cos \theta} = \frac{p \tan^2 \theta + q}{(p - q) \tan \theta} \quad . \quad . \quad (2)$$

If ψ is the angle between the resultant stress and the direction of p

$$\psi = \phi - \theta$$

and it is easy trigonometry to show that $\tan \psi = \tan (\phi - \theta) = \frac{q}{p} \cot \theta$ by using equation (2).

In the case of two unlike stresses, q being tensile for example, the above results hold with a change in sign of q.

55. Ellipse of Stress.—Draw two concentric circles, as in Fig. 58, whose radii OX, OY are respectively equal to q and p, and let OF be the radius inclined at an angle θ to OY, OAB the radius inclined at an angle $(90^\circ - \theta)$ to OY as in Fig. 58. Draw BD and AE perpendicular to OY, AC perpendicular to BD, and join OC.

Then $\quad \mathrm{OD} = \mathrm{OB} \cos (90^\circ - \theta) = p \sin \theta$

and $\quad \mathrm{CD} = \mathrm{AE} = \mathrm{OA} \sin (90^\circ - \theta) = q \cos \theta$

$$\mathrm{OC} = \sqrt{\mathrm{OD}^2 + \mathrm{AE}^2} = \sqrt{p^2 \sin^2 \theta + q^2 \cos^2 \theta} = f_r \quad \text{from (1)}$$

also $$\tan \mathrm{DOC} = \frac{\mathrm{AE}}{\mathrm{OD}} = \frac{q \cos \theta}{p \sin \theta} = \frac{q}{p} \cot \theta = \tan \psi$$

therefore $$\text{angle DOC} = \psi$$

and $$\text{angle COF} = \phi$$

and it follows that C is on an ellipse whose major axis is $2p$ and minor axis $2q$.

This ellipse is called the Ellipse of Stress.

From it we can obtain the resultant stress on a plane in any given direction, for by drawing OB at right angles to the given direction, and BD parallel to XX to meet the ellipse in C, we get OC as the resultant intensity of stress on the required plane, and the angle COF is the angle between this resultant stress and the given plane.

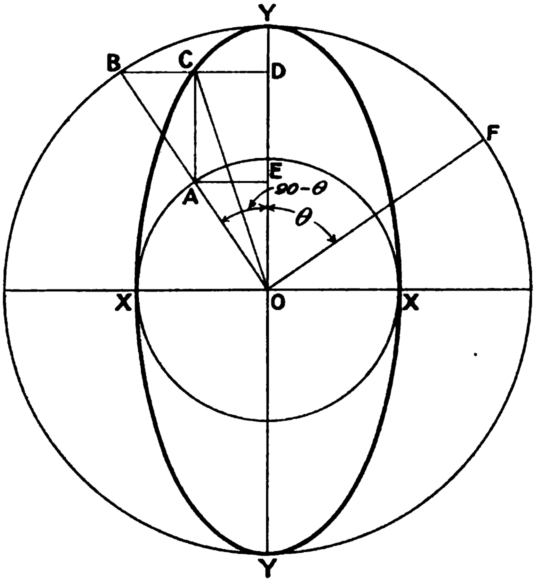


FIG. 58.

CHAPTER VIII

FRAMED STRUCTURES

56. Frames.—A frame is a structure which consists of struts and ties, pin-jointed together, and designed to be loaded mainly at the joints, and though as a whole it may be under bending, its individual members are in compression or tension. A perfect frame is one which has just sufficient members to keep it in equilibrium under all systems of loading. If it has too few or too many members, then it is said to be imperfect. Frames in one plane are called plane frames, and in two or more planes space frames.

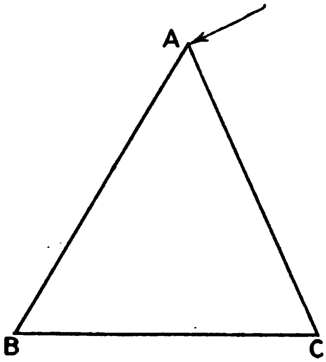


FIG. 59.

The simplest plane framed structure is a triangle, with pin joints at A, B, and C in Fig. 59. If such a structure be supported at B and C, for example, any load applied in the plane of the frame at A will be resisted by pure tensile or compressive loads in AB and AC. The point A is, it will be noticed, fixed save for a slight change of position due to the elongation or compression of AB and AC under the applied load.

ACBD in Fig. 60 has too few members and is an incomplete frame. Under certain conditions of loading it will be in equilibrium, but usually it will not. A load applied at A in the direction AB, for example, would deform the structure completely.

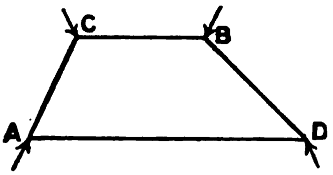


FIG. 60.

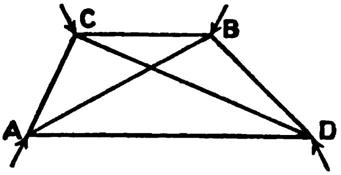


FIG. 61.

(From Morley's "Theory of Structures." Longmans, Green & Co.)

ACBD in Fig. 61 has too many members and is a redundant frame. It consists of six bars, any of which is capable of taking either tension or compression. In this case AB or CD could be safely omitted, leaving a structure sufficiently braced against any system of loading.

If AB, CD were only capable of taking tension the structure would be just braced, for either one or the other tie would slack off, the remaining one forming the brace. This actually happens in an aeroplane. Consider the front truss of the single bay aeroplane shown in Fig. 62. In normal flight the flying wires AD, A′D′, BE′, B′E are in play, while the weight wires BC, B′C′ are inoperative. In a steep nose dive the direction of the air forces on the front truss is reversed, with the result that the wires BC, B′C′ come into play and the others become slack. If, however, AD and A′D′ had been members capable of taking either tension or compression, then the wires BC, B′C′ would have been redundant in nose diving, the air reactions at A and A′ being taken up by AD and A′D′ in compression.

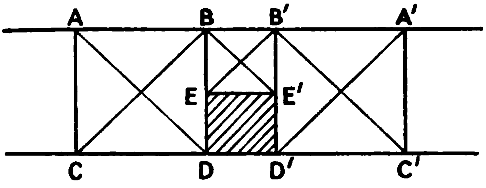


Fig. 62.

Again, consider the front elevation of the bracing of the tail plane spar shown in Fig. 63. The spar is attached to the fuselage CC′EE′ at the points C and C′; DG is the fin post, to which the bracing wires DB, DA, DB′, DA′ from the tail plane spar are attached. AE, BE, A′E′, B′E′ are steel tubes from the spar to the bottom rails of the fuselage. The above structure is over-braced for down loads, for though the bracing wires AD and so on would, in the absence of the stay tubes, take these loads, the presence of the tubes renders them unnecessary.

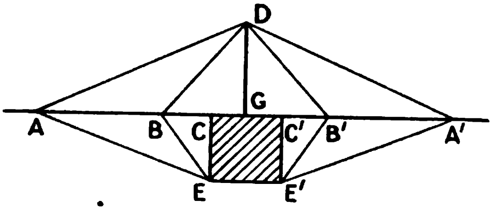


Fig. 63.

It is very usual to overbrace aeroplane structures, for the duplication of members gives an added safety to the structure if failure of one of the bracing members occurs. The specifications of British fighting aeroplanes, for example, definitely called for the duplication of certain parts, as, for example, the flying wires and tail plane bracing, the provision of which usually caused a redundancy, although this duplication was ignored in the actual strength calculations.

A redundant frame, as a rule, adds to the strength of the structure as a whole, though this is not always true, for in any redundant framework the load in one member may cause extra loads in any or all of the other members. Overbraced frameworks have the great disadvantage that the loads in the members cannot be calculated by simple processes, though in Chapter XXII. an outline is given of a method of finding the loads in such structures.

An alternative method of dealing with a redundant frame is to divide it up into two perfect frames with certain common members and calculate the loads in each separately, then adding them together algebraically. This is not accurate, but it is a close approximation in many cases. In the case of such a redundancy as shown in the example of the tail plane bracing above, it is usual to make the structure of sufficient strength, ignoring the superfluous members.

The number of members in a perfect plane frame is $2n-3$, where n is the number of joints or nodes.

This can be shown as follows. For a triangle three members and three nodes are necessary. For every additional node two extra members are required. If there are n nodes, therefore, the total number of bars to make the structure perfect is $2n-3$.

A number of types of aeroplane structures is shown in Plates XIII, XIV, and XV.

57. Methods of determining Stresses in Structures.—The forces, including the reactions, acting on the aeroplane structure having been found by methods described in later chapters, the stresses in the members themselves may be found by one or a combination of three methods—

(1) The stress diagram.
(2) The method of sections.
(3) Resolution.

We will consider these three ways of dealing with the problem in order.

58. Stress Diagrams.—To the engineer probably the most satisfactory method of determining the forces in the individual members of a structure is by means of the stress diagram.

The stress diagram is obtained by drawing a polygon of forces for the external forces on the frame and for each joint of the frame, and fitting them together in succession. The method will be more readily understood from one or two examples. Fig. 64 shows the semi-elevation of a biplane, under the vertical reactions as indicated. The lettering adopted is that of Bow's notation, in which letters are placed in the spaces between members, and in the spaces separated by the lines of action of the applied loads. The vertical reaction at the top outer support, for example, would be spoken of as the reaction CD, and the load in the intermediate interplane strut would be called the load HJ, and so on.

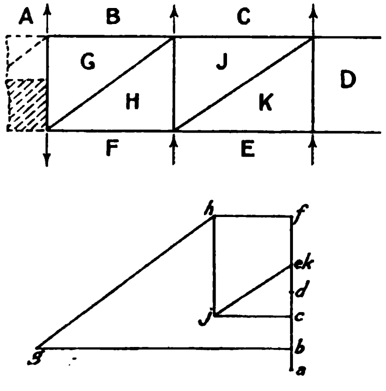


FIG. 64.

Begin by setting the external reactions out on some convenient scale on a line *af*; to this scale *ab* represents the reaction AB, and so on.

Starting from the reaction DE, the corner KED is clearly in equilibrium under the reaction DE, and a corresponding equal and opposite thrust in KD, there being no load in KE, since it is at right angles to the line of

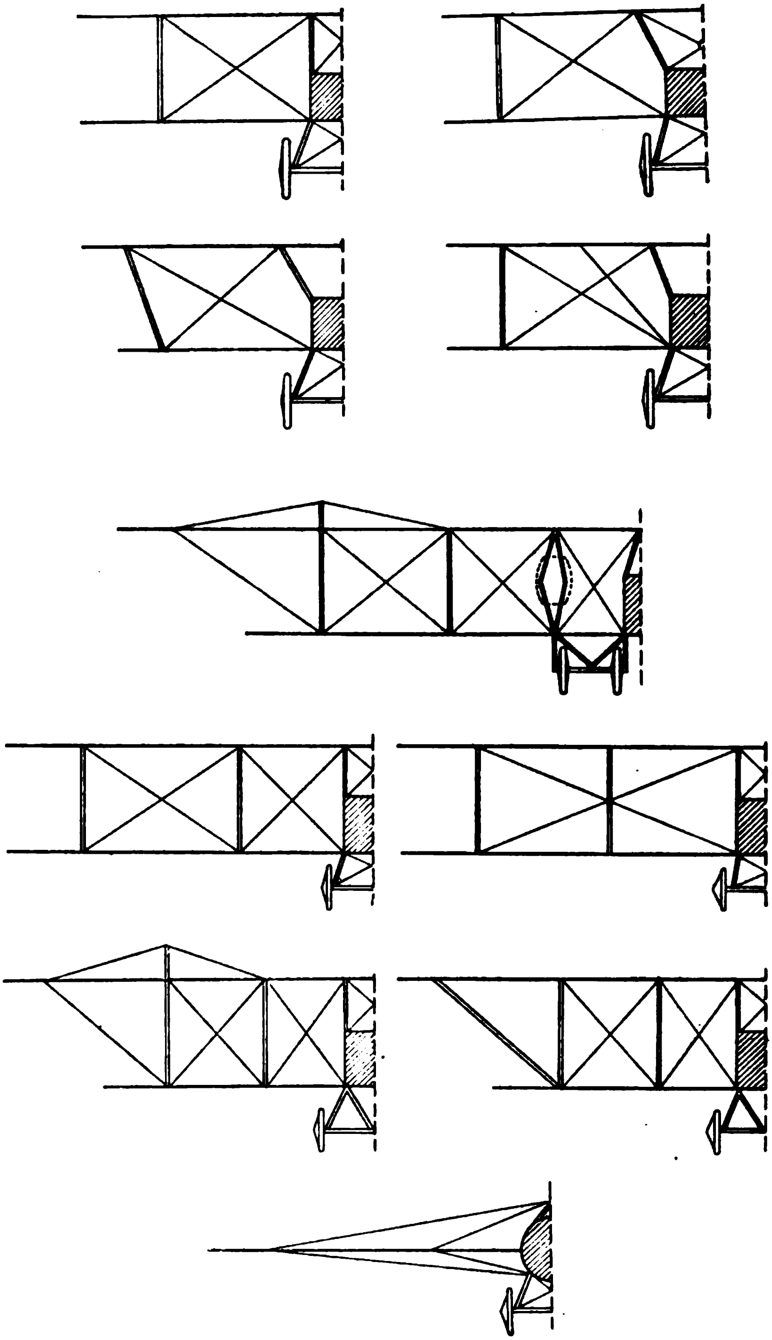


Plate XIII.—Typical Main Plane Bracings.

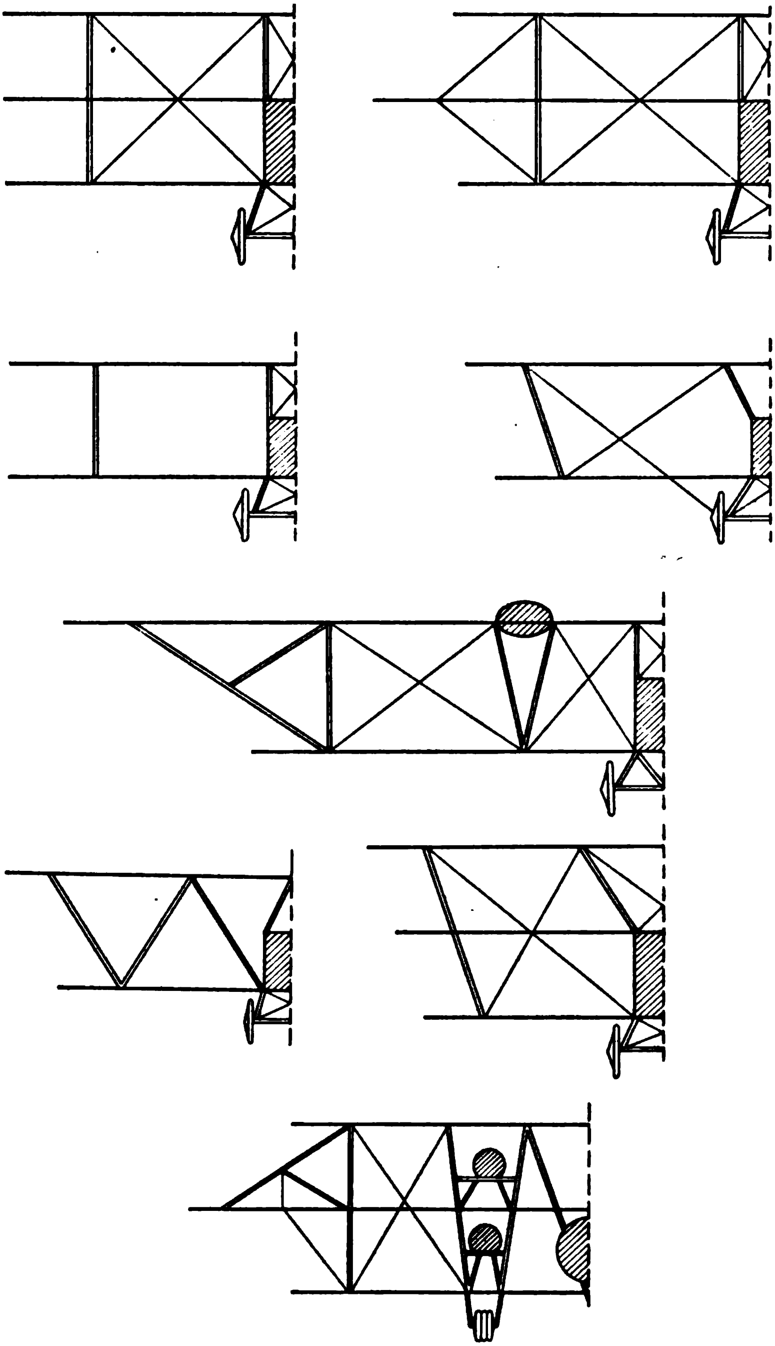


Plate XIV.—Typical Main Plane Bracings.

the external reaction DE. Therefore, if from *d* is marked off on the vertical scale a length *dk* corresponding to DK, then *dk* will represent both in magnitude and direction the thrust KD. It will be clear that *k* coincides with *e*.

Now from *k* draw *kj* parallel to KJ. Then *kj* represents the load KJ in direction. The next thing is to find its magnitude. This is done by drawing *jc* parallel to JC, meeting *jk* in *j*. The line *jk* then represents the load KJ both in magnitude and direction, and also *jc* represents JC, for these lines represent the forces in direction, and they must represent them in magnitude, for the point *j* is common to both, and that can only be the case where the lines meet.

So by drawing *hj* parallel to HJ and *fh* parallel to FH we get the magnitude and direction of the loads JH and FH, and so finally the loads GB, HG in the same manner. All that is necessary to determine the actual magnitudes of all the loads in the members is to read them off from the lower figure, usually called the stress diagram or reciprocal figure, to the scale to which the original reactions were set down.

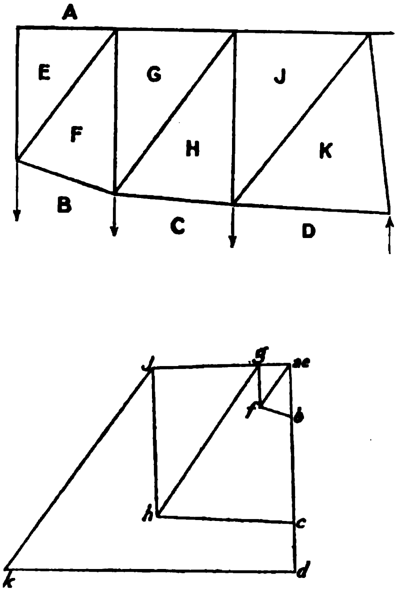


FIG. 65.

Example for the Forepart of a Fuselage.—The top diagram of Fig. 65 represents the side elevation of one-half of the forepart of a fuselage, the loads due to the engine and so on being AB, BC, and CD, acting parallel to one another.

Draw *abcd* parallel to AB, BC, or CD, and mark off, as before, to some convenient scale, *ab* =AB, *bc* = BC, and *cd* = CD.

It is clear that there is no load in AE, and therefore on the stress diagram *e* coincides with *a*.

From *b* draw *bf* parallel to BF, and from *e* draw *ef* parallel to EF, and let them meet in *f*. Then *bf*, *ef* represent in magnitude and direction the loads BF, FE, since *aebf* is the polygon of forces for the point AEFB. So by drawing *ch* parallel to CH and *gh* parallel to GH, and so on, we get the loads in all the members of the truss.

In both the above cases we have taken all the reactions or forces parallel to one another, but if, for instance, in the last example, AB had not been parallel to BC, the original external load line would have been a broken one, each part being parallel to the line of the corresponding applied force.

To distinguish whether a member is in tension or compression the following simple rule should be followed.

Consider the point AEFB. The reaction AB is known in direction, in this case vertically downwards. On the stress diagram put an arrow on *ab* in this direction, and continue round the polygon in the same direction.

Now transfer the direction of these arrow heads to the corresponding members, meeting at the point under consideration. Then if the arrow head on the actual members points towards the point or node, the member is in compression, and if away, in tension. BF, for example, is in compression, and EF in tension. So for the members FG, GH, HC, remembering that we now know the directions of the arrow heads, *i.e.* whether the loads are tensile or compressive, in FB and EF, and that, therefore, we know which way to start round the polygon of forces for that particular system.

59. Method of Sections.—The stresses in the various members of a braced structure may also be found by means of the method of sections, a method first proposed by Rankine, and developed by Ritter, by whose name it is often known.

In principle this method is extremely simple. It is an assertion of the fact that if a braced structure be divided into two by an ideal section, then the loads in the members of the framework cut by the section must balance the external forces acting on either side of the section. The method often saves much laborious calculation when it is desired to find the loads in any particular members of a structure. An actual example will make this clear.

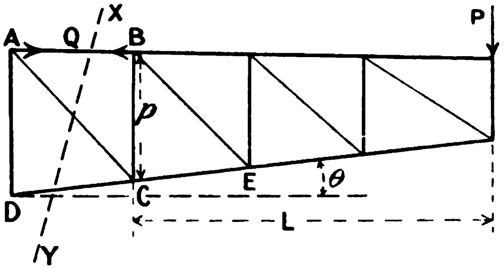


FIG. 66.

Fig. 66 represents the side elevation of a fuselage with a tail load P, as indicated. Consider any panel ABCD and take a section XY across it. The section chosen should not cut more than three members.

Then the external force P is kept in equilibrium by the loads in AB, AC, and CD across this section. Two of these, namely AC, CD, meet at C, and therefore, by taking moments about C, we obtain an equation in P and the load in AB, giving us the value of the latter immediately.

If Q is the load in AB, p the length of the perpendicular from C on AB, L the length of the perpendicular from C on the line of action of P

$$Qp = LP$$

$$Q = \frac{LP}{p}$$

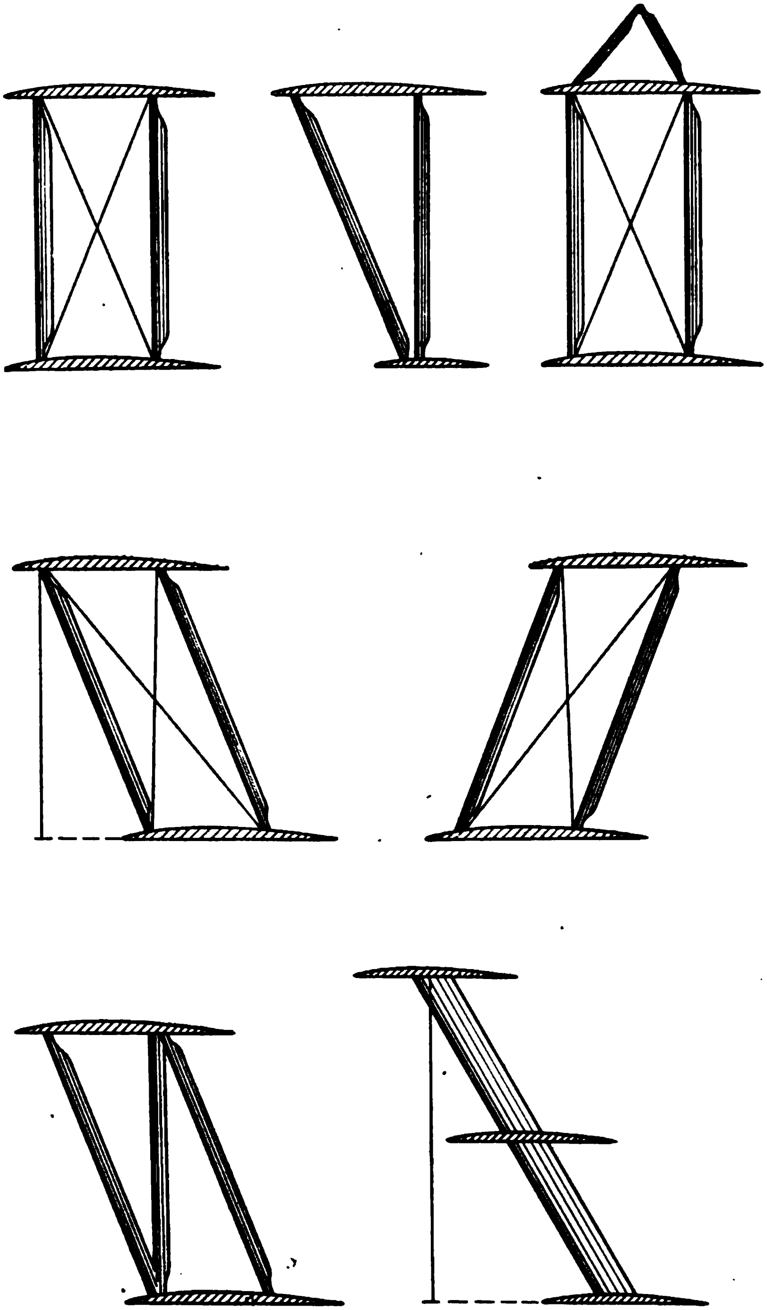


Plate XV.—Typical Main Plane Bracings.

So if L_1 is the length of the perpendicular from A on the line of action of P, p_1 the length of the perpendicular from A on CD, and Q_1 the load in CD, by taking moments about A

$$Q_1p_1 = L_1P$$
$$Q_1 = \frac{L_1P}{p_1}$$

Q is clearly tensile and Q_1 compressive. Now that Q and Q_1 are known the load in AC can be obtained by taking moments about B, for the resulting equation will contain Q_1, P, their moment arms, and the unknown load in AC.

60. Method of Resolution.—This method consists in resolving the resultant force at any point in a structure or across a particular section along the members of the frame meeting at that point or cut by the section. If there are more than two members meeting at a point, then all the remainder must be known in order to obtain the loads in the remaining two by this method.

For example, in Fig. 66 the loads in AC and CD have been found by the method of sections. Then, since the point C is in equilibrium, we have by resolution perpendicular to DCE

$$(\text{load in BC}) \cos\theta = (\text{load in AC}) \sin \text{ACD}$$

which gives the load in BC; and by resolution perpendicular to BC,

$$(\text{load in DC} - \text{load in CE}) \cos\theta = (\text{load in AC}) \sin \text{ACB}$$

which gives the load in CE.

By starting with the point of application of P, and considering the equilibrium of each point in turn, the loads in all the members of the fuselage could have been found, but the method would have been much more cumbersome than either the stress diagram or the method of sections. Where members meet at right angles to one another, however, it is often quicker to employ resolution methods.

For any given structure a combination of methods is often best.

CHAPTER IX

GEOMETRICAL PROPERTIES OF PLANE SECTIONS

In this chapter it is assumed that the reader has a knowledge of elementary statics and dynamics and geometry, and we therefore quote a few general theorems only, omitting elementary definitions and explanations.

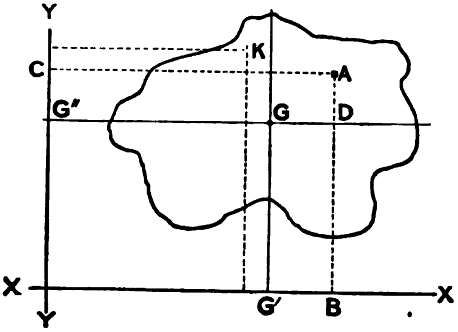


Fig. 67.

61. General Method of finding Position of Centre of Gravity.—Suppose it is required to find the centre of gravity of an irregular section as shown in Fig. 67.

Let XX be any axis in the plane of the figure, and let AB be the perpendicular distance of any small element of area a from XX.

Then aAB is the moment of the area a about XX. The moment of the whole area about the line XX is the sum of all the moments of the small elements of area of which the figure is made up, about XX, *i.e.* the moment of the whole area about XX $= \Sigma a\text{AB}$.

If G is the centre of gravity of the area, and GG′ is the perpendicular from G on to XX, then GG′ × (area of figure) is the moment of the whole area about XX.

i.e. $$\text{GG}' \times \text{area of whole figure} = \Sigma a\text{AB}$$

or $$\text{GG}' = \frac{\Sigma a\text{AB}}{\text{total area of figure}} = \frac{\Sigma a\text{AB}}{\text{A}}, \text{ say}$$

This gives the distance of the centre of gravity from XX. So its distance can be found from any other line YY, say, not parallel to XX, and therefore its exact position is known.

The sum $\Sigma a\text{AB}$ is sometimes called the first moment of the area about the line XX.

If $y = \text{F}(x)$ is the equation of the curve, then the area may be expressed in the form $\int \text{F}(x)dx$, the integral being taken between suitable limits.

The positions of the centres of gravity of a number of sections commonly met with in aeronautical engineering are given on pp. 83 and 84.

Practically, if the centre of gravity is only wanted for rough calculations, it may be obtained by balancing methods, by cutting out the figures in thin cardboard or sheet metal. Balance this cut-out figure on a knife edge along three different lines in its plane, and where they meet gives the position of the centre of gravity. It is advisable to balance along at least three lines as a check. Or the figure may be suspended in turn from three points, not in a straight line, by thread, and where the lines or lines produced of the threads meet again gives the position of the centre of gravity.

62. Centre of Gravity of an Aeroplane.—The position of the centre of gravity of an aeroplane is so constantly required in calculations on its strength and stability that a method of obtaining it should be known, and is given in this chapter for sake of convenience.

If all the weights of the component parts are known, and the positions of their centres of gravity from some fixed datum line, then it is easy to calculate the position of the centre of gravity of the aeroplane as a whole, for it is only a matter of finding the position of the resultant of a number of parallel forces.

The position may, however, be found experimentally as follows.

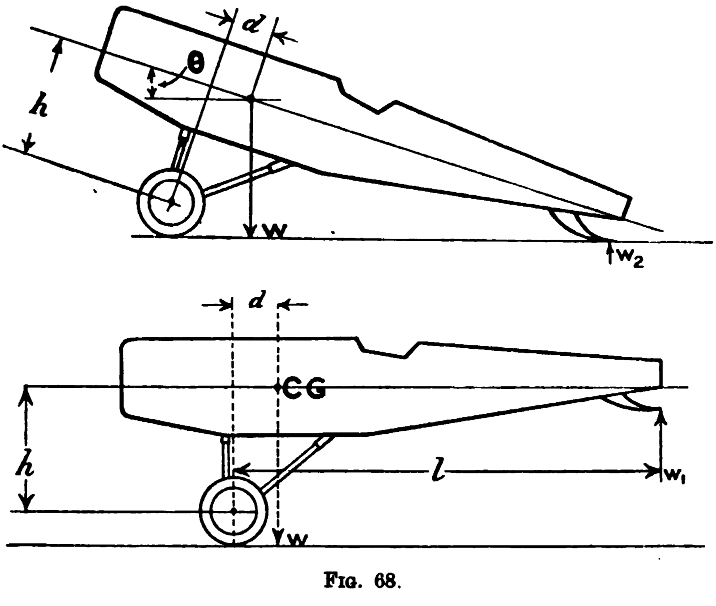


FIG. 68.

Let the height of the centre of gravity from the axle of the wheels be h, when the aeroplane is horizontal, as in Fig. 68, and d its horizontal distance from the vertical through the axle. Let W be the total known weight of the aeroplane, and let θ be the angle made by the fuselage to the horizontal when the tail skid is resting on the ground, as in the upper figure.

Let w_1 be the up load on the tail skid when the aeroplane is horizontal as in Fig. 68, and w_2 the load on the skid when it is resting. Both these loads may be measured by means of a spring balance.

Then if l be the distance from the wheel base to the point of support of the tail

$$w_1 l = \mathrm{W}d$$

$$d = w_1 l/\mathrm{W} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

also $$\mathrm{W}(d \cos\theta + h \sin\theta) = w_2(l \cos\theta + h \sin\theta)$$

or $$h = \frac{w_2 - w_1}{\mathrm{W} - w_2} l \cot\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Also the centre of gravity is clearly on the longitudinal plane of symmetry of the aeroplane, and its exact position is therefore known from equations (1) and (2). If the total weight of the aeroplane is not known, weighbridges should be placed under the chassis. The sum of the readings on these and the tail load is the weight required.

63. Moment of Inertia.—The product of a mass or area and the square of its mean distance from any given point or axis is called the second moment of the mass, or area about the point or axis, or more commonly the moment of inertia, although this term is strictly only true when dealing with mass.

The moment of inertia of the irregular figure on p. 79 about XX is therefore—

$$\Sigma a\mathrm{AB}^2$$

It is usual to use the letter I to denote the moment of inertia, and about the line XX the moment of inertia is written $\mathrm{I_{XX}}$,

i.e. $$\mathrm{I_{XX}} = \Sigma a\mathrm{AB}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\mathrm{I_{YY}} = \Sigma a\mathrm{AC}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Now suppose the whole area to be concentrated at some point K so as to give the same moments of inertia about XX and YY as given by (1) and (2).

Then if k_{xx} and k_{yy} are the lengths of the perpendiculars from K on XX and YY respectively, k_{xx} and k_{yy} are called the radii of gyration of the area about XX and YY respectively, and

$$\mathrm{A}k_{xx}^2 = \mathrm{I}_{xx} = \Sigma a\mathrm{AB}^2$$

also $$\mathrm{A}k_{yy}^2 = \mathrm{I}_{yy} = \Sigma a\mathrm{AC}^2$$

To find the moment of inertia of the area about a line through the centre of gravity of the figure parallel to XX, say, we proceed as follows:—

$$\mathrm{I}_{xx} = \Sigma a\mathrm{AB}^2$$

$$= \Sigma a(\mathrm{AD} + \mathrm{DB})^2$$

$= \Sigma a(\mathrm{AD}+x)^2$, where x is the distance of the centre of gravity from the line XX

$$= \Sigma a(\mathrm{AD}^2 + 2\mathrm{AD}\,.\,x + x^2)$$

$$= \Sigma a\mathrm{AD}^2 + 2\Sigma a\mathrm{AD}\,.\,x + \Sigma a x^2$$

$$= \mathrm{I_{GG''}} + 2x\Sigma a\mathrm{AD} + x^2\Sigma a$$

$= \mathrm{I}_{GG''} + 2x$ [first moment of area about line through its centre of gravity] $+ x^2\mathrm{A}$

$= \mathrm{I}_{GG''} + \mathrm{A}x^2$

i.e. $\mathrm{I}_{gg''} = \mathrm{I}_{xx} - \mathrm{A}x^2$

So $\mathrm{I}_{gg'} = \mathrm{I}_{yy} - \mathrm{A}y^2$, where y is the distance of the centre of gravity from the line YY′

or dividing each term by A

$$k^2_{gg''} = k^2_{xx} - x^2$$
$$k^2_{gg'} = k^2_{yy} - y^2$$

where $k_{gg''}$, $k_{gg'}$ are the radii of gyration about axes through the centre of gravity, and k_{xx}, k_{yy} about parallel axes at distances x and y from the centre of gravity.

These are most important results, for it is usually the moment of inertia about a line through the centre of gravity that is wanted in practice. From these results, therefore, if we know the moment of inertia about any line, we can immediately find it about a corresponding parallel line through the centre of gravity or vice versa.

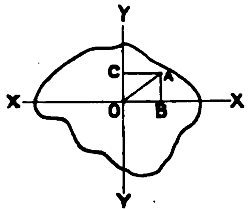


FIG. 69.

The moment of inertia of an area about an axis through the centre of gravity perpendicular to its plane is called the polar moment of inertia, and it is simple to show that it is equal to the sum of the moments of inertia about any two lines at right angles drawn through the centre of gravity in the plane of the area.

Let the polar axis meet the plane at O, Fig. 69, and let XX, YY be two axes at right angles through O. Let A be any element of area a; AB, AC, perpendiculars on the axes.

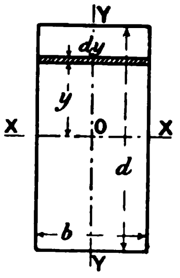


FIG. 70.

Then $\mathrm{AO}^2 = \mathrm{AB}^2 + \mathrm{AC}^2$

therefore $\Sigma a\mathrm{AO}^2 = \Sigma a\mathrm{AB}^2 + \Sigma a\mathrm{AC}^2$

$= \mathrm{I_{XX}} + \mathrm{I_{YY}}$

From this it follows that the sum of the moments of inertia about any two lines at right angles through the centre of gravity is equal to the sum about any other pair of lines at right angles through the same point.

This is a useful property to remember. From it we obtain, for example, that the moments of inertia of a square about all lines through the intersection of the diagonals are equal, and so on.

We shall now proceed to find the moments of inertia of a few simple figures.

64. Moment of Inertia of a Rectangle.—Let b, d be the breadth and depth of the rectangle. Take rectangular axes XOX, YOY through O, the centre of gravity. Consider a narrow strip of thickness dy parallel to XOX, and at a distance y from it.

Then the area of this strip $= b . dy$;

or
$$I_{xx} = \int_{-\frac{d}{2}}^{+\frac{d}{2}} y^2 . b . dy = \frac{b}{3}\left(y^3\right)_{-\frac{d}{2}}^{+\frac{d}{2}} = \frac{bd^3}{12}$$

so
$$I_{yy} = \frac{b^3 d}{12}$$

and
$$I_{DC} = I_{xx} + bd\left(\frac{d}{2}\right)^2 = \frac{bd^3}{12} + \frac{bd^3}{4} = \frac{bd^3}{3}$$

From this the moment of inertia of the usual hollow box section shown in Fig. 71 is deduced as

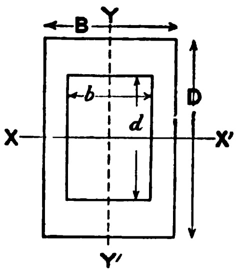


FIG. 71.

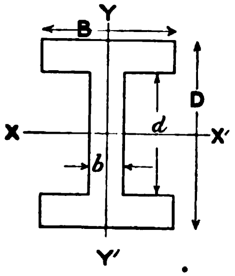


FIG. 72.

$$I_{xx} = \tfrac{1}{12}(BD^3 - bd^3)$$

and for the I section, Fig. 72,

$$I_{xx} = \tfrac{1}{12}[BD^3 - (B - b)d^3]$$

65. Moment of Inertia of a Triangle.—Let ABC be the triangle, and XX a line through the centre of gravity parallel to the base BC. Let the height of the triangle be h, and the length of the base b.

Then the area of any strip distant y from the base, and of thickness dy, is $\frac{b}{h}(h - y)dy$.

Take BC as axis of reference.

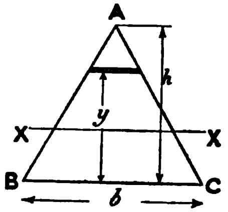


FIG. 73.

$$I_{BC} = \int_0^h \frac{b(h-y)}{h} . y^2 . dy = \frac{bh^3}{12}$$

From this
$$I_{xx} = I_{BC} - \frac{1}{2}bh\left(\frac{h}{3}\right)^2 = \frac{bh^3}{12} - \frac{bh^3}{18} = \frac{bh^3}{36}$$

Most figures can be divided up into rectangles, triangles, and segments of circles or ellipses, and the moment of inertia may be obtained without much difficulty. The moments of inertia of a number of figures about various axes are given on pp. 90 and 91.

66. Moment of Inertia of Streamline Sections.—The moment of inertia of a streamline section is one so constantly occurring in aeronautical engineering calculations that it is worth while to consider it at some length.

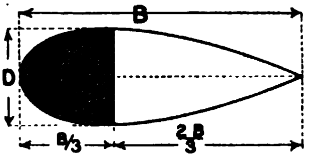


Fig. 74.

Nearly all streamline sections may be considered as made up of two portions, a head and a tail, such that the fore and aft dimension of the head is one-half that of the tail.

Let the dimensions of the streamline section be as in Fig. 74, the shaded portion representing the head.

Then the shapes of all streamlines approximate very closely to one of the following groups of outline curves:—

(1) Head elliptical, tail elliptical.
(2) Head elliptical, tail parabolic.
(3) Head semicircular, tail parabolic.
(4) A curve given by the formula

$$y=\frac{1}{2}\left(\frac{M+1}{B}\right)^{M+1}\left(\frac{x}{M}\right)^{M}(B-x)$$

These will be considered in turn, the moment of inertia being found about the major axis.

(1) It is easily shown from the moment of inertia of an ellipse that the moment of inertia of this form of streamline is

$$\frac{\pi BD^3}{64}=\frac{BD^3}{20{\cdot}4}$$

and its area is

$$\frac{\pi BD}{4}={\cdot}785\ BD$$

(2) Here the moment of inertia of the head

$$=\frac{\pi BD^3}{3\times 64}$$

and of the tail

$$=\frac{8BD^3}{3\times 105}$$

so that the total moment of inertia

$$={\cdot}04175BD^3=\frac{BD^3}{23{\cdot}95}$$

and the area

$$=\frac{\pi BD}{12}+\frac{4BD}{9}={\cdot}7BD$$

(3) The moment of inertia of the head is $\dfrac{\pi D^4}{2 \times 64}$

and of the tail $= \dfrac{8BD^3}{3 \times 105}$

and the total moment of inertia $= \cdot 0626D^4$

or, since $\dfrac{B}{3} = \dfrac{D}{2}$, $= \dfrac{BD^3}{23 \cdot 95}$

and the area $= \cdot 71BD$

(4) $$y = \frac{1}{2}\left(\frac{M+1}{B}\right)^{M+1}\left(\frac{x}{M}\right)^{M}(B - x)$$

The equation represents two curves, one for the head and one for the tail, but with different parameters. It is the equation for a streamline section used at the National Physical Laboratory. The width is 1 and the breadth B, the maximum ordinate occurring as in Fig. 75.

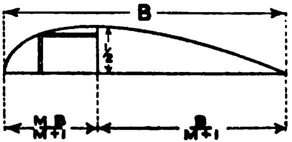


Fig. 75.

The value of the parameter M to give the least resistance, and so the best streamline form, is for the head ·45 and for the tail ·36. The value of B for the tail must be chosen corresponding to this value of M, so as to give the same value of $\dfrac{B}{M+1}$ as for the head. It should be noted that the origin of co-ordinates is not the same for the head as for the tail.

We have the moment of inertia

$$\int y_1^2 y_1 dx_1 + \int y_2^2 y_2 dx_2$$

where (x_1, y_1) (x_2, y_2) are the co-ordinates for the head and tail respectively. The integrals are simple ones and on substitution of the best values of the parameter M, we get the moment of inertia of the best streamline section to be $BD^3/25 \cdot 2$ and its area ·69BD.

It will be noticed that the areas of the last three streamline forms approximate to ·7BD, and the moments of inertia to $BD^3/24$ or $BD^3/25$, and these approximations are accurate enough for practical use in nearly all streamline forms at present in use.

The streamline shape dealt with in (1) is somewhat thicker than the usual form, and consequently the moment of inertia and the area are slightly greater than for the last three.

67. Moments of Inertia of Thin Walled Sections.—A valuable practical method of finding the moment of inertia of a hollow section is by differentiation.

For example, suppose the moment of inertia of a solid streamline section is $BD^3/25$. Then the moment of inertia of a streamline tube whose walls are of thickness $dt = \theta$ is given by $dI = \frac{D^3}{25} . dB + \frac{3BD^2}{25} . dD$, and may be written

$$\left(\frac{D^3}{25} + \frac{3BD^2}{25}\right)2\theta$$

$$= \frac{D^2}{25}(D + 3B)2\theta$$

Again, the moment of inertia of a circle about a diameter is

$$\frac{\pi D^4}{64}$$

Differentiating, the moment of inertia of a tube of thickness θ is

$$\frac{\pi D^3 \theta}{8}$$

68. Graphical Method of finding Moment of Inertia of any Section.—In the case of figures which cannot be divided into rectangles, triangles, and other simple figures, the moments of inertia of which are known, the following graphical method will be found useful. It is assumed that a planimeter is used for obtaining the necessary areas.

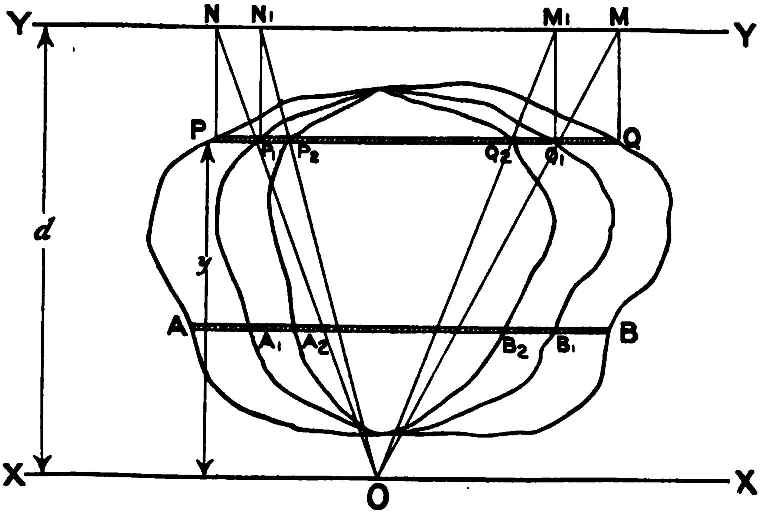


FIG. 76.

Let APQB be any irregular figure, XX the axis about which it is required to find the moment of inertia, YY any other axis parallel to XX and at a distance d from it.

Draw a number of lines PQ, AB . . . across the section parallel to XX, and let PN, QM . . . be perpendicular to YY. Join N, M . . . to any point O in XX, and let ON, OM . . . cut PQ in P_1, Q_1, and so on.

Through the points so obtained draw in the first derived area $P_1Q_1B_1A_1$. Do exactly the same work for this new area and so get the second derived area $P_2Q_2B_2A_2$. Call the original area A, and the derived areas A_1 and A_2 respectively. Let the breadth of these areas at any distance y from XX be x, x_1 and x_2 respectively.

Then the areas of the elementary strips PQ, P_1Q_1, P_2Q_2 or δA, δA_1, δA_2 are equal to xdy, x_1dy, x_2dy.

Also it is clear from the figure that

$$\delta A_1 = \frac{y}{d} . \delta A$$

therefore

$$\Sigma \delta A_1 = \Sigma \frac{y}{d} \delta A$$

$$= \frac{1}{d} \Sigma y \delta A = \frac{1}{d} \Sigma yxdy$$

i.e.

$$\int x_1 dy = \frac{1}{d} \int yxdy$$

or the area A_1 is proportional to the moment of the area A about XX.

The centre of gravity of the area A is at a distance $\bar{y}$ from XX given by

$$\bar{y} = \frac{\Sigma y \delta A}{\Sigma \delta A} = \frac{A_1}{A} . d$$

so

$$\delta A_2 = \frac{y}{d} \delta A_1 = \frac{y^2}{d^2} \delta A$$

or

$$x_2 dy = \frac{y}{d} x_1 dy = \frac{y^2}{d^2} xdy$$

and

$$\int x_2 dy = \frac{1}{d^2} \int y^2 xdy$$

or the moment of inertia of the area A about XX is proportional to A_2, and is equal to $A_2 \times d^2$

i.e.

$$I_{xx} = A_2 d^2$$

and since the distance of the centre of gravity of A from XX is $\frac{A_1}{A} d$

we get

$$I_{gg} = A_2 d^2 - A\bar{y}^2 = A_2 d^2 - A\left(\frac{A_1}{A}\right)^2 d^2$$

$$= d^2\left(A_2 - \frac{A_1^2}{A}\right)$$

which gives the moment of inertia of the figure graphically.

69. Momental Ellipse, or Ellipse of Inertia.—The principal axes of any section are those at right angles through the centre of gravity, such that $\Sigma a \text{ABAC} = 0$, using the lettering of Fig. 67. This sum is usually called the product moment.

If a section has one axis of symmetry then that is one of the principal axes, and the maximum and minimum moments of inertia of the section are about this axis, and the one at right angles, to it through the centre of gravity.

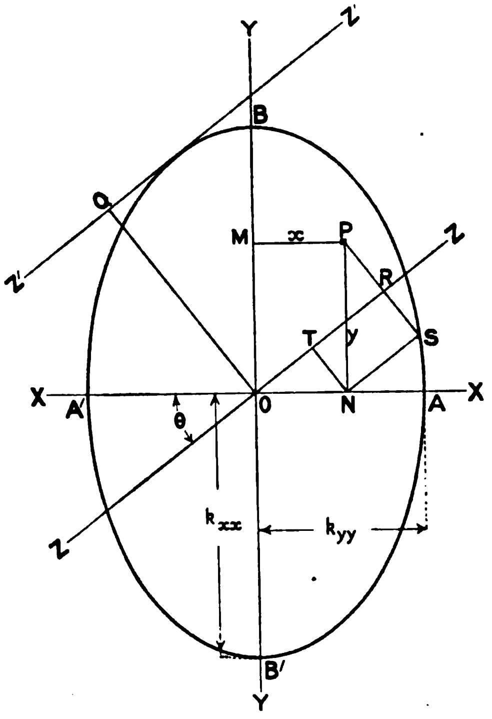


FIG. 77.

Let OX, OY in Fig. 77 be the principal axes of the section, k_{xx}, k_{yy} being the radii of gyration of the section about these axes. Let A'BAB' be an ellipse, with O as centre, such that $OA = k_{yy}$, $OB = k_{xx}$. This is the momental ellipse.

It is easy to find the radius of gyration about any line ZZ inclined at an angle θ to OX. Draw Z'Z', the tangent to the ellipse parallel to ZZ, and let OQ be the perpendicular from O on this tangent. Then OQ is the required radius of gyration.

Proof.—Let P be any point, and draw PM, PN perpendicular to BB' and AA' respectively, and let NT be the perpendicular from N on ZZ, PRS the perpendicular from P to ZZ, and let NS be drawn parallel to ZZ, meeting PS in S.

$$\begin{aligned} \text{Then}\quad I_{zz} &= \Sigma a\,.\,PR^2 = \Sigma a(PS - RS)^2 = \Sigma a(PS - NT)^2 \\ &= \Sigma a(y \cos\theta - x \sin\theta)^2 \\ &= \sin^2\theta\,.\,\Sigma a\, x^2 + \cos^2\theta \Sigma a\, y^2 - 2 \sin\theta \cos\theta \Sigma axy \end{aligned}$$

Now $\Sigma axy = 0$, $\Sigma ax^2 = I_{yy}$, $\Sigma ay^2 = I_{xx}$

therefore $\quad I_{zz} = I_{yy} \sin^2\theta + I_{xx} \cos^2\theta$

Dividing throughout by A

$$k^2_{zz} = k^2_{yy} \sin^2\theta + k^2_{xx} \cos^2\theta$$

And from the properties of the ellipse

$$OQ^2 = OA^2 \sin^2\theta + OB^2 \cos^2\theta$$

Therefore $\quad OQ = k_{zz}$

If a section has no axis of symmetry, the principal axes may be found as follows.

By graphical or other methods find the value of the product moment, and the radii of gyration about any two axes at right angles through the centre of gravity.

Then the angle θ of the principal axes to the chosen axes is given by

$$\tan 2\theta = \frac{2p^2}{k_{xx}^2 - k_{yy}^2}$$

where Ap^2 = the product moment and k_{xx}, k_{yy} are the radii of gyration already found about the chosen axes.

Alternatively find the moment of inertia about two axes XX, YY at right angles passing through the centre of gravity, and also about an axis ZZ through the same point, but inclined at an angle of 45° to XX or YY. Then the angle θ of the principal axes to XX and YY is given by

$$\tan 2\theta = \frac{I_{xx} + I_{yy} - 2I_{zz}}{I_{xx} - I_{yy}}$$

or dividing throughout by A

$$\tan 2\theta = \frac{k_{xx}^2 + k_{yy}^2 - 2k_{zz}^2}{k_{xx}^2 - k_{yy}^2}$$

70. Simpson's Rule.—The following rule for finding the area of any irregular shaped figure is known as Simpson's Rule. It is probably the most useful of all methods for finding areas of irregular figures, other than by means of a planimeter.

Divide the figure up into any odd number of strips of equal width b. Let h, h_1, h_2 . . . be the ordinates of these strips as shown in Fig. 78.

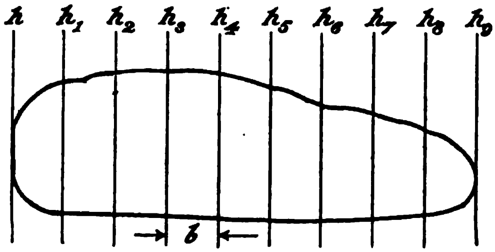


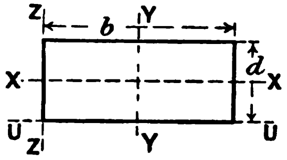
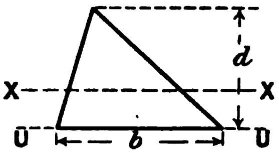
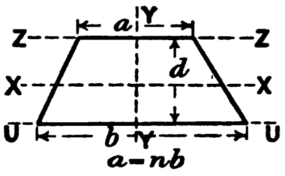
FIG. 78.

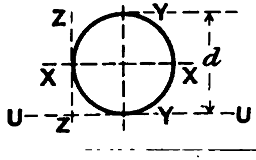
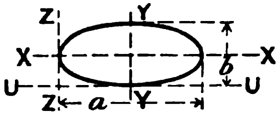
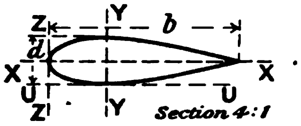
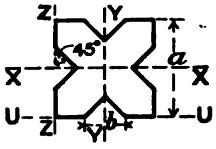
Then the area is given by

$$\frac{b}{3}(h + 4h_1 + 2h_2 + 4h_3 + 2h_4 \ldots h_9)$$

or $b/3$ (first ordinate + last ordinate + four times the sum of the even ordinates + twice the sum of the odd ordinates).

GEOMETRICAL PROPERTIES OF VARIOUS SECTIONS.

Section.	Area.	Position of centre of gravity.		Moments of inertia.			
		From UU.	From ZZ.	I_{UU}.	I_{ZZ}.	I_{XX}.	I_{YY}.
	bd	$\frac{d}{2}$	$\frac{b}{2}$	$\frac{bd^3}{3}$	$\frac{b^3d}{3}$	$\frac{bd^3}{12}$	$\frac{b^3d}{12}$
	$\frac{bd}{2}$	$\frac{d}{3}$	—	$\frac{bd^3}{12}$	—	$\frac{bd^3}{36}$	—
	$\frac{bd}{2}(n+1)$	$\frac{d}{3}\left(\frac{2n+1}{n+1}\right)$	$\frac{d}{3}\left(\frac{n+2}{n+1}\right)$	$\frac{bd^3}{12}(3n+1)$	$\frac{bd^3(n+3)}{12}$	$\frac{bd^3}{36}\left(\frac{n^2+4n+1}{n+1}\right)$	$\frac{b^3d}{48}(n^3+n^2+n+1)$

Section.	Area.	Position of centre of gravity.		Moments of inertia.			
		From UU.	From ZZ.	I_{UU}.	I_{ZZ}.	I_{XX}.	I_{YY}.
	$\frac{\pi d^2}{4}$	$\frac{d}{2}$	$\frac{d}{2}$	$\frac{5\pi d^4}{64}$	$\frac{5\pi d^4}{64}$	$\frac{\pi d^4}{64}$	$\frac{\pi d^4}{64}$
	$\frac{\pi ab}{4}$	$\frac{b}{2}$	$\frac{a}{2}$	$\frac{5\pi ab^3}{64}$	$\frac{5\pi a^3b}{64}$	$\frac{\pi ab^3}{64}$	$\frac{\pi a^3b}{64}$
 Z Y b d X X U U Z Y Section 4:1	·7bd	$\frac{d}{2}$	·40b to ·475b	—	—	$\frac{bd^3}{24}$ to $\frac{bd^3}{26}$	—
	a^2-b^2	$\frac{a}{2}$	$\frac{a}{2}$	—	—	$\frac{a^4}{12}-\frac{a^2b^2}{12}$ $-\frac{b^2(a-b)^2}{24}$	as I_{XX}

CHAPTER X

BENDING MOMENTS, SHEAR FORCES, AND STRESSES IN BEAMS

71. Shear Force and Bending Moment.—The main structural parts of an aeroplane, like those of any other engineering structure, as explained in Chapter VIII., consist of (1) struts, (2) ties, and (3) beams. In this chapter it is proposed to explain how to obtain the stresses in beams when the loading or external forces acting on them is known. The corresponding investigations for struts and ties are dealt with in Chapter XVIII.

The **shear force** at any section of a beam, usually called for short the shear, is the algebraic sum of all the external forces, including the reactions, acting on either side of the section resolved normal to the beam.

The **bending moment** at any section of a beam is the algebraic sum of the moments of all the applied forces, including the reactions, on either side of the section. Bending moment is usually written B.M. or M for convenience.

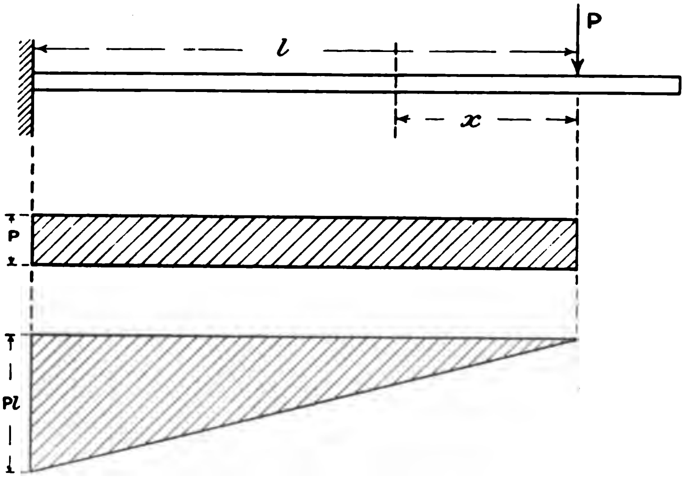


FIG. 79.

We will consider the shears and B.M.'s at various points on beams, and also obtain the shear force and B.M. diagrams for variously loaded types of beams such as are likely to occur in aeroplane structures.

These diagrams may be drawn, and the variation of the shear force and bending moment along the beam shown graphically, by plotting the value of the shear and bending moment at points on the beam against the span of the beam.

72. Cantilever with Concentrated Load.—Consider the simplest case of all, a cantilever, *i.e.* a beam encastré at one end and free at the other, with a single concentrated load on it. This case occurs, for example, in the ribs of a rudder or on the rudder post of many aeroplanes.

Let P be the concentrated load applied at a distance l from the fixed support.

Then at any distance x from the point of application of the load the shear is P, *i.e.* the shear is constant throughout, and is represented by a rectangle of height P.

The B.M. at x is Px, *i.e.* it is proportional to x, and so the B.M. diagram is a triangle of maximum height Pl (Fig. 79).

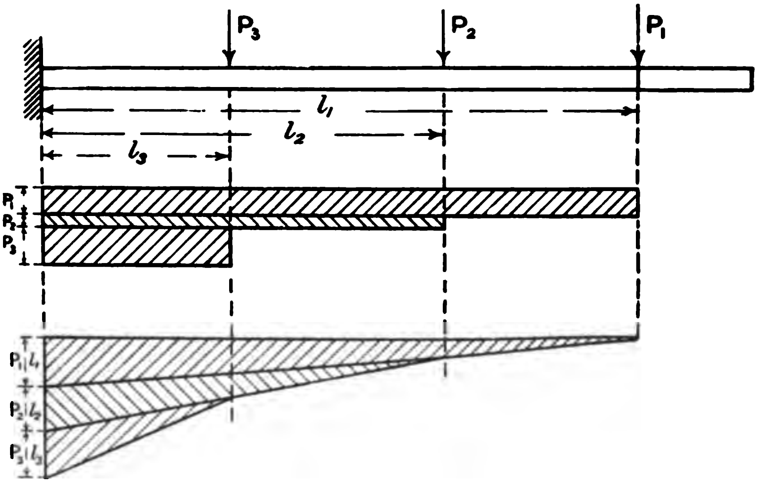


FIG. 80.

In the case of a cantilever with more than one concentrated load it is clear, from the definitions of shear and bending moment, that the shear and bending moment diagrams for each load have only to be added together. An example is given for three loads, the cross-hatching making the method of obtaining the results quite clear (Fig. 80).

73. Cantilever with Uniform Load.—This is an extremely important case in aeroplane structures. It occurs in the calculations on the strength of the overhangs of main plane spars, tail plane spars, rudder ribs, and so on. This, and the following case of a cantilever under triangular loading, should be thoroughly grasped.

Let the total uniformly loaded length of the cantilever be l inches, and the load be w lbs. per inch run.

Then at a distance x from the unloaded end the shear is wx lbs., *i.e.* the shear force diagram is a triangle whose maximum height is wl.

The bending moment at this point is $wx.\frac{x}{2}$, *i.e.* the total load up to the point multiplied by the distance of its C.G. from the point

$$=\frac{wx^2}{2}$$

So the outline curve of the bending moment diagram is proportional to x^2, *i.e.* is a parabola with its vertex at the free end of the cantilever (Fig. 81).

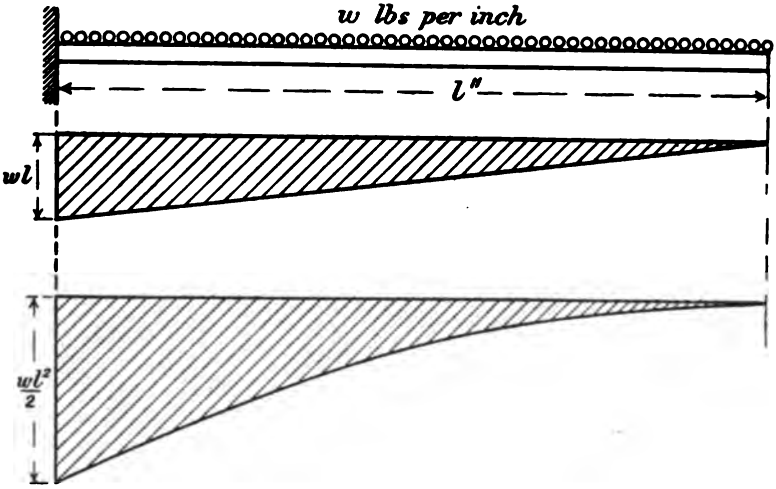


FIG. 81.

74. Cantilever with Triangular Loading.—This is an important case occurring in the calculations on the strength of control surfaces. At any point x from the free end the shearing force is

$$\frac{wx}{l}\times\frac{x}{2}=\frac{wx^2}{2l}$$

This varies as x^2, so the shear force diagram is a parabola, with its vertex at the free end.

The bending moment at any point x is

$$\frac{wx^2}{2l}\cdot\frac{x}{3}=\frac{wx^3}{6l}$$

This varies as x^3, and the shape of the curve is as shown in Fig. 82, *i.e.* a parabola of the third order.

75. Freely Supported Beam on Two Supports.—Let AB (Fig. 83) be a freely supported beam, resting on supports A, B.

Let a concentrated load P be applied at a point distant a from A and

b from B. Then by taking moments about each support in turn we obtain

$$R_A = \frac{Pb}{a+b} \text{ and } R_B = \frac{Pa}{a+b}$$

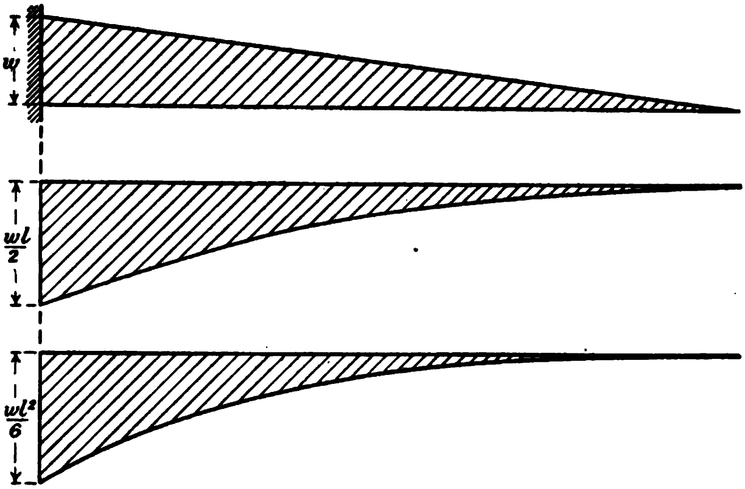


FIG. 82.

It is clear that at any point between A and the point of application of the load the shear is

$$R_A = \frac{Pb}{a+b}$$

i.e. the shear is constant.

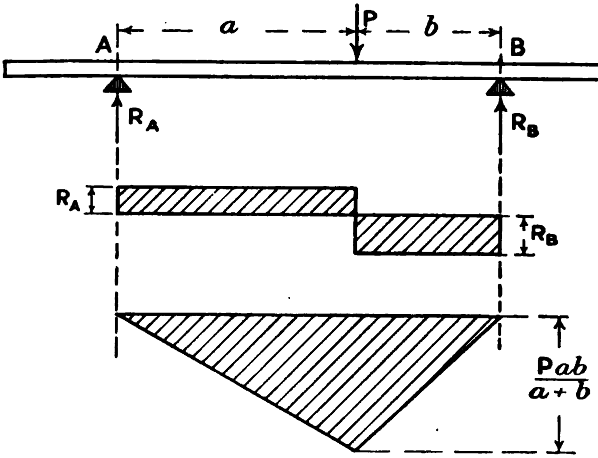


FIG. 83.

At any point between the load and B the shear is

$$\frac{Pb}{a+b} - P = -\frac{Pa}{a+b} = -R_B$$

So that the shear between the point of application of the load and B is also constant and equal to $-R_B$, *i.e.* it is of *opposite* sign to that between A and the point of application of the load.

The B.M. at any point x between A and P is

$$R_A . x = \frac{Pbx}{a+b}$$

This is proportional to x, so that the bending moment diagram is a triangle, the B.M. at P being equal to $\frac{Pab}{a+b}$. Similarly the bending moment diagram for the remaining part of the beam is a triangle, and the complete diagram is as in Fig. 83.

76. Freely Supported Beam under Uniform Loading.—Let the length of the beam AB (Fig. 84) be l inches, and the load be w lbs. per inch run.

Then $$R_A = R_B = \frac{wl}{2}$$

At any point x from A the shear force is

$$R_A - wx = w\left(\frac{l}{2} - x\right)$$

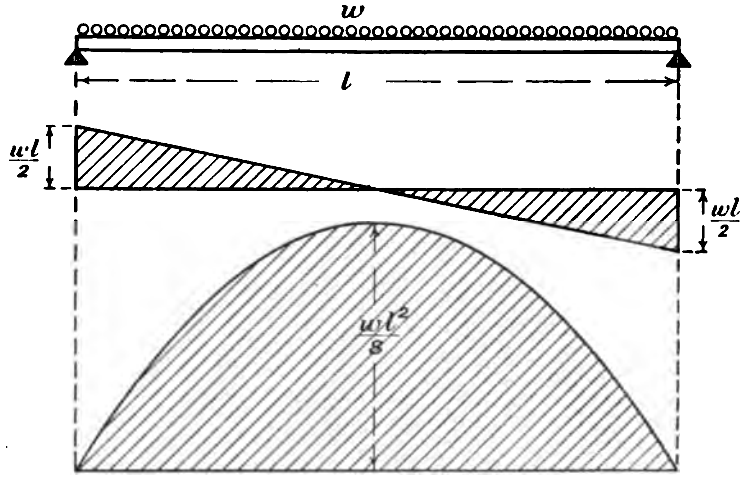


FIG. 84.

This changes sign at the centre, and the shear force diagram is shown in Fig. 84.

The B.M. at any point x from A is

$$R_A . x - wx . \frac{x}{2} = \frac{wlx}{2} - \frac{wx^2}{2}$$

So that the B.M. diagram is a parabola, the maximum B.M. occurring at the centre of the beam, and being equal to $\frac{wl^2}{8}$.

For a freely supported beam with two equal overhangs and uniformly loaded, we have the diagrams shown in Fig. 85.

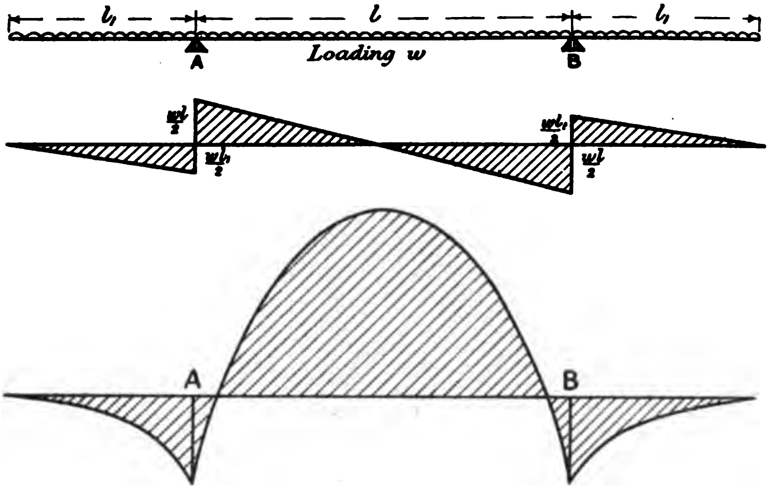


FIG. 85.

It will be noticed in the last diagram of this figure that the bending moments for the overhangs and for the spans are of opposite sign, and provided $l > 2l_1$ there will be two points in the span where the B.M. is zero. These points are called points of zero bending, points of inflection, or points of contraflexure.

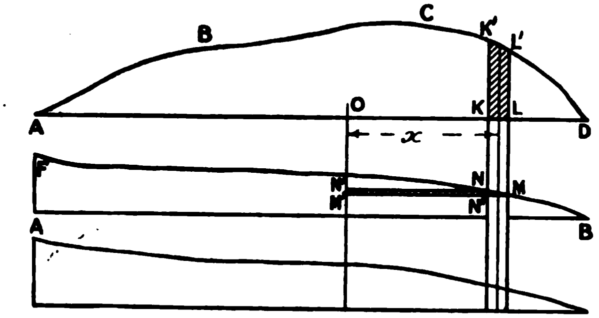


FIG. 86.

77. Relation between Load, Shear, and B.M. Diagrams.—It is important to know how the load, shear, and B.M. diagrams are connected, and how they may be deduced from one another. In Fig. 86 let AD be the points of support of a beam, ABCD representing the loading.

Consider any point O in the span, and let a short portion of the load K′L′LK be at a distance x from O. Then the shear at O due to K′L′LK is clearly equal to the area K′L′LK, and therefore the total difference of shears at O, D is equal to the area of the load curve up to the point O. In other words, the shear at any point is the integral of the loading;

i.e.

$$S = \int w dx, \quad w = \frac{dS}{dx}$$

The constant of integration depends upon the reactions at the supports, *i.e.* the method of fixing the beam.

Let BMNFA be the shear curve. Let MN be the points on the shear curve corresponding to KL on the load curve.

Then the B.M. at O due to the load K′L′LK

$$= K'L'LK . x = NN'' . x$$

since the difference of the ordinates at M and N gives the load KLL′K′, *i.e.* NMM′N′ represents the B.M. at O due to the load K′L′LK, and therefore the total B.M. at O equals the area of the shear diagram up to O. That is to say, the B.M. at any point is the integral of the shear force.

i.e.

$$\text{B.M.} = \int S dx = \int\int w dx dx, \quad S = \frac{dM}{dx}$$

suitable constants being introduced, which depend on the way the ends of the beam are fixed.

These are extremely important relations constantly occurring in the calculations on the strength of aeroplane spars, and the reader should become thoroughly familiar with them. He should plot the shear curve for various types of loading and show graphically that the shear at any point, *i.e.* the ordinate of the shear curve at that point, is equal to the area of the load curve up to that point, and similarly for the relation between the B.M. curve and the shear curve.

From the above it follows that the shear is zero where the B.M. is a maximum, a fact of some importance in the splicing of spars, especially built-up and box spars. It is also important to know the points of zero bending, for the flanges of built-up and box spars should be spliced at these points, and the webs at the points of maximum bending, *i.e.* minimum shear.

78. Theory of Simple Bending.—From the foregoing pages the shear and bending moment at any point along a beam may be found for any type of loading, and the relation must now be found between the external forces acting on the beam and the internal stresses which keep it in equilibrium.

The assumptions made in the theory of simple bending, and throughout this chapter, are

(1) The beam is stressed within the elastic limits of the material.
(2) Young's Modulus is the same for tension and compression.
(3) A plane cross section at right angles to the plane of bending always remains plane.

(4) There is no resultant thrust or pull across the cross section of the beam.

(5) The cross section of the beam is symmetrical about an axis through its centroid parallel to the plane of bending.

In Fig. 87 let ED, BC be two cross-sections of the beam close together, and after bending let them be as shown at E′D′, B′C′. They will clearly not be parallel since, due to bending, the fibres parallel and close to CD will have stretched, while those parallel and close to EB will have shortened. It is also clear that there is some plane between CD and EB where the material is neither stretched nor compressed. This plane is called the neutral plane or surface, and its line of intersection with the cross-section of the beam, the neutral axis of the section.

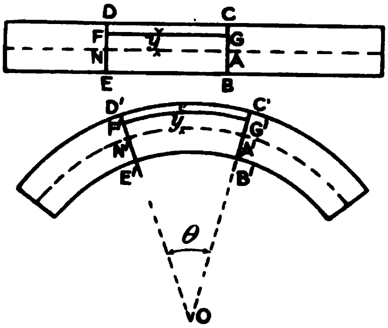


Fig. 87.

Let the sections E′D′, C′B′, inclined after bending at a small angle θ to one another, meet in a line perpendicular to the plane of the paper. Let this line intersect the plane of the paper in O. Let NA be the line in which the neutral surface cuts the plane of the paper before bending, and N′A′ be that line after bending. Let y be the distance of any layer of the material FG parallel to the neutral surface from that surface.

Then if R is the radius of curvature we have

$$\frac{\mathrm{F'G'}}{\mathrm{N'A'}} = \frac{(\mathrm{R}+y)\theta}{\mathrm{R}\theta} = \frac{\mathrm{R}+y}{\mathrm{R}}$$

The strain at the layer F′G′ is

$$e = \frac{\mathrm{F'G'} - \mathrm{FG}}{\mathrm{FG}} = \frac{\mathrm{F'G'} - \mathrm{N'A'}}{\mathrm{N'A'}} = \frac{(\mathrm{R}+y)\theta - \mathrm{R}\theta}{\mathrm{R}\theta} = \frac{y}{\mathrm{R}}$$

The longitudinal tensile stress intensity is

$$\mathrm{E}e = \mathrm{E}\frac{y}{\mathrm{R}} = p, \text{ say.}$$

This is equal to the compressive stress at the same distance below the neutral surface, *i.e.* the intensity of the direct longitudinal stress at any point in the cross-section is proportional to the distance of that point from the neutral axis, reaching a maximum at the boundary farthest from the neutral surface.

It is shown in standard works on the theory of structures that the neutral axis of a beam subjected to simple bending passes through the centroid of the section.

79. Moment of Resistance.—The longitudinal internal forces, which are tensile on one side of the neutral surface and compressive on the other, clearly form a couple, which must at any section, since the beam is in equilibrium, be equal and opposite to the bending moment at that section. This couple is called the moment of resistance.

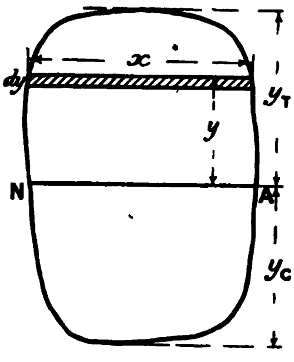


FIG. 88.

In Fig. 88 let the shaded area be any elementary strip at a distance y from the neutral axis. The total stress on the elementary area is $p.xdy$, and the moment of this stress is $p \,.\, xdy \,.\, y = pxydy$.

Total moment throughout the section is

$$M = \int pxydy = \int \frac{E \,.\, y}{R} \,.\, xydy$$

$$= \frac{E}{R}\int xy^2dy$$

$$= \frac{E}{R} \,.\, I$$

where I is the moment of inertia of the section about the neutral axis;

therefore $$\frac{M}{I} = \frac{E}{R} = \frac{p}{y}$$

This last relation is extremely important.

The maximum intensities of stress are at the outer boundaries of the surface, and if these are f_t and f_c respectively,

$$f_t = \frac{My_t}{I} \quad \text{and} \quad f_c = \frac{My_c}{I}$$

where y_t, y_c are the distances of the most highly stressed tensile and compressive fibres from the neutral axis.

The quantity I/y, where y is the distance of the neutral axis from the most highly stressed fibre, is called the modulus of the section, and is usually denoted by Z, so that we have the relation $f = M/Z$. There are two moduli for every section which is not symmetrical about the neutral axis.

80. Combined Bending and Direct Stress.—In aeroplane spars there is not only the stress due to bending caused by the lift reactions, but there is usually also a direct stress due to the end load caused by the type of bracing used. The resultant stress across the section will be obtained by adding together that due to bending and that due to the end load, which may be either tensile or compressive.

Let the end load be P, and to fix ideas let it be a thrust. Then if A is the area of the cross-section of the spar, the stress due to P is P/A, and is uniformly distributed over the section. The maximum compressive stress due to the bending of the spar is My_c/I, as shown in the previous paragraph, and the maximum tensile stress due to bending is My_t/I, where M is the bending moment at the section corrected for the buckling effect of the end load as explained in para. 92.

$$\text{Therefore total maximum compressive stress} = \frac{My_c}{I} + \frac{P}{A}$$

$$\text{and the total maximum tensile stress} = \frac{My_t}{I} - \frac{P}{A}$$

If P is a tensile load, then the above equations hold with the sign of P changed.

81. Distribution of Shear Stress.—It has been shown in Chapter VII. that every shear stress has a complementary shear stress of equal intensity at right angles to it.

In an aeroplane spar there is a shear at right angles to the longitudinal axis of the spar caused by the external forces, and therefore a corresponding shear along the axis of the spar of equal intensity. This latter shear stress is extremely important, and must be taken into account when designing the thickness of the webs of spars.

The distribution and value of the shear stress at any section of a beam may be found as follows.

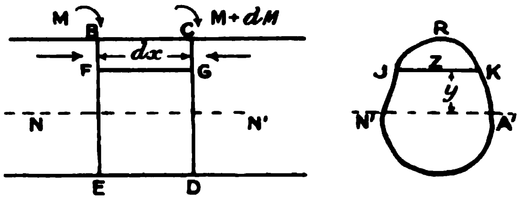


FIG. 89.

In Fig. 89 let BE, CD be two cross-sections of the beam at a small distance dx from one another, and let the bending moments at these sections be M and $M + dM$ respectively. Let the breadth of the section at any height y be $z = JK$.

Then the longitudinal stress intensity at height y above the neutral axis NN′ is, as already shown,

$$p = \frac{My}{I}$$

where I is the moment of inertia of the section about N′A′.

Now BFGC is in equilibrium.

The longitudinal thrust on any element of cross-section at BF is $p\,.\,z\,.\,dy$, where $z\,.\,dy$ is the area of this element.

$$= \frac{My}{I}\,.\,z\,.\,dy$$

so the thrust at CG on an element at the same height is

$$\frac{(M + dM)y}{I}\,.\,z\,.\,dy$$

Therefore the excess of thrust on the element of area at CG over that at BF is the difference of the above quantities.

i.e.

$$\frac{dM.y}{I}.z.dy$$

and the total difference of thrusts on the areas CG, BF is

$$\int_{y}^{y_0}\frac{dMy}{I}.z.dy$$

where $y_0 = \mathrm{BN}$.

But since BFGC is in equilibrium the excess of thrust must be balanced by the longitudinal shearing force across the surface FG. Let q be the intensity of the shear stress across FG.

The shear force across FG is

$$q.z.dx = \int_{y}^{y_0}\frac{dMy}{I}.z.dy$$

$$= \frac{dM}{I}\int_{y}^{y_0}yzdy$$

therefore

$$q = \frac{dM}{dx}\cdot\frac{1}{Iz}\int_{y}^{y_0}yzdy = \frac{S}{Iz}\int_{y}^{y_0}yzdy$$

where S is the total shear force on the cross-section of the beam.

Now

$$\int_{y}^{y_0}yzdy$$

is the moment of the area JRK about N′A′ and is equal to AY, where A is the area of JRK and Y the distance of its centroid from the neutral axis.

The shear stress is a maximum at the neutral axis and is equal to $\frac{S.AY}{Ib}$, where b is the breadth of the section at the neutral axis and AY is the moment of the area of the section above the neutral axis.

The distribution of shear stress is given for the two sections most commonly met with in aeroplane spars, *i.e.* rectangular and I sections.

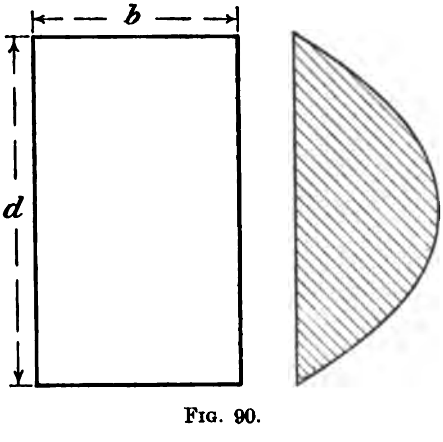


FIG. 90.

Rectangular Section.—

$$q=\frac{S}{I}\int_{y}^{\frac{d}{2}} y\,.\,dy$$

since $z=b=$ const.

$$=\frac{S}{2I}\left(\frac{d^2}{4}-y^2\right)$$

$$=\frac{6S}{bd^3}\left(\frac{d^2}{4}-y^2\right)$$

since $$I=\frac{bd^3}{12}$$

This is a parabola, the maximum shear stress being $\frac{3S}{2bd}$ at the neutral axis (Fig. 90).

I *Section.*—Let dimensions be as in Fig. 91. Then the shear stress intensity at any height y above the neutral axis is

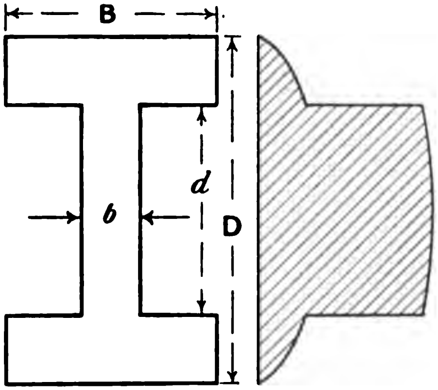


FIG 91.

$\frac{S}{Ib}\times$ moment of area above height y about the neutral axis.

The shear stress diagram is shown in Fig. 91.

On the inner edge of the flange

$$q=\frac{S}{8I}(D^2-d^2)$$

and just inside the web

$$q=\frac{S}{8I}(D^2-d^2)\frac{B}{b}$$

and at the neutral axis the maximum shear stress is

$$\frac{S}{I}\left(\frac{D^2-d^2}{8}\cdot\frac{B}{b}+\frac{d^2}{8}\right)$$

82. Deflection of Beams.—It has been already shown that for a beam of uniform section we have the relation

$$\frac{E}{R}=\frac{M}{I} \quad \text{therefore} \frac{1}{R}=\frac{M}{EI}$$

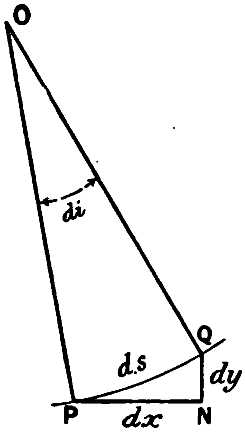


FIG. 92.

If in Fig. 92 PQ$=ds$ is a small length of the beam, di the angle between the tangents to the beam at each end of this small length, and dx, dy have their usual significance

$$\frac{1}{R}=\frac{di}{ds}=\frac{di}{dx}$$

when i is very small

$$=\frac{d}{dx}\left(\frac{dy}{dx}\right)$$

since $\frac{dy}{dx}=\tan i=i$

when i is very small

$$\frac{1}{R}=\frac{d^2y}{dx^2}$$

therefore $\frac{dy}{dx}=\text{slope of beam}=\int\frac{1}{R}dx=\int\frac{M}{EI}dx$

between suitable limits;

and $y=\text{deflection}=\int\int\frac{M}{EI}dxdx$

between suitable limits.

From this and the previous articles the following relations are obtained—

$$w=\frac{dS}{dx}=\frac{d^2M}{dx^2}=EI\frac{d^4y}{dx^4} \quad . \quad . \quad . \quad . \quad (1)$$

$$S=\frac{dM}{dx}=EI\frac{d^3y}{dx^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$M=EI\frac{d^2y}{dx^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\text{slope}=i=\int\frac{Mdx}{EI} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$\text{deflection}=y=\int\int\frac{Mdxdx}{EI} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

These five relations are exceedingly important. From them, for example, if the loading on an aeroplane spar is known, the shear force, bending moment, slope and deflection can be obtained by successive integration, the proper constant of integration being added at each step. Alternatively, if the bending moment is given for every point along the spar, the loading, shear, deflection and slope may be deduced.

The deflections of variously loaded beams will now be considered.

83. Uniformly Loaded Cantilever.—This case is important, for it is necessary that the designer should have the wing tips of his aeroplane as rigid as possible. With a knowledge of the deflection of a uniformly loaded cantilever he can deduce approximately the deflections of his spar overhangs, making due allowance for the stiffening effect of fabric and secondary members.

Let l = length of cantilever
w = load per unit run.

Then the B.M. at x from the free end is

$$M = \frac{wx^2}{2} = EI\frac{d^2y}{dx^2}$$

therefore
$$\frac{dy}{dx} = \frac{1}{EI}\int\frac{wx^2}{2} + \text{const.}$$
$$= \frac{wx^3}{6EI} + \text{const.}$$

When $x = l \quad \frac{dy}{dx} = 0$

therefore constant of integration $= -\frac{wl^3}{6EI}$

therefore
$$\frac{dy}{dx} = \frac{wx^3}{6EI} - \frac{wl^3}{6EI}$$
$$\text{deflection} = y = \frac{wx^4}{24EI} - \frac{wl^3x}{6EI} + \text{const.}$$

when $x = l$, deflection $= 0$

therefore
$$\text{constant} = \frac{wl^4}{6EI} - \frac{wl^4}{24EI} = \frac{wl^4}{8EI}$$

therefore
$$\text{deflection} = \frac{wx^4}{24EI} - \frac{wl^3x}{6EI} + \frac{wl^4}{8EI}$$

The maximum deflection is when $x = 0$ and equals

$$\frac{wl^4}{8EI}$$

84. Uniformly Loaded Beam Simply Supported.—Let ACB be the deflected beam of length l and loaded uniformly w per unit length. Take origin at A.

Then, as in para. 76, B.M. at any point distant x from A is

$$\frac{w}{2}(lx - x^2)$$

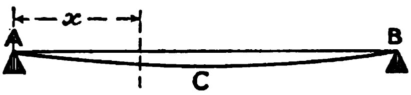


FIG. 93.

Therefore

$$\text{slope} = \frac{dy}{dx} = \frac{w}{2\text{EI}}\int(lx - x^2)dx$$

$$= \frac{w}{2\text{EI}}\left(\frac{lx^2}{2} - \frac{x^3}{3}\right) + \text{const.}$$

$$\frac{dy}{dx} = 0 \text{ when } x = \frac{l}{2}$$

Therefore

$$\text{constant} = -\frac{w}{2\text{EI}}\left(\frac{l^3}{8} - \frac{l^3}{24}\right)$$

$$= -\frac{wl^3}{24\text{EI}}$$

and the deflection

$$= \int\frac{w}{2\text{EI}}\left(\frac{lx^2}{2} - \frac{x^3}{3} - \frac{l^3}{12}\right)dx + \text{const.}$$

$$= \frac{w}{2\text{EI}}\left(\frac{lx^3}{6} - \frac{x^4}{12} - \frac{xl^3}{12}\right) + \text{const.}$$

$$\text{deflection} = 0 \text{ when } x = 0 \quad \text{therefore constant} = 0$$

The deflection is clearly a maximum when $x = \frac{l}{2}$

$$\text{and} = \frac{wl^4}{2\text{EI}}\left(\frac{1}{48} - \frac{1}{192} - \frac{1}{24}\right)$$

$$= -\frac{5wl^4}{384}$$

This result could have been obtained by using the equation $\text{EI}\frac{d^4y}{dx^4} = w$ and integrating four times, adding the proper constant each time.

A table of standard deflections will be found on p. 127, at the end of the chapter.

85. Built-in or Encastré Beam.—A built-in or encastré beam is one such that the slope of the beam at one or more of the supports is completely fixed. In that case the deflections, B.M.'s, and so on, may be calculated, introducing the condition that the slope of the beams is fixed at the point of support. This was assumed above, for example, when finding the deflection of a cantilever under uniform load.

Consider the case of a beam on two supports, each end being fixed so that the slope of the beam at these is horizontal, and take the origin at one support.

Then

$$\text{EI}\frac{d^4y}{dx^4} = w$$

$$\text{EI}\frac{d^3y}{dx^3} = wx + \text{A}$$

$$\text{EI}\frac{d^2y}{dx^2} = \frac{wx^2}{2} + \text{A}x + \text{B}$$

$$\text{EI}\frac{dy}{dx} = \frac{wx^3}{6} + \frac{\text{A}x^2}{2} + \text{B}x + \text{C}$$

$$\text{EI}\,y = \frac{wx^4}{24} + \frac{\text{A}x^3}{6} + \frac{\text{B}x^2}{2} + \text{C}x + \text{D}$$

where A, B, C, D are constants of integration.

Now when $x = 0 \quad y = 0$

therefore $\mathrm{D} = 0$

when $x = l \quad y = 0$

therefore
$$\frac{wl^4}{24} + \frac{\mathrm{A}l^3}{6} + \frac{\mathrm{B}l^2}{2} + \mathrm{C}l = 0 \quad \ldots\ldots \quad (1)$$

when $x = 0 \quad \dfrac{dy}{dx} = 0$

therefore $\mathrm{C} = 0$

when $x = l \quad \dfrac{dy}{dx} = 0$

therefore
$$\frac{wl^3}{6} + \frac{\mathrm{A}l^2}{2} + \mathrm{B}l = 0 \quad \ldots\ldots \quad (2)$$

From (1) and (2) we get $\mathrm{A} = -\dfrac{wl}{2}$

$$\mathrm{B} = +\frac{wl^2}{12}$$

therefore
$$\mathrm{EI}\frac{dy}{dx} = \frac{wx^3}{6} - \frac{wlx^2}{4} + \frac{wl^2x}{12}$$
$$= 0 \text{ when } x = \frac{l}{2},\ 0 \text{ or } l$$

therefore
$$\mathrm{EI}y = \frac{wx^4}{24} - \frac{wlx^3}{12} + \frac{wl^2x^2}{24}$$

This is a maximum when $x = \dfrac{l}{2}$.

Then
$$\mathrm{EI}y = \frac{wl^4}{384} - \frac{wl^4}{96} + \frac{wl^4}{96}$$

and
$$y = \frac{wl^4}{384\mathrm{EI}}$$

one-fifth of the deflection of the freely supported beam.

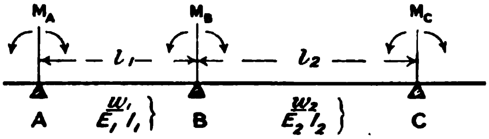


FIG. 94.

86. Theorem of Three Moments.—Let A, B, C (Fig. 94) be three consecutive supports of a continuous beam, of spans l_1, l_2, carrying distributed loads w_1, w_2 per unit length respectively. Let Young's moduli for the two spans be E_1 and E_2, and the moments of inertia of the sections of the spars be I_1 and I_2 respectively. Then the bending moment at any point in AB or BC is the algebraic sum of the bending moments of a pin-jointed beam and those caused by the fixing moments at the supports; *i.e.* $M = M_1 + M_2$, where M_1 is the moment for the pin-jointed beam, and M_2 is the moment as a result of the fixing moments at the support. As a rule, M_1 and M_2 are of opposite signs.

Consider the span BC. Take B as origin, and BC for the axis of x.

Then $$M_1 = \tfrac{1}{2}w\,(l_2x - x^2)$$

If M_A, M_B, M_C are the fixing moments at the supports we have

$$M = E_2I_2\frac{d^2y}{dx^2} = \frac{w}{2}l_2x - \frac{w}{2}x^2 - M_B - (M_C - M_B)\frac{x}{l_2}$$

for the span BC.

Integrating,

$$E_2I_2\frac{dy}{dx} = \frac{w_2l_2x^2}{4} - \frac{w_2x^3}{6} - M_Bx - \frac{(M_C - M_B)x^2}{2l_2} - E_2I_2i_B$$

where i_B is the value of $\frac{dy}{dx}$ at B, *i.e.* where $x = 0$.

Integrating again

$$E_2I_2y = \frac{w_2l_2x^3}{12} - \frac{w_2x^4}{24} - \frac{M_Bx^2}{2} - \frac{(M_C - M_B)x^3}{6l_2} - E_2I_2i_Bx$$

the constant of integration being 0 since $y = 0$ when $x = 0$.

When $x = l_2$ $y = 0$

therefore $$0 = \frac{w_2l_2^3}{12} - \frac{w_2l_2^3}{24} - \frac{M_Bl_2}{2} - \frac{(M_C - M_B)l_2}{6} - E_2I_2i_B$$

or $$6E_2I_2i_B = \frac{w_2l_2^3}{4} - 2M_Bl_2 - M_Cl_2 \quad . . . . (1)$$

So, by considering the span BA, taking B again as origin, but in this case taking x as positive when measured to the left

$$-6E_1I_1i_B = \frac{w_1l_1^3}{4} - 2M_Bl_1 - M_Al_1 \quad . . . . (2)$$

Dividing (1) by E_2I_2 and (2) by E_1I_1 and adding we get

$$\frac{M_Al_1}{E_1I_1} + \frac{2M_Bl_1}{E_1I_1} + \frac{2M_Bl_2}{E_2I_2} + \frac{M_Cl_2}{E_2I_2} = \frac{w_1l_1^3}{4E_1I_1} + \frac{w_2l_2^3}{4E_2I_2}$$

As a general rule $E_1 = E_2$; $I_1 = I_2$; and $w_1 = w_2 = w$, say, and this reduces to the well-known form

$$M_Al_1 + 2M_B(l_1 + l_2) + M_Cl_2 = \frac{w(l_1^3 + l_2^3)}{4}$$

By successive application of the above formula the fixing moments for a continuous beam over any number of supports may be found. For if there are N supports, say, then N − 2 equations can be written down by the use of the theorem of three moments, and the two remaining equations come from known conditions of the problem, *i.e.* the end conditions must be given, or the moment at one or more supports known from other considerations.

When the fixing moments at the supports are known, the reactions at the supports may be calculated by taking the moments of internal and external forces about the various supports, or by writing down the shears on each side of the support and adding them algebraically together, so getting the total reaction.

The reaction at B, for example, is

the shear to the left of B + the shear to the right of B

$$= \frac{wl_2}{2} + \frac{M_B - M_C}{l_2} + \frac{wl_1}{2} + \frac{M_B - M_A}{l_1}$$

and so on for the other supports.

As an example take the top front spar of the biplane shown in Fig. 95.

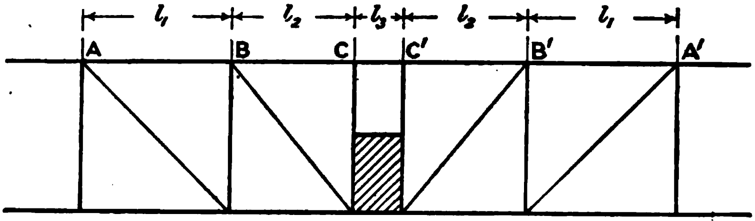


FIG 95.

We have

$$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 = \frac{w}{4}(l_1^3 + l_2^3) \quad . \quad . \quad . \quad (1)$$

neglecting the effect of end loads in the spars.

Also taking the span BCC′, since $M_C = M_C'$ by symmetry

$$M_B l_2 + M_C(2l_2 + 3l_3) = \frac{w}{4}(l_2^3 + l_3^3) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and since M_A is known from the conditions of the overhang loading, from (1) and (2) we immediately get M_B and M_C, and so all the fixing moments on the spar are known.

Or, again, C might have been a pin-joint, in which case $M_C = 0$, and we have the equation

$$M_A l_1 + 2M_B(l_1 + l_2) = \frac{w}{4}(l_1^3 + l_2^3)$$

which gives the value of M_B.

87. General Form of Equation of Three Moments.—It is necessary that the aeroplane designer should be able to calculate the strength of a continuous beam under any form of loading, for he will find many members of his aeroplane acting as beams, and loaded in a variety of ways. The rear spar of a tail plane, for example, not only has a continuous load on it, but also a series of concentrated loads from the elevator hinges.

Let ADB, BEC (Fig. 96) be the bending moment diagram for the spans AB, BC of a continuous beam, considered as pin-jointed at A, B, and C. Let AF, BG, HC represent the fixing moments at A, B, and C respectively, the shaded area representing the complete bending moment diagram for the continuous beam. Let the area of ADB be A_1, and the distance of its centre of gravity from A be x_1. So let the area BEC be A_2, and the distance of its centre of gravity from C be x_2.

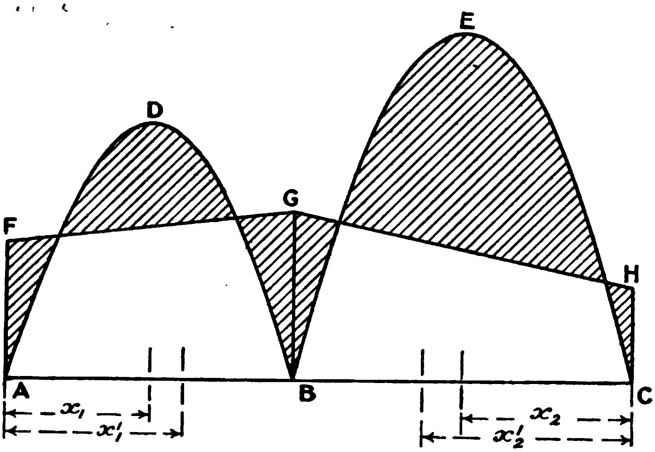


FIG. 96.

These areas will be negative quantities for downward loading, bending moments being reckoned positive which produce convexity upwards.

Let the distances of the centres of gravity of the areas AFGB $= A_1'$, and BGHC $= A_2'$ be x_1', x_2' from A and C respectively.

Taking A as origin, and calling x positive, measured from A towards B we have

$$x\frac{d^2y}{dx^2}=\frac{Mx}{EI}$$

Integrating between $x=0$ and $x=l_1$,

$$\left(x\frac{dy}{dx}-y\right)_0^{l_1}=l_1i_B=\frac{1}{EI}\int_0^{l_1}(M_1+M_2)x\,dx=\frac{1}{EI}(A_1x_1+A_1'x_1') \quad . \ (1)$$

So, taking C as origin, and calling x positive, measured from C towards B

$$\left(x\frac{dy}{dx}-y\right)_0^{l_2}=l_2i_B=\frac{1}{EI}(A_2x_2+A_2'x_2') \quad . \ . \ . \ . \ (2)$$

But since the slope at B is the same, from (1) and (2) we get

$$\frac{A_1x_1+A_1'x_1'}{l_1}=-\frac{A_2x_2+A_2'x_2'}{l_2} \quad . \ . \ . \ . \ . \ (3)$$

the sign of the right-hand side being changed on account of the change in the sign of x.

By taking moments about A we get

$$A_1'x_1' = \frac{l_1^2}{6}(M_A + 2M_B) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and similarly

$$A_2'x_2' = \frac{l_2^2}{6}(M_C + 2M_B) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Substituting from (4) and (5) in (3) we get

$$\frac{6A_1x_1}{l_1} + \frac{6A_2x_2}{l_2} + M_Al_1 + 2M_B(l_1 + l_2) + M_Cl_2 = 0$$

This is the general form of the equation of three moments.

The tacit assumption was made in the above proof that all the supports were collinear both before and after loading. If the supports are not in the same straight line after loading let B fall a distance y_a below A, and y_c below C. Then it can be shown that the general equation becomes

$$\frac{6A_1x_1}{l_1} + \frac{6A_2x_2}{l_2} + M_Al_1 + 2M_B(l_1 + l_2) + M_Cl_2 + 6EI\left(\frac{y_a}{l_1} + \frac{y_c}{l_2}\right) = 0$$

As a particular application of the general theorem consider a beam under a triangular distribution of loading. Let ABCD (Fig. 97) be the beam, w_a, w_b, w_c the intensities of loading at the supports A, B, C.

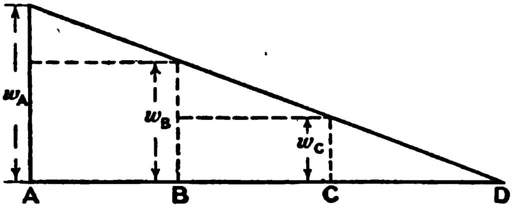


FIG. 97.

The B.M. due to the load on BC can be divided into two parts—

(1) Due to a uniform distributed load w_c.

(2) Due to a distributed load varying from 0 to $(w_b - w_c)$.

The moment of the B.M. diagram about C is the sum of the moments of (1) and (2).

Moment of (1) $\quad \frac{2}{3} \cdot l_2 \cdot \frac{w_cl_2^2}{8} \cdot \frac{l_2}{2} = \frac{w_cl_2^4}{24}$

To find the moment of (2) let $(w_b - w_c) = w_0$

Then the B.M. at a distance x from C is

$$\frac{w_0}{6}\left(\frac{x^3}{l_2} - l_2x\right)$$

The moment of the B.M. diagram $= \frac{w_0}{6}\int_0^{l_2}\left(\frac{x^3}{l_2} - l_2x\right)x\,dx = \frac{w_0l_2^4}{45}$

Therefore total moment of moment

$$= \frac{w_cl_2^4}{24} + \frac{(w_b - w_c)l_2^4}{45} = \frac{l_2^4}{360}(7w_c + 8w_b)$$

Bay AB.—To find the moment of the B.M. diagram due to (2).

Area of the B.M. diagram

$$= \frac{w_0'}{6}\int_0^{l_1}\left(\frac{x^3}{l_1} - l_1x\right)dx$$

$$= \frac{w_0'l_1^3}{24}$$

where $w_0' = w_a - w_b$.

The moment about B $= \frac{w_0'l_1^4}{45}$

Therefore the distance of the C.G. from B $= \frac{w_0'l_1^4}{45} \cdot \frac{24}{w_0'l_1^3} = \frac{24l_1}{45}$

Therefore the distance of the C.G. from A $= \frac{21}{45}l_1 = \frac{7l_1}{15}$

The moment about A $= \frac{7l_1}{15} \cdot \frac{w_0'l_1^3}{24}$

$$= \frac{7w'_0l_1^4}{360}$$

The total moment of the area about A

$$= \frac{7w_0'l_1^4}{360} + \frac{w_bl_1^4}{24}$$

$$= \frac{l_1^4}{360}(7w_a + 8w_b)$$

Substituting in the general equation we obtain the result

$$\frac{7}{60}w_al_1^3 + \frac{8}{60}w_b(l_1^3 + l_2^3) + \frac{7}{60}w_cl_2^3 + M_Al_1 + 2M_B(l_1 + l_2) + M_Cl_2 = 0$$

88. Offset Moments.—In aeroplane construction it frequently happens that the actual points taken on a spar for the points of support are not the true ones, but are offset some distance from their actual positions. For example, in Fig. 98 AB represents a portion of the top spar of a biplane, C is the interplane strut, DE a flying wire, EF, E′F′ bolts through the spar to which are fastened the wiring lug and strut socket EE′.

The practical point of support of the spar is at E, which takes the direct pull of the wire; but in calculations it is usual to take the point of support at H, the intersection of the neutral axis of the spar with the centre line of the strut. In that case the wire has a vertical offset pull measured by the distance KG, and a horizontal offset pull, measured by the distance HG.

Or the offset may be looked at in another way, as in Fig. 99. From this point of view, if the line of the pull of the wire cuts the neutral axis NA of the spar in K, and the centre line of the strut B cuts it in L, then the point of support may be taken as at K, and the strut load is offset at L.

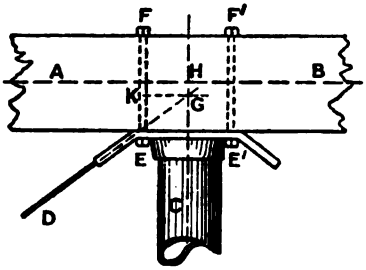


FIG. 98.

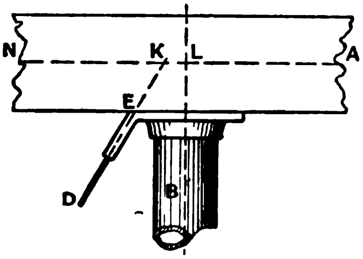


FIG. 99.

Both in analytical and graphical methods it will be found that the most convenient way of considering offsets is to suppose the line of the wire produced to meet the axis of the corresponding strut, and to measure the distance of this point of intersection from the neutral axis of the spar, and it is the method which will be adopted in the following proofs of the effects of offsets. But whichever method is adopted, the bending moment at any particular point on the spar, except between E and K, should be the same in all cases, and it will serve as a useful check on the results of any set of calculations to obtain the same results, treating the points of support in both the ways suggested above.

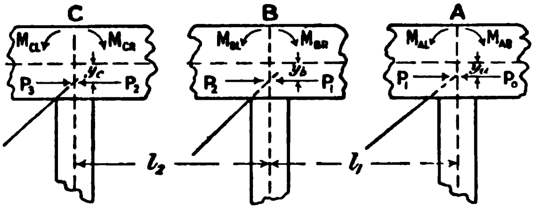


FIG. 100.

Let ABC (Fig. 100) be any three consecutive points of support of an aeroplane top spar. Let the flying wires cut the axes of the corresponding centre lines of the interplane struts at points which are distant y_a, y_b, y_c from the neutral axis of the spar; y is counted positive when measured below the neutral axis of the spar, and negative above. Let P_0, P_1, P_2 be the end loads in the spars due to the pull of the wires, P_0 being the end load to the right of A in the figure.

Then due to the offset pulls of the wires the fixing moments each side of the supports are unequal. Let these fixing moments to the right and left of each support be M_{AR}, M_{AL} : M_{BR}, M_{BL} : M_{CR}, M_{CL} respectively.

It is easy to show that the equation of three moments becomes

$$M_{AL}l_1 + 2M_{BR}l_1 + 2M_{BL}l_2 + M_{CR}l_2 = \frac{w}{4}(l_1^3 + l_2^3)$$

for the spars, where w, as usual, is the loading per unit length on the spar. If there are N supports the repeated application of the above equation gives us N — 2 equations. There are, however, 2N unknowns, and the additional equations are found by considering the equilibrium of each point of support. At A we have, taking moments about the intersection of strut and spar axes for a small vertical slice of the spar containing the point—

$$M_{AR} + P_0 y_a = M_{AL} + P_1 y_a$$

or

$$M_{AR} - M_{AL} = y_a(P_1 - P_0)$$

so

$$M_{BR} - M_{BL} = y_b(P_2 - P_1)$$

and so on, giving further N equations. The remaining two are found from the end conditions. M_{AR}, for example, would be known if A were the outermost point of support of the spar, and M_{CL} would be found by symmetry if C were the last support before the centre line of the machine.

It should be noted here that the values of P_0, P_1, P_2 . . . may be found to a near enough approximation in the first instance by applying the theorem of three moments, neglecting offsets, and so finding the reactions and end loads. They are the values of the end loads due to the tensions in the flying wires only, and do not include drag loads. If the end loads then found by taking account of offsets are widely different from the first approximation, another must be made, taking the new offset end loads for the next approximation. A second approximation always comes near enough for practical purposes. If P_0, P_1 . . . are tensile loads the above equations are modified by writing $-P_0$ for P_0, $-P_1$ for P_1, and so on. The general effect of the offset wires as shown in Fig. 100 is to decrease the largest fibre stresses just inside the support, and increase the smallest, *i.e.* they tend to equalize them. If the wires had cut above the neutral axis they would have tended to increase the largest fibre stresses and decrease the smallest.

89. Graphical Method.—It will be found useful to be able to find the offset moments graphically, and this can be done as follows. Fig. 101 represents the bending moment diagram over the supports A, B, C, D, . . .

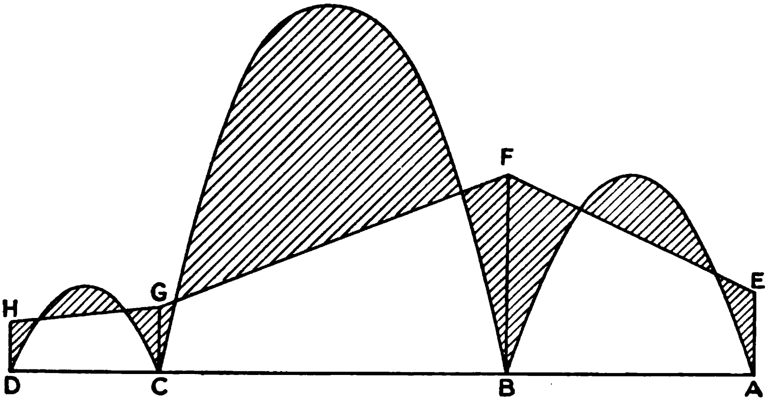


Fig. 101.

of an aeroplane spar. This diagram is found in the usual way, as shown in para. 86, the fixing moments at A, B, C, D . . . being found by the theorem of three moments, neglecting offsets. The free bending moment diagram is then drawn, *i.e.* the moments in each span, assuming pin-joints at the supports, and the two combined. The shaded area shows the resulting bending moment diagram.

Using the notation on p. 113, the offset moment, *i.e.* the difference of the moments to the right and left of any support, is of the form $(P_2 - P_1)y$. Call these offset moments m_A, m_B, m_C, m_D. Then the bending moment diagram for these offset moments, assuming each bay is pin-jointed, and the offset moment applied just to the left of each support, is as in Fig. 102.

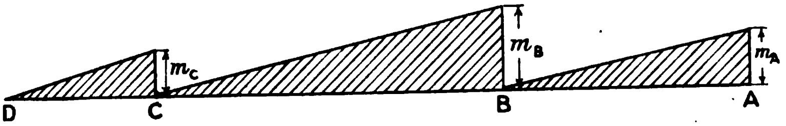


FIG. 102.

Since, however, the spar is continuous this diagram is modified. From the generalized equation of three moments we have

$$\frac{6A_1x_1}{l_1} + \frac{6A_2x_2}{l_2} + M_Al_1 + 2M_B(l_1 + l_2) + M_Cl_2 = 0$$

adopting the notation used on p. 110. Applying this general equation to the above figure we get

$$M_Al_1 + 2M_B(l_1 + l_2) + M_Cl_2 = m_Al_1 + 2m_Bl_2$$

where M_A, M_B . . . are the fixing moments just to the right of each support due to the continuity of the spar, for offset moments only. From the end conditions, and the continued application of the equation, we can find all the fixing moments due to the offset couples. This gives the bending moment diagram, Fig. 103.

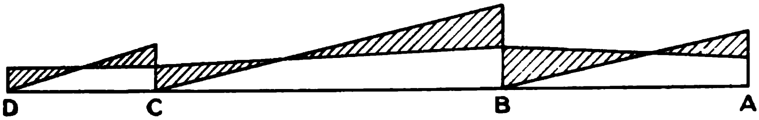


FIG. 103.

This diagram must now be combined with that showing the bending moments obtained for the spar when offsets were neglected, and we get finally the diagram shown in Fig. 104.

This graphical method must be used carefully, or the wrong sign may be taken in adding the two diagrams together. The simplest way of ensuring that the correct signs have been taken is as follows.

Consider the effect of the offset at the outer strut position, A say. If the line of the flying wire cuts the neutral axis of the spar on the outer side of the strut, the effect of this offset is, clearly, to increase the bending moment in the centre of the outer bay. This means that the fixing moment at the support must be decreased, *i.e.* the offset moment must be plotted below E. The rest of the diagram follows naturally.

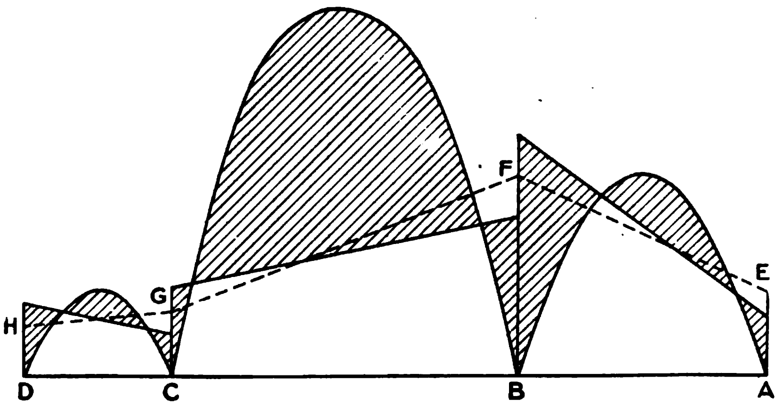


FIG. 104.

90. Continuous Beam with End Loads.—The above methods of finding the bending moments in the spars of an aeroplane are accurate enough for all preliminary design purposes, and serve as a good check upon the first rough out of the spar sizes. But these methods will not be found satisfactory in all cases, for the spars of an aeroplane are not only subjected to bending, but also to heavy end loads in some conditions of flight, and these end loads seriously affect the fixing moments at the points of support, as well as those in the spans.

The complete solution of the problem of a continuous beam, under a distributed load and end loads, was first given by Messrs. H. Booth and H. Bolas in 1915.[1] The solution was given in too cumbersome a form for general use, but the following year Mr. Arthur Berry published a simplification of the method due to Messrs. Booth and Bolas, and this simplified method is, after a little practice, as quick as those already outlined. It is this method which should be adopted by the aeroplane designer as a final check upon his work.

Throughout the following proof the offset pulls of the wires are neglected, and it is further assumed that the connections between the interplane struts and the spars are in the same straight line, and that these connections can be treated as pin-joints.

We shall consider the top spar ABC . . . of a biplane, A being the outermost support (Fig. 105). As in the notation used throughout this chapter M_A, M_B, M_C . . . will be used to denote the fixing moments at A, B, C . . . considered positive when they tend to produce convexity upwards. The deflection at any point is denoted by y, the loading per unit run by w. Dimensions are as in Fig. 105. The distance of any point in a member is measured from the centre point of the span, and is counted positive in the direction B to A or C to B.

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[1] "Some Contributions to the Theory of Engineering Structures, with Special Reference to the Problem of the Aeroplane," by H. Booth and H. Bolas. Issued by the Air Department, Admiralty, April, 1915.

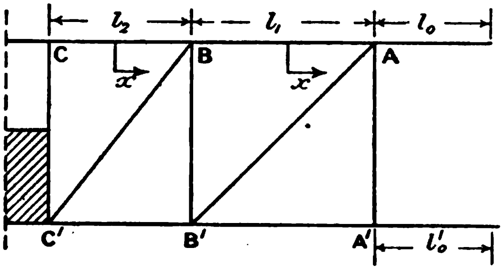

FIG. 105.

It will simplify the equations if the following contractions are used :—

$\mu = \sqrt{\dfrac{P}{EI}}$, where P is the compressive end load in the span under consideration, E and I have their usual significance, and $a = \dfrac{l}{2}.\mu$, a being measured in radians.

Suffixes are used to denote the particular span under consideration, though where there is no risk of confusion these are dropped.

The bending moment at any point = M

$$= EI\frac{d^2y}{dx^2}$$

Take moments about any point in AB and we obtain the equation

$$EI\frac{d^2y}{dx^2} + Py = \frac{w}{2}\left(\frac{l}{2} - x\right)^2 + S_A\left(x - \frac{l}{2}\right) + M_A$$

where S_A is the shear at A.

Differentiate this equation twice and we get

$$\frac{d^2M}{dx^2} + \mu^2 M = w \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The integral of this equation is of the form

$$M = A \sin \mu x + B \cos \mu x + \frac{w}{\mu^2} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where A and B are constants of integration.

But when $x = l/2 \quad M = M_A$

and when $x = -l/2 \quad M = M_B$

Therefore $M_A = A \sin a + B \cos a + \dfrac{w}{\mu^2}$

$$M_B = -A \sin a + B \cos a + \frac{w}{\mu^2}$$

Therefore $A = \dfrac{M_A - M_B}{2 \sin a}$

$$B = \frac{M_A + M_B}{2 \cos a} - \frac{w}{\mu^2 \cos a}$$

Substituting in (2)

$$M=\frac{M_A-M_B}{2}.\frac{\sin\mu x}{\sin a}+\frac{M_A+M_B}{2}.\frac{\cos\mu x}{\cos a}+\frac{w}{\mu^2}\left(1-\frac{\cos\mu x}{\cos a}\right)\quad .\quad (3)$$

and integrating this equation twice

$$\begin{aligned}EIy=&-\frac{M_A-M_B}{2}.\frac{\sin\mu x}{\mu^2\sin a}-\frac{M_A+M_B}{2}.\frac{\cos\mu x}{\mu^2\cos a}+\frac{w\cos\mu x}{\mu^4\cos a}\\&-\frac{w}{2\mu^2}\left(\frac{l^2}{4}-x^2\right)+A_1\left(x+\frac{l}{2}\right)+B_1\left(x-\frac{l}{2}\right)\quad .\ .\ .\quad (4)\end{aligned}$$

where A_1 and B_1 are constants of integration.

But $y=0$ when $x=\pm\,l/2$.

Therefore $$0=-\frac{M_A}{\mu^2}+\frac{w}{\mu^4}+A_1l$$

and $$0=-\frac{M_B}{\mu^2}+\frac{w}{\mu^4}-B_1l$$

Substituting in (4)

$$\begin{aligned}EIy=&-\frac{M_A-M_B}{2}.\frac{\sin\mu x}{\mu^2\sin a}-\frac{M_A+M_B}{2}.\frac{\cos\mu x}{\mu^2\cos a}+\frac{w}{\mu^4}\left(\frac{\cos\mu x}{\cos a}-1\right)\\&-\frac{w}{2\mu^2}\left(\frac{l^2}{4}-x^2\right)+(M_A-M_B)\frac{x}{l\mu^2}+\frac{M_A+M_B}{2\mu^2}\quad .\ .\ .\ .\quad (5)\end{aligned}$$

Differentiating

$$\begin{aligned}EI\frac{dy}{dx}=&-\frac{M_A-M_B}{2}.\frac{\cos\mu x}{\mu\sin a}+\frac{M_A+M_B}{2}.\frac{\sin\mu x}{\mu\cos a}-\frac{w}{\mu^3}.\frac{\sin\mu x}{\cos a}+\frac{wx}{\mu^2}\\&+\frac{M_A-M_B}{l\mu^2}\quad .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\quad (6)\end{aligned}$$

Now the slope at B is the same, whether we consider it from the point of view of the span AB, or the span BC. Therefore, by using equation (6) for both spans in turn and equating the slopes when $x=-\,l_1/2$ and $x=+\,l_2/2$, we have

$$\begin{aligned}&\frac{1}{I_1}\left(-\frac{M_A-M_B}{2}.\frac{\cot a_1}{\mu_1}-\frac{M_A+M_B}{2}.\frac{\tan a_1}{\mu_1}+\frac{w_1}{\mu_1^3}\tan a_1-\frac{w_1l_1}{2\mu_1^2}+\frac{M_A-M_B}{l_1\mu_1^2}\right)\\&=\frac{1}{I_2}\left(-\frac{M_B-M_C}{2}.\frac{\cot a_2}{\mu_2}+\frac{M_B+M_C}{2}.\frac{\tan a_2}{\mu_2}-\frac{w_2}{\mu_2^3}\tan a_2+\frac{w_2l_2}{2\mu_2^2}+\frac{M_B-M_C}{l_2\mu_2^2}\right)\end{aligned}$$

This equation, on rearranging, can be written

$$\begin{aligned}&\frac{l_1M_A}{I_1}\left(\frac{3}{2}.\frac{2a_1\operatorname{cosec}2a_1-1}{a_1^2}\right)+\frac{l_2M_C}{I_2}\left(\frac{3}{2}.\frac{2a_2\operatorname{cosec}2a_2-1}{a_2^2}\right)\\&\quad+\frac{2l_1M_B}{I_1}.\frac{3}{4}.\frac{1-2a_1\cot 2a_1}{a_1^2}+\frac{2l_2M_B}{I_2}.\frac{3}{4}.\frac{1-2a_2\cot 2a_2}{a_2^2}\\&\quad=\frac{w_1l_1^3}{4I_1}\,3\,.\,\frac{\tan a_1-a_1}{a_1^3}+\frac{w_2l_2^3}{4I_2}.\,3\,.\,\frac{\tan a_2-a_2}{a_2^3}\end{aligned}$$

or if

$$f(a)=\frac{3}{2}\cdot\frac{2a\operatorname{cosec}2a-1}{a^2}$$

$$\phi(a)=\frac{3}{4}\cdot\frac{1-2a\cot 2a}{a^2}$$

$$\psi(a)=3\,.\,\frac{\tan a-a}{a^3}$$

we get

$$\frac{l_1}{I_1}M_Af(a_1)+\frac{l_2}{I_2}M_Cf(a_2)+2M_B\left\{\frac{l_1}{I_1}\phi(a_1)+\frac{l_2}{I_2}\phi(a_2)\right\}$$
$$=\frac{w_1l_1^3}{4I_1}\psi(a_1)+\frac{w_2l_2^3}{4I_2}\psi(a_2)\quad\ldots\ldots\ldots\ (7)$$

This is the most general form of the equation of three moments for a continuous beam.

If instead of compressions in the spars we had tensions, (7) becomes

$$\frac{l_1}{I_1}M_AF(a_1)+\frac{l_2}{I_2}M_CF(a_2)+2M_B\left\{\frac{l_1}{I_1}\Phi(a_1)+\frac{l_2}{I_2}\Phi(a_2)\right\}$$
$$=\frac{w_1l_1^3}{4I_1}\Psi(a_1)+\frac{w_2l_2^3}{4I_2}\Psi(a_2)\quad\ldots\ldots\ldots\ (8)$$

where

$$F(a)=\frac{3}{2}\cdot\frac{1-2a\operatorname{cosech}2a}{a^2}$$

$$\Phi(a)=\frac{3}{4}\cdot\frac{2a\coth 2a-1}{a^2}$$

$$\Psi(a)=3\,.\,\frac{a-\tanh a}{a^3}$$

If $a=0$ the corresponding functions become 1, and the ordinary equation of three moments is at once deduced from (7) by putting $P_1=P_2=0$. If one span is in tension and the other in compression, then the corresponding series of functions must be used. Suppose, for example, AB is in tension and BC in compression. Then (7) becomes

$$\frac{l_1}{I_1}M_AF(a_1)+\frac{l_2}{I_2}f(a_2)+2M_B\left\{\frac{l_1}{I_1}\Phi(a_1)+\frac{l_2}{I_2}\phi(a_2)\right\}=\frac{w_1l_1^3}{4I_1}\Psi(a_1)+\frac{w_2l_2^3}{4I_2}\psi(a_2)$$

The functions $f(a)$, etc., have been tabulated by Mr. Berry, to whom the result in (7) is due, for a wide range of values, and the tables are given in Appendix I., as well as approximate formulæ for carrying them still further. As a approaches 90° the calculations should be carried out with a much greater degree of accuracy than when it is small, as the high value of a indicates that the Euler failing load of the bay, if it were pin-jointed, is being approached. The bay may in some circumstances be supported by the adjacent bays, but as a general rule $a=90°$ means failure of the structure.

If the flying wires are offset, then using the notation of Fig. 100, the generalized equation becomes

$$\frac{l_1}{\mathrm{I}_1}\mathrm{M}_{\mathrm{AL}}f(a_1)+\frac{l_2}{\mathrm{I}_2}\mathrm{M}_{\mathrm{CR}}f(a_2)+2\mathrm{M}_{\mathrm{BR}}\frac{l_1}{\mathrm{I}_1}\phi(a_1)+2\mathrm{M}_{\mathrm{BL}}\frac{l_2}{\mathrm{I}_2}\phi(a_2)$$

$$=\frac{w_1{l_1}^3}{4\mathrm{I}_1}\psi(a_1)+\frac{w_2{l_2}^3}{4\mathrm{I}_2}\psi(a_2)\quad.\;.\;.\;.\;.\;.\;.\;.\;.\quad(9)$$

Maximum Bending Moment.—From equation (3) on p. 118, putting $\mu=\frac{2a}{l}$ and rearranging, we get that the bending moment at any point in AB is given by

$$\mathrm{M}=\frac{wl^2}{4a^2}-\left(\frac{wl^2}{4a^2}-\frac{\mathrm{M}_{\mathrm{A}}+\mathrm{M}_{\mathrm{B}}}{2}\right)\frac{\cos\mu x}{\cos a}+\frac{\mathrm{M}_{\mathrm{A}}-\mathrm{M}_{\mathrm{B}}}{2}\cdot\frac{\sin\mu x}{\sin a}$$

It is easy to show that M is a numerical maximum when

$$\tan\mu x=-\frac{\frac{\mathrm{M}_{\mathrm{A}}-\mathrm{M}_{\mathrm{B}}}{2}}{\frac{wl^2}{4a^2}-\frac{\mathrm{M}_{\mathrm{A}}+\mathrm{M}_{\mathrm{B}}}{2}}\cot a\quad.\;.\;.\;.\;.\quad(10)$$

(10) gives the distance from the middle of the span of the point where the bending moment is greatest.

If μx obtained from this equation is numerically greater than a there is no maximum, the bending moment increasing steadily from M_{A} to M_{B}. But if μx is less than a

$$\mathrm{M}_{\max}=\frac{wl^2}{4a^2}-\frac{\frac{wl^2}{4a^2}-\frac{\mathrm{M}_{\mathrm{A}}-\mathrm{M}_{\mathrm{B}}}{2}}{\cos\mu x\cos a}\quad.\;.\;.\;.\;.\quad(11)$$

The corresponding formulæ for a member under tension are

$$\tanh\mu x=-\frac{\frac{\mathrm{M}_{\mathrm{A}}+\mathrm{M}_{\mathrm{B}}}{2}}{\frac{wl^2}{4a^2}+\frac{\mathrm{M}_{\mathrm{A}}+\mathrm{M}_{\mathrm{B}}}{2}}\coth a$$

and

$$\mathrm{M}_{\max}=-\frac{wl^2}{4a^2}+\frac{\frac{wl^2}{4a^2}+\frac{\mathrm{M}_{\mathrm{A}}+\mathrm{M}_{\mathrm{B}}}{2}}{\cosh\mu x\cosh a}$$

and when a spar is subjected to neither compression nor tension we obtain the well-known results

$$\mathrm{M}=\frac{w}{2}\left(x^2-\frac{l^2}{4}\right)+\frac{\mathrm{M}_{\mathrm{A}}+\mathrm{M}_{\mathrm{B}}}{2}+(\mathrm{M}_{\mathrm{A}}-\mathrm{M}_{\mathrm{B}})\frac{x}{l}$$

$\mathrm{M}_{\max}$ occurs where $x=-\frac{\mathrm{M}_{\mathrm{A}}-\mathrm{M}_{\mathrm{B}}}{wl}$

and is given by $\mathrm{M}_{\max}=-\frac{wl^2}{8}+\frac{\mathrm{M}_{\mathrm{A}}+\mathrm{M}_{\mathrm{B}}}{2}-\frac{(\mathrm{M}_{\mathrm{A}}-\mathrm{M}_{\mathrm{B}})^2}{2wl^2}$

Also from the above results we get

$$f_s = \frac{M_{max}}{Z} + \frac{P}{A}$$

where Z and A are the modulus and area of the section respectively, and f_s is the total maximum stress.

A further modification has been obtained by Messrs. Webb and Thorne,[1] and is given here. It has the advantage that algebraical functions are used instead of trigonometrical ones, but it has the disadvantage that as the number of bays increases the degree of the equation in N, the load factor, rises rapidly. These formulæ, however, are accurate within a very small percentage.

With the usual notation the theorem of three moments may be written, for a spar in compression,

$$\begin{aligned}\frac{1}{(Q_1 - P_1)l_1}\Big\{M_{AL}\Big(1 + 0{\cdot}2\frac{P_1}{Q_1}\Big) + 2M_{BR}\Big(1 - 0{\cdot}38\frac{P_1}{Q_1}\Big) + \frac{1}{4}w_1l_1^2\Big(1 - 0{\cdot}014\frac{P_1}{Q_1}\Big)\Big\}\\ + \frac{1}{(Q_2 - P_2)l_2}\Big\{M_{CR}\Big(1 + 0{\cdot}2\frac{P_2}{Q_2}\Big) + 2M_{BL}\Big(1 - 0{\cdot}38\frac{P_2}{Q_2}\Big)\\ + \frac{1}{4}w_2l_2^2\Big(1 - 0{\cdot}014\frac{P_2}{Q_2}\Big)\Big\} = 0\end{aligned}$$

where $$Q = \frac{\pi^2 EI}{l^2}$$

The B.M. in the middle of the bay is

$$M_{mid} = \frac{Q_1}{Q_1 - P_1}\Big\{\frac{1}{2}(M_{AL} + M_{BR})\Big(1 + 0{\cdot}26\frac{P_1}{Q_1}\Big) + \frac{1{\cdot}02w_1l_1^2}{8}\Big\}$$

and $$M_{max} = M_{mid} + \frac{1}{2}\cdot\frac{(M_{BR} - M_{AL})^2}{w_1l_1^2}$$

The distance x of this maximum from A is given by

$$x = \frac{l_1}{2} + \frac{M_{BR} - M_{AL}}{w_1l_1}$$

The distances x_1, x_2 of the points of inflection are given by

$$x_1 = \frac{l_1}{2} + {\cdot}9\Big(\frac{M_{BR} - M_{AL}}{w_1l_1}\Big) - 1{\cdot}2\sqrt{\frac{M_{max}}{w}}$$

and $$x_2 = \frac{l_1}{2} + {\cdot}9\Big(\frac{M_{BR} - M_{AL}}{w_1l_1}\Big) + 1{\cdot}2\sqrt{\frac{M_{max}}{w}}$$

---

[1] "Wing Spar Stresses," by H. A. Webb, M.A., A.F.R.Ae.S., and H. H. Thorne, B.A. B.Sc., *Aeronautics*, January 1, 1919.

For a spar in tension, the modified form of the theorem of three moments becomes

$$\frac{1}{(Q_1+P_1)l_1}\left\{M_{AL}\left(1-\cdot12\frac{P_1}{Q_1}\right)+2M_{BR}\left(1+\cdot32\frac{P_1}{Q_1}\right)+\frac{1}{4}w_1l_1^2\left(1+\cdot0125\frac{P_1}{Q_1}\right)\right\}$$

$$+\frac{1}{(Q_2+P_2)l_2}\left\{M_{CR}\left(1-\cdot12\frac{P_2}{Q_2}\right)+2M_{BL}\left(1+\cdot32\frac{P_2}{Q_2}\right)\right.$$

$$\left.+\frac{1}{4}w_2l_2^2\left(1+\cdot0125\frac{P_2}{Q_2}\right)\right\}=0$$

$$M_{mid}=\frac{Q_1}{Q_1+P_1}\left\{\frac{1}{2}(M_{AR}+M_{BL})\left(1-\cdot21\frac{P_1}{Q_1}\right)+\cdot98\frac{w_1l_1^2}{8}\right\}$$

$$M_{max}=M_{mid}+\frac{1}{2}\frac{(M_{AR}-M_{BL})^2}{w_1l_1^2}$$

$$x=\frac{l}{2}+\frac{M_{BR}-M_{AL}}{w_1l_1}$$

The above formulæ give sufficient points on the bending moment diagrams to enable them to be plotted to a high degree of accuracy. The errors in the fixing moments found by this method are less than ·5 per cent., in cases tested on actual aeroplanes where P/Q varied from ·05 to ·83. The errors in M_{mid} and M_{max} are less than 5 per cent.; and, finally, the errors in the positions of the points of inflection are less than 4 per cent.

91. Pin-Joints in Spars.—An important case occurs when, owing to considerations of design, a pin-joint is made in an aeroplane spar at any place other than near one of the positions of the points of inflection, or zero bending. It was first pointed out by Mr. J. Case that if the pin-joint were not at or near one of the points very large stresses might be caused as a result.

Consider the spar shown in Fig. 106, for example.

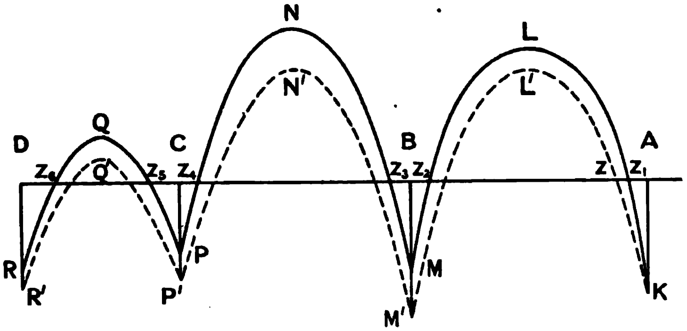

FIG. 106.

Let ABCD . . . be the points of support, A being the outer, and KLMNPQR the bending moment curve for the spar supposed continuous. Then z_1, z_2, z_3 . . . are the points of zero bending. Now suppose it were necessary to place a pin-joint at Z. This would mean that since M_A is fixed, the bending moment diagram would be KL′M′N′P′Q′R′, so that the fixing moments at B, C . . . may be very much increased by the introduction of an actual pin-joint. Of course if the pin-joint had been placed between Z_2 and B the fixing moments at the supports would have been relieved.

Another point to notice about the choice of a position for the pin-joint is that if the spar is in compression the slope of the deflection curve is discontinuous, running up to a peak, and this again adds to the bending moments.

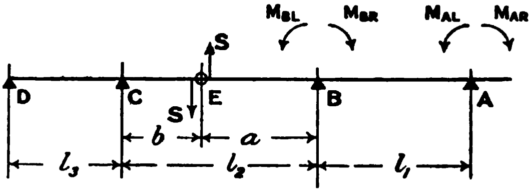

FIG. 107.

Let ABCD (Fig. 107) be an aeroplane spar under a uniform load w per unit length, A being the outermost support. Suppose there is a pin-joint E in the bay BC. Let lengths and notation be as in the figure.

As already shown we have for AB

$$\mathrm{EI}\frac{d^2y}{dx^2} = \mathrm{M_A}\cdot\frac{l-x}{l} + \mathrm{M_B}\cdot\frac{x}{l} - \frac{w}{2}(lx - x^2)$$

and $$\mathrm{EI}y = \frac{\mathrm{M_A}}{6l}(l-x)^3 + \frac{\mathrm{M_B}}{6l}x^3 - \frac{w}{24}(2lx^3 - x^4) + \mathrm{A}_1x + \mathrm{B}_1$$

where A_1, B_1 are constants of integration.

Applying this equation to each of the spans in turn, and also writing down the condition that $y = \delta$, say at E, and 0 at the points of support, with the further conditions that $\frac{dy}{dx}$ is continuous at B and C, we obtain the equations

$$\frac{\mathrm{M_A}l_1}{\mathrm{I}_1} + 2\mathrm{M_B}\left(\frac{l_1}{\mathrm{I}_1} + \frac{a}{\mathrm{I}_2}\right) = \frac{w}{4}\left(\frac{l_1^3}{\mathrm{I}_1} + \frac{a^3}{\mathrm{I}^3}\right) + \frac{6\mathrm{E}\delta}{a}$$

$$2\mathrm{M_C}\left(\frac{b}{\mathrm{I}_2} + \frac{l_3}{\mathrm{I}_2}\right) + \frac{\mathrm{M_D}l_3}{\mathrm{I}_3} = \frac{w}{4}\left(\frac{b^3}{\mathrm{I}_2} + \frac{l_3^3}{\mathrm{I}_3}\right) + \frac{6\mathrm{E}\delta}{b}$$

$$\mathrm{S} = \frac{\mathrm{M_C}}{b} - \frac{wb}{2} = \frac{wa}{2} - \frac{\mathrm{M_B}}{a}$$

where S is the shear at the pin-joint.

The equations including the end load effect can readily be deduced from the work in para. 90.

The reactions, distances between the points of inflection, and shears at any point may now be found by the method described earlier in the chapter.

92. Perry's Formula.—The strength of the main plane spars for failure in the plane of bending is found to a very close approximation by means of a formula suggested by Professor Perry,[1] for the strength of pin-jointed struts with lateral load. A beam with end load can, from another point of view, be considered as a pin-jointed strut with lateral load, as far as regards the part between the points of inflection, the pin-joints being at the points of inflection of the beam.[2]

On such a strut the bending moment at any point consists of that due to the lateral load plus that due to the end load. Let δ be the deflection at any point, and M_x the bending moment there due to the lateral load. Then if P is the thrust or end load, the total bending moment

$$M = M_x + P\delta$$

In the case of a uniformly distributed load, the case we have in an aeroplane spar, the bending moment diagram approximates closely to a curve of cosines, and so we can write the bending moment due to the lateral load in the form, between the points of inflection,

$$M_x = \frac{1}{8} Wl \cos\frac{\pi x}{l}$$

where W is the total distributed load on this part of the beam, l is the length of the beam between the points of inflection, and x is measured from a point as origin midway between the points of inflection. At the points of inflection $x = l/2$ and $M_x = 0$, and at the origin $x = 0$ and $M_x = Wl/8$, the correct bending moments for the beam at these points. Between the middle and the ends, the formula suggested gives a slightly less bending moment than that due to a uniform loading.

From this

$$EI\frac{d^2\delta}{dx^2} + P\delta + \frac{Wl}{8}\cos\frac{\pi x}{l} = 0$$

The solution of this equation is

$$\delta = \frac{\frac{Wl}{8}\cos\frac{\pi x}{l}}{\frac{EI\pi^2}{l^2} - P}$$

The deflection is greatest midway between the points of inflection, and here we get

$$\delta_{\max} = \frac{Wl^3}{8(\pi^2 EI - Pl^2)}$$

$$M_{\max} = \frac{Wl}{8} + \frac{PWl^3}{8(\pi^2 EI - Pl^2)}$$

$$= \frac{Wl}{8}\left(1 + \frac{P}{\frac{\pi^2 EI}{l^2} - P}\right)$$

---

[1] *Phil. Mag.*, March, 1892.

[2] The application of this method to aeroplanes was pointed out by Major A. R. Low, R.A.F., in April, 1914.

Now $\frac{\pi^2 EI}{l^2}$ is the Euler failing load for a pin-jointed strut of length l, and is usually denoted by Q.

Then
$$M_{\max} = \frac{Wl}{8}\left(\frac{Q}{Q-P}\right)$$

$$\text{Maximum stress due to bending} = \frac{Wl}{8}\left(\frac{Q}{Q-P}\right)\frac{y}{I}$$

where y is the distance of the most highly stressed fibre from the neutral axis, and I is the moment of inertia of the section.

Also the stress due to the end load is P/A, where A is the area of the section.

Therefore
$$\text{total stress} = \frac{Wl}{8}\left(\frac{Q}{Q-P}\right)\frac{y}{I} + \frac{P}{A}$$

If f is the total ultimate stress, and N is the factor of loading required, w the loading per unit length of the spar, we get

$$f = N \cdot \frac{wl^2}{8} \cdot \frac{y}{I}\left(\frac{Q}{Q-NP}\right) + \frac{NP}{A}$$

a quadratic in N, the smaller root of which gives the factor on the spar under unit load P, and distributed load w per unit length. From this equation either the load factor on the spar when stressed to a given value may be obtained, or the stress for a given number of times unit load.

Perry's approximate formula is equivalent to replacing sec θ in the exact solution[1] by $\frac{1}{1-\left(\frac{2\theta}{\pi}\right)^2}$. The formula is exact when $\theta = 0$, and is a good approximation when θ is small; but the error increases as θ increases, and when $\theta = \frac{\pi}{2}$ the percentage error, calculated on the value of sec θ, is 21·5. Hence the approximation is a poor one when, as may happen in aeroplane spars, the compression with the load factor applied is not far from Euler's collapsing load.

The following treatment is due to Mr. Arthur Berry.

A very much closer approximation to sec θ is given by

$$S = \frac{\left\{1 + C\left(\frac{2\theta}{\pi}\right)^2\right\}}{1-\left(\frac{2\theta}{\pi}\right)^2}$$

where
$$C = \frac{4}{\pi} - 1 = \cdot 273$$

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[1] See Morley's "Strength of Materials," Longmans, Green and Co.

$\frac{S}{\sec\theta}$ is 1 for $\theta=0$, and $\theta=\frac{\pi}{2}$, and for intermediate values the maximum percentage error, for θ about 60°, is less than one per cent.

Consider now the case of a beam with equal fixing moments at the two ends. The notation of paragraph 90 is followed.

From equation 11 on p. 120 we have for N times loading

$$M_{max}=\left(NM-\frac{wl^2}{4a^2}\right)\sec\sqrt{Na}+\frac{wl^2}{4a^2}$$

Also the maximum fibre stress is given by

$$f=M_{max}\cdot\frac{y}{I}+\frac{NP}{A}$$

where y, I, etc., have their usual significance.

Eliminating M_{max} between these two equations we have

$$\left\{\left(NM-\frac{wl^2}{4a^2}\right)\sec\sqrt{Na}+\frac{wl^2}{4a^2}\right\}\frac{y}{I}+\frac{NP}{A}-f=0$$

This is a rigorous equation for N.

Using the approximate formula for $\sec\sqrt{Na}$ we have

$$\left\{NM-\frac{wl^2}{4a^2}\right\}\cdot\left\{\frac{1+NC\left(\frac{2a}{\pi}\right)^2}{1-N\left(\frac{2a}{\pi}\right)^2}+\frac{wl^2}{4a^2}\right\}\frac{y}{I}+\frac{NP}{A}-f=0$$

or

$$N^2\left(\frac{2a}{\pi}\right)^2\left\{\frac{P}{A}-\frac{CMy}{I}\right\}-N\left\{\frac{My}{I}-(C+1)\left(\frac{2}{\pi}\right)^2\frac{wl^2y}{4I}+\frac{P}{A}+\left(\frac{2a}{\pi}\right)^2f\right\}+f=0$$

Replacing $\left(\frac{2a}{\pi}\right)^2$ by P/Q, and C, $\frac{C+1}{\pi^2}$ by their numerical values we have finally

$$\frac{N^2P}{Q}\left(\frac{P}{A}-\frac{\cdot 273\ My}{I}\right)-N\left(\frac{My}{I}-\frac{\cdot 129\ wl^2y}{I}+\frac{P}{A}+f\frac{P}{Q}\right)+f=0$$

If M_{max} is negative the sign of y must be changed throughout.

In the case in which the fixing moments at the ends of the beam are not equal, but are, say, M_A and M_B, there is no corresponding simple formula. If, however, we replace M by $\frac{M_A+M_B}{2}$ the formula so modified is still valid if we interpret f no longer as the true maximum fibre stress, but the fibre stress halfway between the ends of the beam. If M_A-M_B is small this does not differ much from the true maximum.

DEFLECTION OF BEAMS

Cantilever of length *l* with end load W	Deflection $= \frac{Wl^3}{3EI}$
Cantilever of length *l* with evenly distributed load W	„ $= \frac{Wl^3}{8EI}$
Freely supported beam of length *l* with concentrated load W at the centre of span	„ $= \frac{Wl^3}{48EI}$
Freely supported beam of length *l* with evenly distributed load W over whole span	„ $= \frac{5}{384} \cdot \frac{Wl^3}{EI}$
Beam of length *l* encastré both ends, with a concentrated load W at the centre of the span	„ $= \frac{1}{192} \cdot \frac{Wl^3}{EI}$
Beam of length *l* encastré both ends, with an evenly distributed load W over the whole span	„ $= \frac{1}{384} \cdot \frac{Wl^3}{EI}$
Beam of length *l* under a constant bending moment M	„ $= \frac{Ml^2}{8EI}$

CHAPTER XI

FORCES ON THE MAIN PLANE STRUCTURE

93. Interference between Wings.—In Chapter II. it was explained how the wings raised the aeroplane from the ground. It was pointed out there that this lift was due to the particular shape of the aerofoil section, and also that the distribution of pressure between the top and bottom surfaces of the aerofoil was roughly in the ratio of 3 to 1; in short, that upon the upper surface of the aerofoil depended to a greater extent than upon the lower, its lifting efficiency. This is borne out strongly in the modern aeroplane, which consists of two or more lifting surfaces superimposed upon each other. It has been found from experiment, that unless the two surfaces of a biplane are placed at a distance apart greater than one and three-quarters to twice the chord, there is an interference of one plane with the other, and the nearer the upper plane is brought to the lower the less is the efficiency of the lower lifting surface. In practice, therefore, it would seem that the best thing would be to keep the planes at least a distance of one and three-quarter times the chord apart, so that both planes are lifting to their fullest capacity. Unfortunately, however, other considerations come into play here, considerations of weight and resistance. The greater the gap, the greater naturally the length of the interplane struts, and the heavier they must be to withstand the loads coming upon them. It must be remembered, too, that the strength of a strut of constant cross-section varies inversely as the square of its length. Moreover, a long strut naturally offers greater resistance to the air than a short strut does, so from this consideration, too, it is impossible to make the gap too big unless at a great sacrifice of speed. Practical experience has shown that, taking weight, resistance, and interference into consideration together, the best gap between the lifting surfaces of a biplane is one which is ·8 to 1·2 times the average chord. With this gap there is still a slight interference of the lift of one plane with the other, the effect being greater on the lower plane. The ratio of lift per unit area of the top plane to that of the bottom, in most aeroplanes, is about 1·2 to 1, and this ratio is one which is now generally adopted as fitting in with the average well-designed aeroplane structure.

In the case of a triplane the ratio of lifts per unit area of the top plane, middle plane, and bottom plane to one another is usually taken as 1·2 : ·8 : 1, the middle plane clearly being interfered with on both surfaces. A reasonable estimate for a quadruplane follows 1·2 : ·8 : ·8 : 1·0, always provided, of course, that the gap is not too small.

94. Hunsaker's Triplane Tests.—The free flow of air over the middle wing of a triplane is, as already stated, greatly interfered with by the top and bottom wings, and a series of tests carried out by Hunsaker,[1] in 1916, show that the ratio of lifts of the top : middle : bottom planes is 1·2 : ·8 : 1 is not accurate for fine angles of attack, *i.e.* for high-speed flight. The following table gives relative values of L and L/D for the three planes at various angles of incidence.

Angle of incidence.	Lift, upper.	Lift, middle.	Lift, lower.	$\frac{L}{D}$ upper.	$\frac{L}{D}$ middle.	$\frac{L}{D}$ lower.
0	2·68	1·0	1·82	3·63	1·0	2·30
2	2·14	1·0	1·76	3·18	1·0	2·13
4	1·91	1·0	1·64	2·59	1·0	1·69
8	1·56	1·0	1·36	1·49	1·0	1·37
12	1·56	1·0	1·31	1·30	1·0	1·34
16	1·49	1·0	1·20	1·22	1·0	1·17

It will be observed that at fine angles of incidence, *i.e.* C.P. back, the ratio of the lifts is roughly 1·34 : ·56 : 1, and for large angles of attack, *i.e.* C.P. forward 1·24 : ·8 : 1. When calculating the strength of a triplane for top speed, therefore, the ratio of the lifts should be more in the neighbourhood of 1·3 : ·6 : 1. These tests further showed that the drag upon the three planes is the same for all.

95. Wing Tip Loading.—As a result of a series of tests carried out at the Royal Aircraft Establishment in 1917, it was found that the load distribution along the span of the wings is practically independent of the shape of the ends of the wings. This load distribution curve is given in Fig. 108. It will be noticed that the curve is plotted in terms of the

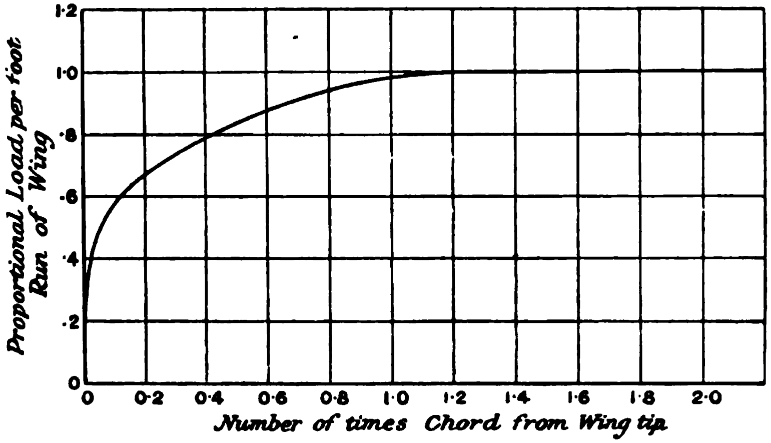

FIG. 108.

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[1] "Aerodynamical Properties of the Triplane," J. C. Hunsaker and T. H. Huyt, *Engineering*, July 21, 1916.

distances from the wing tip expressed as fractions of the chord. For all shapes the loading is constant after a distance of about 1·2 times the chord along the span has been reached.

The distance of the centre of pressure from the leading edge was found, also, to be a constant fraction of the chord of the aerofoil for any particular angle of attack, *i.e.* if the angle of attack of the plane was such that the C.P. was at ·3 of the chord from the leading edge for the major part of the span, then its locus at the ends of the wings was ·3 of the chord of each section. That is to say, the path of the centre of pressure along the span follows the shape of the wing tip, and ends at the extreme tip. From this it at once follows that if a wing tip is raked, aft, say, to fix ideas, the front spar takes less of the load and the rear spar more, than if the wing were cut square. Due account must be taken of these differences when calculating the distributed loads on the spars, their effective lengths, and so on.

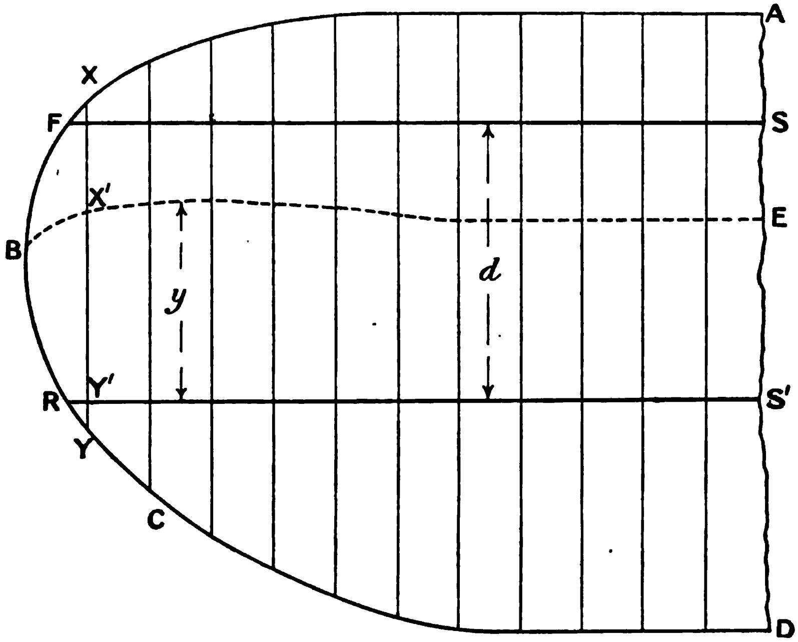

FIG. 109.

The following method shows how the actual shearing force and bending moment may be calculated for any position of the centre of pressure. From the shear we get the effective overhang, that is to say, the length of the overhang which would give the same shear if loaded uniformly, and from this we can get the effective span of the planes. It should be noted here that the effective length of the overhang, though giving the correct shear at the outer point of support, gives, as a rule, too low a bending moment at that point. The bending moment must be found as indicated below.

Let Fig. 109 be a plan view of the wing tip AFCD. Let FS, RS' be the front and rear spars respectively, EB the locus of the C.P. drawn for some definite fraction of the chord. Let d be the distance between spar centres, y the distance of any point in EB from the rear spar. Let lines be drawn at right angles to the spars at distances ·1, ·2, etc., of the chord from the extreme wing tip. From Fig. 109, and the load curve diagram in Fig. 108, the load curve may be drawn for the wing tip and for each spar in turn (Figs. 110, 111). The load curve for the front spar is found by plotting the locus of y/d times the ordinate of the load curve. For example, at the section XY we have $y/d = X'Y'/d$, and the ordinate of the load curve being ·6, the product $\cdot 6X'Y'/d$ gives the intensity of loading on the front spar across this section. So the intensity of the load on the rear spar across this section is $\frac{\cdot 6(d - X'Y')}{d}$, and the load curves are drawn for the two spars. These curves are shown in Fig. 111. From them we get at once by integration, curves giving the shearing force at any point

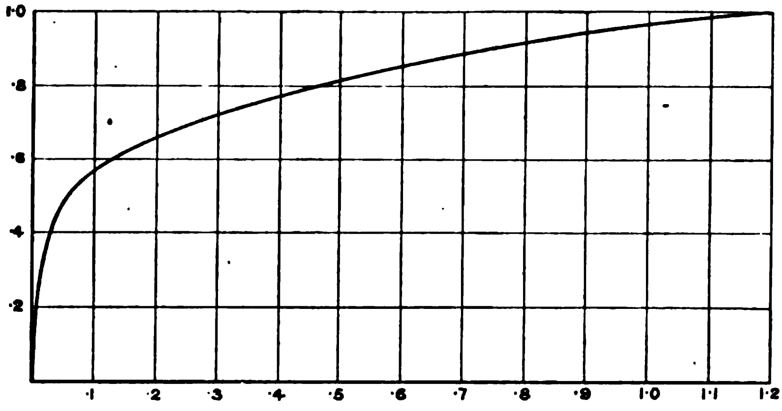

Fig. 110.

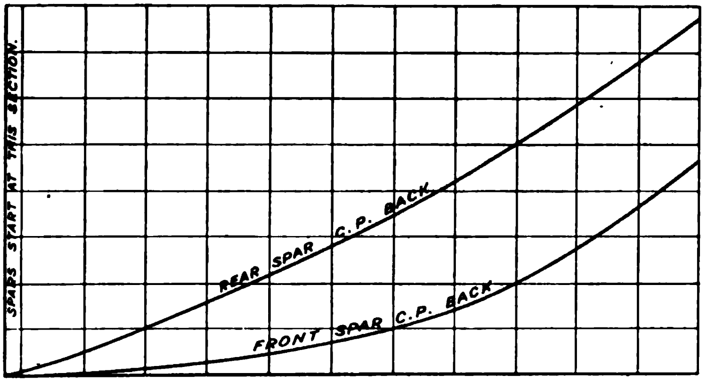

Fig. 111.

on the spars, and by integrating these latter curves we get finally the curves giving the bending moment at any point on the spars. These two sets of curves are shown in Figs. 112, 113. For those who are not familiar with methods of graphical integration it may be noted here that to integrate a curve all that is necessary is to plot a fresh curve the ordinate of which at every point is equal to the area between the original curve and the axis up to the ordinate at that point.

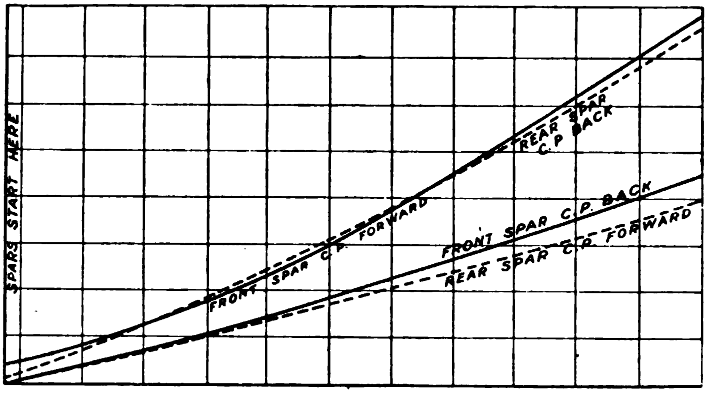

FIG. 112.

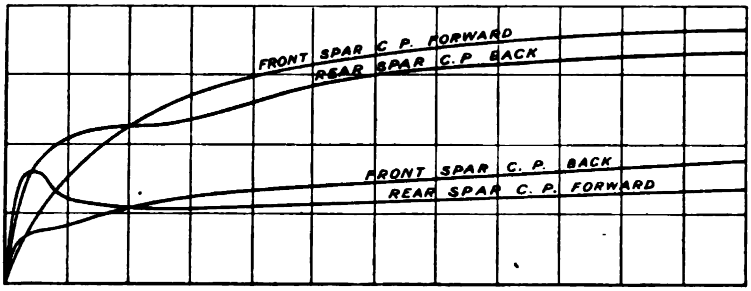

FIG. 113.

Fig. 114 gives curves of the total shear force and bending moment on a wing tip across any section, and it will be found sometimes more convenient to use these curves and apply the corrections for rake, which are given below for four standard wing tips.

In the table on p. 133, l = the length of the overhang from the outer support to the extreme wing tip.
d = distance between the spars.
w = load per unit run for the wing as a whole.
c = chord.

A and B are constants that have been calculated for the four wing tips shown in Fig. 115.

It will be found that nearly all wing tips fall into one or other of these shapes. The shears and bending moments should be found from the shear and bending moment diagram in Fig. 114, and the corrections given in the table applied. The corrections are, for the front spar when the C.P. is at ·28 of the chord, and for the back spar when the C.P. is at ·5 of the chord. These two positions of the C.P. are given, as they usually correspond to the two cases giving the worst loads on the front and rear trusses in normal flight.

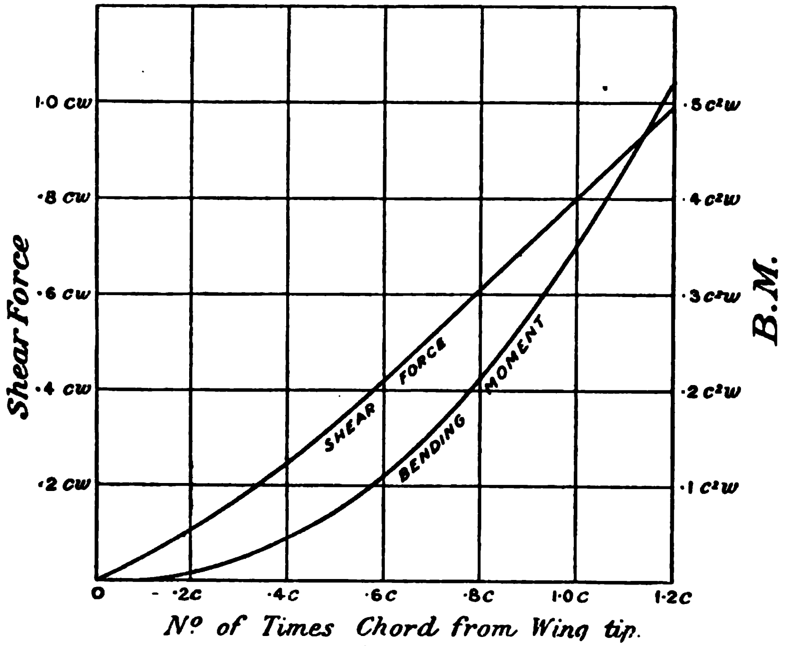

FIG. 114.

The bending moments and shears are found on the assumption that the wing tip is rectangular, and the following corrections applied for rake:

	Shear	Moment
Front	$-\frac{A}{d}c^2w$	$-\frac{(Al-B)}{d}c^3w$
Rear	$+\frac{A}{d}c^2w$	$\frac{(Al-B)}{d}c^3w$

Wing tip.	C.P. forward.		C.P. back.	
	A.	B.	A.	B.
1	0	0	0	0
2	·049	·0064	·036	·005
3	·117	·03	·084	·023
4	0	—·0012	—·02	—·005

96. Corrections for Balanced Wing Flaps.—The above methods of finding the shear and bending moments at the outer points of support of a wing are extremely laborious, when the wing flaps are balanced. The reason for this is that the whole of the load on the balanced portion of the flap, whatever the position of the centre of pressure, must come on the rear spar. Unfortunately there are not enough experimental data on balanced wing flaps to determine accurately the necessary corrections in a simple form. The method given below is an approximate one only, and has been tested in the following way. A number of balanced wing flaps were chosen and the shear and the bending moments on the spars for various positions of the centre of pressure found by actual integration. The suggested approximate method was then tried, and found to be in all cases not more than 10 per cent. wrong for the C.P. in the worst position for the spar concerned, *i.e.* C.P. forward for the front spar and C.P. back for

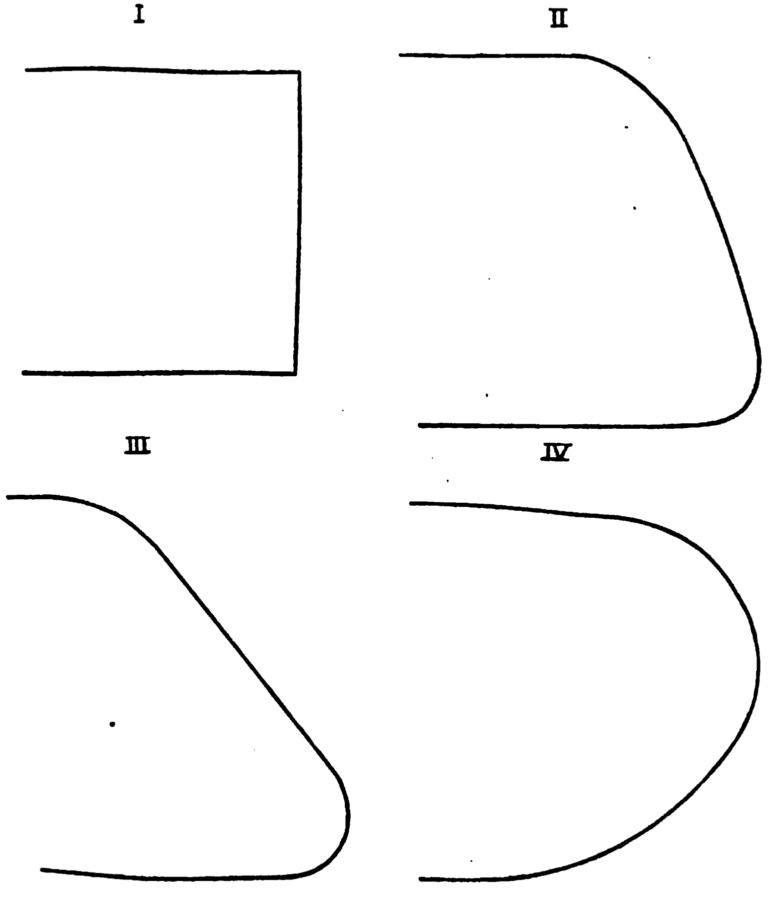

FIG. 115.

the rear spar. Moreover, the discrepancies were on the right side, *i.e.* the shear and bending moments were on the whole too heavy. The greatest discrepancy occurred on the rear spar with the C.P. forward, and the case is not of great importance. Here again, however, it was on the right side for the overhang.

Let PP′ and QQ′ be the rear and front spars of the wing tip under consideration. Let d be the distance between the spars, a the length of the unbalanced portion from the outer point of support, and b the length of the balanced portion, as in Fig. 116.

Then, for the front spar read the shear and bending moment for a wing tip, of length $(a+b)$ from the curves in Fig. 114, using the w for the front spar. Let this shear be S_1 and the bending moment M_1.

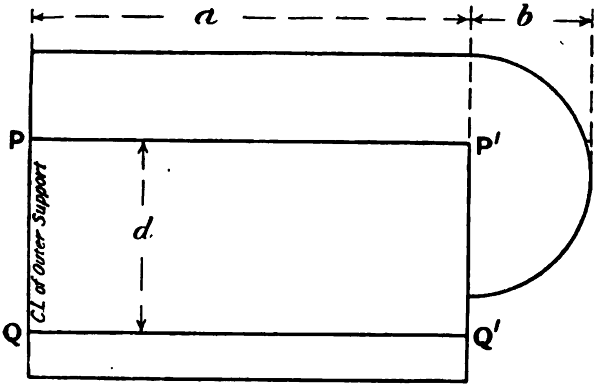

Fig. 116.

Read the shear and bending moment for the balanced portion of length b from the same curves, using the same w as before.

Let these be S_2 and M_2.

Then the shear at Q is $S_1 - S_2$, and the bending moment at Q is $M_1 - M_2 - aS_2$.

For the rear spar read the shear and bending moment for the unbalanced portion a as if the wing tip were square, using w for the rear spar. Next read the total shear for the balanced portion b from the curves. Take the shear as concentrated at the centroid of the balanced portion so that its moment about P is $\left(a+\frac{b}{2}\right) \times$ shear.

The total w for the wing should be used in this latter case, and not the w for the rear spar.

Add these shears and moments to obtain the total shear and bending moments at P.

97. Spar Loading.—The aeroplane should be balanced out as explained in para. 32, and in all the following remarks on finding the loading on the spars it will be assumed that the total lift on the wings, to which reference is made, is the corrected lift, and not the weight of the aeroplane, which is often, though erroneously, taken in aeroplane strength calculations. Before finding the loads on the spars it is necessary to find what load each plane carries, and this can only be found when the total effective span or the effective area of each wing is known. To find the effective span of the wing we proceed as follows. From the shear curve in Fig. 114, the effective overhang is obtained. For example, if the length of the extreme wing tip, from the outermost point of support, is ·8*c*, then from the shear curve it is seen that the shearing force at the point of support is ·6*cw*. This latter gives the shear on the effective overhang on the assumption that it is uniformly loaded as on the rest of the plane, and so the effective overhang, as it is generally called, is ·6*c*, though the actual overhang is ·8*c*.

The rest of the span presents little difficulty in considering the lift, but care should be taken to exclude all non-lifting parts of the main planes. The bottom centre section of a biplane, for example, seldom lifts, as it is usually interfered with by the fuselage. The top centre section, too, may be cut away to allow a better view for the pilot, or there may be a gravity tank interfering with the lift at that point. In cases like this it is better to use the effective area, rather than to work in spans. This is the better method also if the chord varies in different parts of the wing, *e.g.* if the flap projects behind the trailing edge of the rest of the plane.

The total effective span, then, is twice the effective overhang, plus the total distance between the outer supports, less, of course, non-lifting spans. The total effective area is twice the effective overhang times the chord, plus the total lifting area between the outermost supports. When it comes to considering the strength of any part of the plane that is partly cut away, as the top centre section in many aeroplanes, due account must be taken of the decrease in the distributed load on the spars, due to the decreased lifting area at that section. In such cases it is best to consider the loading on the plane per unit area across that section, and so deduce the loading per unit length on the spars. The weight of the wings is acting in the opposite direction to the air reactions, and so is relieving them. This weight can be deducted at points where it occurs, but this would lead to laborious calculations, giving results no more accurate than those made on the usual assumption that the weight of the wing structure may be taken off as a continuous load.

If W is the total lift on the main planes, and w is the weight of the wings, then the upward load transmitted by the wings is $(W - w)$, and is divided between the planes in the proportions quoted in para. 93. In the case of a biplane, for example, if W_B be the load per unit area on the bottom plane, then $1{\cdot}2\ W_B$ is the load per unit area on the top plane, and

$$(1{\cdot}2\ A_T + A_B)W_B = W - w$$

where A_T and A_B are the effective areas of the top and bottom planes respectively. From this we at once obtain that the load per unit length on the top plane is $\frac{1{\cdot}2\,(W - w)}{1{\cdot}2\,A_T + A_B}$ × chord of top plane, and the load per unit length on the bottom plane is $\frac{W - w}{1{\cdot}2\,A_T + A_B}$ × chord of bottom plane.

The main plane spars of the aeroplane carry this load, and it is conveyed to them by means of the fabric and the ribs, as explained in Chap. XVII. The spars are, for all practical purposes, uniformly loaded beams over the greater part of their length. In the case of the rear spar, the aileron hinges cause concentrated loads, with a corresponding decrease in the distributed load on that part of the rear spar, and, of course, concentrated loads may occur at other points in special instances, as, for example, when bombs are carried in the middle of a bay. In these cases due account must be taken in applying the various methods which follow, of the effect of any special distribution of load on the fixing moments, shearing forces, and so on.

The distributed load on the spar is found as follows, taking the top wing of a biplane as an example.

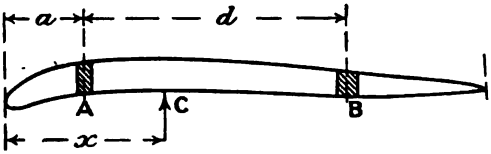

FIG. 117.

Fig. 117 shows the section of the aerofoil of the top plane, with spars at A and B and the centre of pressure at C. Dimensions are as in the figure. Then if w_T is the load per unit length of the span on the top plane, the distributed load per unit length on the front spar is $\left(\frac{a + d - x}{d}\right)w_T$, and on the rear spar $\left(\frac{x - a}{d}\right)w_T$.

Having found the loading on the spars, the bending moments, reactions, etc., are found by the methods described in Chap. X.

98. Secondary Failure in Spars.—The spars of an aeroplane may fail by buckling in the plane of the wing, and since the spars are connected by ribs, they will fail together.

The following treatment of this type of spar failure is due to Messrs. Booth and Bolas, and was published by them from the Admiralty Air Department in 1915.

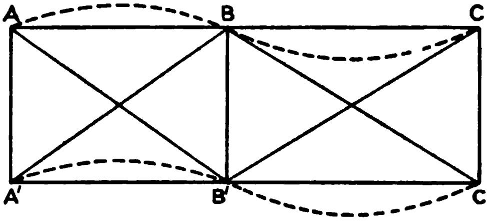

FIG. 118.

Let Fig. 118 represent the structure of a wing, where AC, A′C′ are the spars, and AA′, BB′, CC′ the drag struts. Then the failure in question will be in the direction of the plane of A′C′CA. Owing to the presence of the ribs (which act as ties between the two spars, and also as struts owing to the effective stabilizing of the fabric), the spars AB, A′B′ must deflect together in the same direction and by the same amounts. The same applies to BC and B′C′. At the same time, failures of the type indicated by the curved lines in Fig. 118 are possible, from which it is apparent that, considering beams over the length ABC, we cannot justly do other than assume pin-joints, since between B and C the spars may deflect in the opposite direction to that between A and B, and in such a manner as to give equal slopes on each side of B. In such a case the pin-joint assumption is correct. The problem then resolves itself into the solution of the failure of two parallel pin-jointed struts, which are attached together by rigid rods (as in a ladder), the direction of failure being in the plane of the struts. We shall assume that the connecting rods between the struts introduce no bending moment; in other words, that the rods are pin-jointed at their ends.

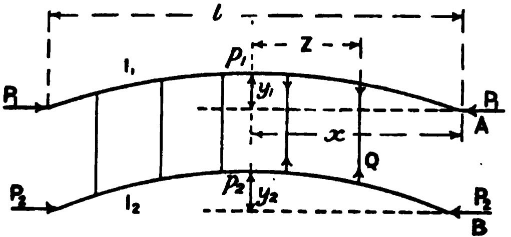

FIG. 119.

In Fig. 119 let :—

I_1 = Moment of inertia of strut A, for failure in the plane under consideration.

I_2 = Moment of inertia of strut B, for failure in the plane under consideration.

P_1 = Compressive load in strut A.

P_2 = Compressive load in strut B.

Q = Load in any inter-connecting rod.

y_1 = Deflection of any point p_1 on A at distance x from end.

y_2 = Deflection of any point p_2 on B at distance x from end.

Z = Distance of inter-connecting rod from p_1, p_2.

The curvatures of the struts after deflection are the same.

Hence
$$y_1 = y_2 = y, \text{ say} \quad \ldots\ldots\ldots \quad (1)$$

Consider strut A.

At the point p_1

the bending moment $= P_1 y + \Sigma QZ$

Therefore
$$EI_1 \frac{d^2y}{dx^2} = -(P_1 y + \Sigma QZ) \quad \ldots\ldots \quad (2)$$

Consider strut B.

At the point p_2

the bending moment $= P_2 y - \Sigma QZ$

Therefore
$$EI_2 \frac{d^2y}{dx^2} = -(P_2 y - \Sigma QZ) \quad \ldots\ldots \quad (3)$$

Therefore, adding (2) and (3) . . . . .

$$E(I_1 + I_2)\frac{d^2y}{dx^2} = -(P_1 + P_2)y \quad \ldots\ldots \quad (4)$$

But this is the equation which would be obtained for a strut with a moment of inertia $(I_1 + I_2)$ having a compressive load $(P_1 + P_2)$.

Hence, in order to calculate the lateral strength of the spars in any drag panel, the lateral moments of inertia of the sections should be added together, and the total failing load found by the ordinary Eulerian expression for pin-jointed ends, thus:—

$$P = \frac{\pi^2 EI}{l^2}$$

Where $$I = I_1 + I_2 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

The compression in the spars which occurs for any one position of the C.P. should then be added together and taken as the working load. The value of P, as given by (5), divided by the working load, gives the total load factor. When estimating the compressions in the front and back spars separately it is usual to calculate this for the front spar when the C.P. is farthest forward, and for the back spar when the C.P. is farthest back. The working load to be taken above is not the sum of the loads thus found, but is the total end load for any given position of the C.P. This varies slightly for different C.P. positions, as will be seen from para. 32, and there is, in addition, a slight difference due to the difference in drag loads when the C.P. is forward and back.

99. Drag Forces.—In addition to the lift forces on the main planes of an aeroplane there are, as explained in Chap. II., drag forces acting. The effect of these forces can be calculated on the assumption that the spars are continuous and supported at the points of attachment of the drag struts and wires. The effect of bending due to drag in the spars is, however, generally negligible compared with that due to lift forces, and it will be quite accurate enough, generally, to calculate the loads due to drag on the assumption that the members are pin-jointed throughout. The direction of the resultant of the drag forces depends upon the position of the centre of pressure, and in most aeroplanes, when the C.P. is in its most forward position, we get anti-drag forces acting on the drag bracing. This is clear from the following considerations.

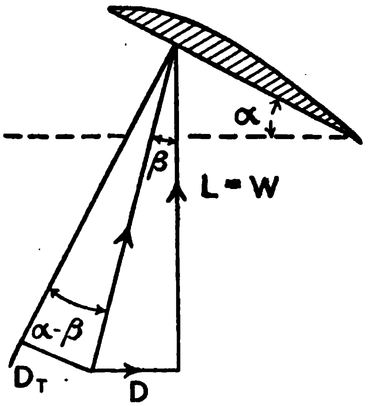

FIG. 120.

In Fig. 120 let W = the weight of aeroplane.
L = the lift on wings acting vertically.
D = the drag on wings acting horizontally.
α = the angle of incidence of the aerofoil.
β = the angle between the vertical and the resultant of the lift and drag.
D_T = the drag parallel to the chord.

Then resolving along the chord of the aerofoil we get

$$D_T = W \sec \beta \sin (\alpha - \beta)$$

If β is greater than α, or L/D less than cot α, we have drag.
If β is less than α, or L/D greater than cot α, we have anti-drag.

100. Secondary Stress in Nose-diving.—In 1918 it was pointed out by Dr. H. P. Hudson that in nose-diving the bending stress from the distributed load due to drag may be a considerable percentage of that due to lift. The resultant load makes an angle with the drag plane, and the deflection of the spar is oblique. The maximum stress due to bending not in a principal plane is correctly found by adding together the maximum stresses due to the two component bendings in the principal planes, and to this must be added the direct stress due to the end load. If, as is usual, there are two or more drag bays in each lift bay, it is accurate enough to treat the drag girder as composed of equal and uniform bays. Let a be the length of one of these. Then the value of the secondary bending moment in the middle of a drag bay is $\frac{W_D \cdot a^2}{24}$, and at a support $\frac{W_D \cdot a^2}{12}$, and the distance between the two points of inflection in one bay is $\frac{a}{\sqrt{3}} = {\cdot}58a$. There is usually one intermediate drag strut in each lift bay; then all the points of the spar usually investigated, namely, the lift supports and the middle of the lift bays, are points of support for the drag; the secondary bending moment is $m = \frac{W_D a^2}{12}$, and the distance between the two adjacent inflections is $\cdot 42a$. If the two adjacent drag bays are not of equal length, it is sufficient to take the average value of a. In the case of three or any odd number of drag bays in one lift bay, the secondary bending moment in the middle of the lift bay is $\frac{W_D a^2}{24}$, and the corresponding distance between inflections is $\cdot 58a$. Since the front and rear spars are connected by ribs and fabric, their deflections in the secondary direction are the same, and are resisted by the sum of their strengths. The proper moment of inertia to use in this connection is $I = I_1 + I_2 =$ the sum of the minimum moments of inertia of the front and rear spars. But as each spar bends about its own neutral axis, the stress in each spar is proportional to the distance from its own centre line; and if b is the breadth of a spar, the secondary modulus for the front spar is $\frac{I}{\frac{1}{2}b_1}$, and for the rear spar $\frac{I}{\frac{1}{2}b_2}$.

To allow for buckling due to end load, we must again treat the drag girder as a whole, and the same total moment of inertia I must be used in calculating the secondary Euler failing load, which for a drag support is $q = \frac{\pi^2 EI}{(\cdot 42a)^2}$. This must be taken in conjunction with the total compression in the girder $p = P_1 + P_2$. If one of the spars is in tension, the corresponding P is negative. It may be necessary to average I and p. The actual secondary bending moment at a point of drag support is then $m\,.\,\frac{q}{q-p}$, and the corresponding secondary stress in the front spar is

$$\frac{\frac{1}{2}b_1}{I}\cdot\frac{mq}{(q-p)}$$

The final expression for the maximum total stress in the front spar is

$$\frac{M_1}{Z_1}\cdot\frac{Q_1}{Q_1-P_1}+\frac{\frac{1}{2}b_1 m}{I}\cdot\left(\frac{q}{q-p}\right)+\frac{P_1}{A_1}$$

and in the rear spar

$$\frac{M_2}{Z_2}\cdot\frac{Q_2}{Q_2-P_2}+\frac{\frac{1}{2}b_2 m}{I}\cdot\left(\frac{q}{q-p}\right)+\frac{P_2}{A_2}$$

If the two spars are of the same breadth, the secondary bending terms are exactly the same for front and rear.

The equation for the load factor is

$$f=\left(\frac{M\,.\,N}{Z}\times\frac{Q}{Q-PN}\right)+\left(\frac{\frac{1}{2}bmN}{I}\times\frac{q}{q-pN}\right)+\frac{PN}{A}$$

which is a cubic for N. The value of N can never exceed either Q/P or q/p, which respectively make the first and second terms infinite.

Generally, the value of m is small, and the secondary term is important only when its denominator $q-pN$ is small. This occurs when the spars are narrow compared with their depth, so that I, and therefore q, is small. If p/q is nearly unity, its value is the controlling circumstance in determining the value of N, which is then just a little less than p/q.

101. Stagger Loads.—In addition to the drag loads previously noted, the drag bracing and spars carry extra loads if the aeroplane is staggered. The general effect of a forward stagger is to add to the effect due to the drag, so putting a tension in the front spars and a compression in the rear.

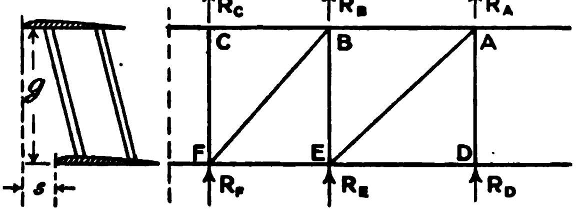

FIG. 121.

Let s be the stagger, g the gap, R_A, R_B . . . the reactions as in Fig. 121. Then, if the stagger loads in the direction of drag are taken as positive, we have at any point B, say, the stagger load due to the strut

$$=-\frac{s}{g}(R_A+R_D+R_B)\quad.\;.\;.\;.\;.\;.\;.\;(1)$$

and the stagger load due to the wire

$$= \frac{s}{g}(R_A + R_B + R_D + R_E) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Therefore total load at B in the direction of the chord is

$$\frac{s}{g} \times R_B$$

$$= \frac{s}{g} \times \text{reaction at B}$$

So the stagger load at any point of support $= s/g$ times the reaction at that point, provided, always, that the interplane struts and wires are equally staggered. If not (1) and (2) are suitably modified, and their sum gives the load on the drag bracing due to stagger at the point in question.

The loads in the interplane struts and wires are increased by stagger in the ratio of their actual lengths to the vertical projections of their lengths, and on the assumption that the reactions were calculated in the first place ignoring stagger.

There are certain other loads which may come on the main lifting surfaces of an aeroplane due to the particular type of structure, and these loads must be carefully considered, especially in any unusual type of construction. We can only deal here with a few of the more common types.

102. **Tail Booms.**—In a pusher type of aeroplane the tail booms are usually attached well out on the wings, and thus may throw considerable loads on the drag bracing. In Fig. 122 the booms are attached to the rear spars at A and B. Now if, for example, there is a down load P on the tail C there is a corresponding anti-drag load at B, a drag load at A, and a relieved tension in the flying wire at A.

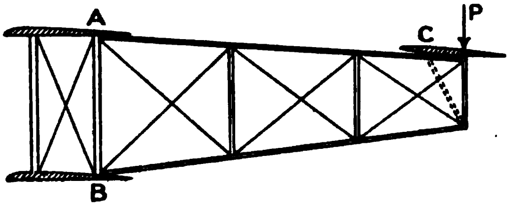

FIG. 122.

These loads, especially in the case of a limiting nose-dive, may be extremely heavy, and must be taken into account.

103. **Pylon Bracing.**—The pylon bracing of an aeroplane comes into operation

(1) In high speed flight.
(2) In landing.

In Fig. 123 ADC is the pylon bracing of the top front spar of an aeroplane. Then in high speed flight, for example, there are forces acting in the directions of the arrows at A, B, C . . . etc., and these put into play the pylon wires DA, DC, and the weight wires of the aeroplane. The vertical components of the loads in the pylon wires AD, CD throw an extra load in the interplane strut BE, as well as putting heavy compressions in the spar ABC. These compressions are serious in a limiting nose-dive, and must be watched carefully. In a triangular pylon the struts tend to put a tension in the corresponding drag strut, and this should be noted when designing the aeroplane.

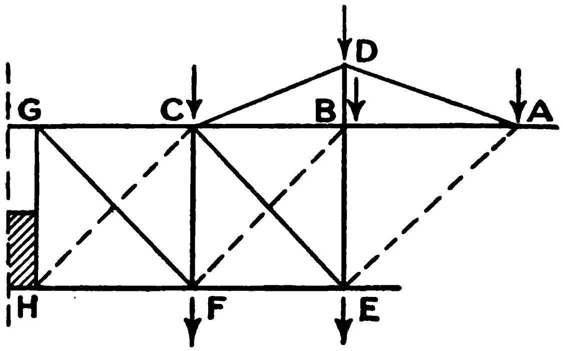

Fig. 123.

104. Multiple Engine Aeroplanes.—A multiple engine aeroplane will be found to present many difficulties which do not occur in the single engine type. In the latter the torque of the engine and the thrust of the propeller need not enter directly into the calculations of the main plane structure. In a multiple engine aeroplane these, however, become of importance, as a large part of the weight has to be carried in the wings, and this affects the loads in the whole structure between the engine nacelles.

The lift upon the wings and the loading upon the spars are found in exactly the same way as for a single engine aeroplane, but at the engine nacelles there are concentrated weights occurring which must be taken into account. In considering these weights account must be taken of the actual attitude of the aeroplane.

These weights are divided between the front and rear trusses, according to the position of the centre of gravity. If this falls between the spars, then the loads in both the spars are relieved between the engine nacelles, for at these the weight of the engine is acting in an opposite direction to the lift reactions; but the engine nacelle may be so overhung that its centre of gravity is forward of the front truss, and in this case the loads on the rear truss are increased, though the loads on the front truss are more greatly relieved than if the centre of gravity of the engine unit came between the spars. The relief on the front truss may be great enough to overcome the total lift reaction at the engine nacelle, in which case the weight-wire of the inner bay comes into play.

In the case of the twin or multiple engine aeroplane it is necessary, in order to obtain accurately the loads on the lift and drag bracing between the engine nacelles, to determine the engine thrusts and torques.

105. Engine Thrust.—It is assumed that the performance of the aeroplane is known, and therefore that the top and bottom speeds have been determined, as well as the total resistance of the aeroplane at these speeds. Let V , V_B be the top and bottom speeds respectively.

Then $$W = k_L \rho S V_T^2$$

therefore $$k_L = \frac{W}{\rho S V_T^2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

the symbols used having their usual significance.

The aerofoil being known, the value of $\frac{k_L}{k_D}$ corresponding to this value of k_L can be obtained from the aerofoil curves. Then $\frac{W}{\left(\frac{k_L}{k_D}\right)}$ = total resistance in lbs. of the wings of the aeroplane at the speed V_T, since $R_W = k_D \rho S V_T^2$.

From the performance data of the aeroplane the total resistance R_T = propeller thrust, is known.

$$\text{also } R_T - R_W = R_B \quad . \; . \; . \; . \; . \; . \; . \quad (2)$$

where R_B is the body resistance of the aeroplane.

Now, the resistance of the body, unlike that of the wings, depends on V^2 only, so if the body resistance found from (2) is reduced in the ratio of $\left(\frac{V_B}{V_T}\right)^2$, we have the body resistance at bottom speed.

From the aerofoil characteristics the value of k_D corresponding to V_B is found, and the resistance of the wings at bottom speed is then—

$$R_W = k_D \, . \, \rho S \, . \, V_B^2$$

106. Engine Torque.—Let P = horse-power of engine.
N = revolutions per min. of the propeller.
T = torque in ft.-lbs.

Then $$P = \frac{2\pi NT}{33{,}000}$$

therefore $$T = \frac{P \times 33{,}000}{2\pi N} \text{ ft.-lbs.}$$

The way the torque is distributed is not a simple matter to decide. In an ungeared engine it will be found accurate enough to assume that the engine feet take equal loads.

If d = distance between engine feet in feet.
$2n$ = number of engine feet.

Then the load coming on each foot $= \frac{T}{dn}$ lbs.

In the case of a geared engine a great deal depends upon the rigidity of the crankcase. If this is perfectly rigid the torque will come equally on the feet, but if not, the feet near the gear box will get a much larger load than the feet farthest away.

107. Application of Engine Loads.—It is not always easy to settle how the various loads due to engine thrust and propeller torque come on the structure, so this will be dealt with here at some length. Every aeroplane will present its own particular problems, and the methods suggested, therefore, can only be general. The following cases will be considered:—

(*a*) Both engines full on, C.P. forward and back.
(*b*) Both engines off.

The case of one engine off is considered in Chap. XV.

Fig. 124 represents the part front elevation of a twin engine aeroplane. The engine is so mounted that it is symmetrical about a centre line of the interplane strut which could be put in place of the engine mounting to complete the structure. There is no stay from the engine mounting to the body of the aeroplane. This form of structure will be considered later.

It will be found necessary to find the centre of head resistance, and this may be obtained by making approximations as to the relative resistances of various units, in the absence of further data. In the particular twin engine biplane under consideration it would be fair to assume that each engine and engine nacelle has a resistance equal to a quarter of the total body resistance, and that the total resistance of the fuselage, tail unit, and chassis is equal to the remainder, *i.e.* half the total body resistance. The wing resistance, which is known, may be divided between the planes in proportion to their effective areas. By taking these various resistances to act approximately at the centre of gravity of the areas, as shown on the front elevation of the aeroplane, we can obtain the centre of head resistance. This centre of head resistance being known, the aeroplane can be balanced out, as explained in para. 32, including now the couple due to the engine thrust.

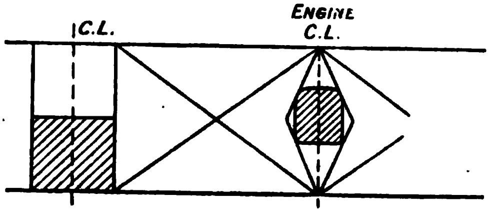

FIG. 124.

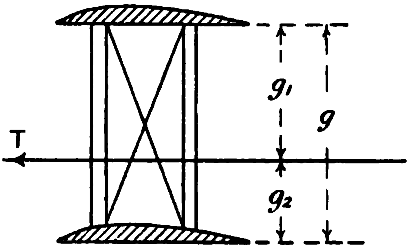

FIG. 125.

In Fig. 125.

If g = gap.

g_1 = the distance of the centre line of the propeller thrust from the top plane.

g_2 = the distance of the centre line of the propeller thrust from the bottom plane.

T = engine thrust.

R_B = resistance of body.

$T_N = T - \frac{1}{4}R_B$.

Then forward pull on the top plane $= \frac{g_2}{g}T_N = T_T$, say,

and forward pull on the bottom plane $= \frac{g_1}{g}T_N = T_B$

Fig. 126 shows a plan view of the top plane of the biplane we are considering. Since we know the total resistance of the wings, as already explained, we know the drag from the wing tip to the centre line of the engine; call this drag D_W. At the engine on the top plane we have a pull T_T, so the load carried to the drag bracing between the centre line of the engine and the centre line of the aeroplane is $(D_W - T_T)$. This, plus the drag of the plane between these points, passes down the incidence wires to the body.

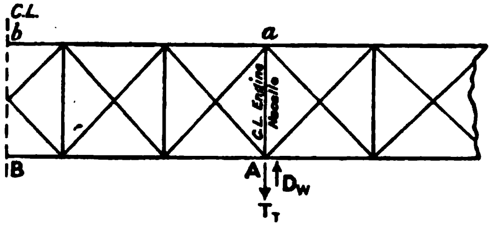

FIG. 126.

If T_T is less than the total drag on the top wing, all the drag wires are in play, and the net drag force passes to the body down the incidence wire from the top rear spar to the bottom front spar.

In either case, this net force, whether drag or anti-drag, is met by the load in the top longerons arising from the tail load, whose moment balances the moments of the engine thrust and the resistance.

The bottom plane is treated in exactly the same way as the top plane.

Both Engines Off.—When both engines are cut out the gliding angle of the aeroplane should be worked out for the particular centre of pressure position being considered and the external loads on the structure balanced out. Instead of the propeller pull we now have a propeller resistance due to "wind-milling." An approximate formula for this resistance is

$$R = \cdot000059\, D^2V^2$$

where R = resistance in lbs.
D = the diameter of the propeller in feet.
V = the speed of the aeroplane in ft. per sec.

This resistance should be divided between the top and bottom planes, exactly as in the case of the propeller thrust. The drag of the wings can now be balanced out at the centre section of the aeroplane.

108. Unsymmetrical Engine Mounting.—If the engine mounting is not symmetrical the distribution of loads must be carefully considered.

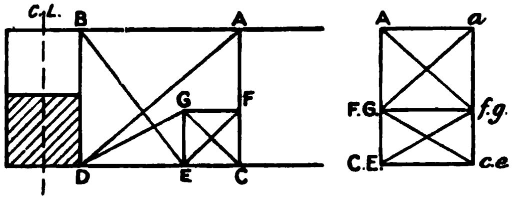

FIG. 127.

Consider the front elevation and incidence bracing at the engines of the aeroplane shown in Fig. 127. DG is a stay tube from the fuselage to the engine bearer attached close to one of the engine feet. The engine bearers are G*g*, F*f*, small letters being used throughout for the rear truss.

At F*f* there is little difficulty in balancing out the loads due to engine thrust, torque, and the weight of the engine, for the vertical components of A*f* and *f*C must be equal. At G, however, there is only the wire G*e*, if, as is generally the case, there are no incidence wires in the plane DG*gd*.

The thrust in GE, due to the tension in G*e*, will be taken up at E either by BE or EF. If the lift reaction at E is greater than the thrust in GE, then the difference must be taken on GD and GC. If, however, there is still a down load at E, it will be taken up by BE and EF. There will be a horizontal component of EF at F, which will put a compression in GF, and consequently a compression in DG, which, in turn, has an upward component at G, so further reducing the down load at E. If the reader is not careful he will find himself here working in a circle. He should assume loads P_1, P_2, P_3 . . . in the various members concerned and write down the equations of equilibrium at G, E, F, C, etc.; and so obtain the loads in GE, GF, etc.

109. Incidence Wiring.—The above methods of calculating the strength of aeroplanes entirely neglect the effects of the incidence bracing. This is not of much importance in normal flight, though it is exceedingly important in nose-diving and in the case of a cut wire. To take account of these wires, even approximately involves an acquaintance with the elementary principles of elasticity. In Chapter XXII. are given methods by which a fairly good approximation can be made to the effect of incidence wiring. The designer will be well advised to make himself familiar with the methods described in that chapter, for there is little doubt that by using them he will be able to save weight in a large aeroplane.

CHAPTER XII

INTERNAL FORCES IN A FUSELAGE

110. Nature of External Loads.—It has been shown in Chapter VI. that the external forces upon the fuselage structure arise from various causes. The following gives the nature of these loads in a concise form for reference when considering the internal stresses caused in this part of the structure.

Load arising from	Nature of load.	When occurring.	Remarks.
Control of aeroplane (aerodynamic).	1. Up load on tail.	Low speed flight.	Factor as for C.P. forward.
	2. Down load on tail.	High speed flight.	Factor as for C.P. back.
	3. Maximum steady down load on tail.	Terminal velocity nose dive.	Factor 1¼ to 1½.
	4. Momentary maximum down load on tail.	Pulling out of high speed flight.	Criterion is strength of wing structure, and fuselage should fail simultaneously with this.
	5. Side load on fin and rudder.	In a turn.	Full rudder at top speed. Often gives torque on fuselage as well as direct load.
Standing loads.	6. Reaction of ground on tail skid.	When taxying on ground or when landing.	Factor required rather arbitrary, 4–6 usual.
Engine loads.	7. Dead weight of power unit.	Always.	Factor required same as for C.P. forward.
	8. Thrust of propeller.	When engine running.	A maximum when engine full out, aeroplane at rest. Once times this load to be taken in conjunction with No. 7.
	9. Torque of propeller.	When engine running.	Balanced by wash-in and wash-out or by aileron control.
	10. Gyroscopic action of propeller and engine.	Engine running on a turn.	Effect upon bearers must be watched. Is balanced by simultaneous loads on rudder and elevators or else different engines running opposite ways balance each other.

It is not necessary to make full calculations for every case in the above table, since many of them give loads identical in quality and varying only in quantity when, of course, the maximum only need be considered. By maximum, in this case, is meant the load inclusive of the factor of safety or load factor demanded since a small unit load when multiplied by a large factor which may be required, may outweigh a much larger unit load which only requires a small factor. The method of dealing with a fuselage should be as follows:—

A. Estimate the loads tending to break the fuselage upward, including the required factor for each case. The loads are generally—

(1) The load on the tail plane at slowest flying speed.
(2) The skid load when standing on the ground.

The greater of these will determine the design of the fuselage to resist upward loads.

B. Estimate the loads tending to break the fuselage downwards.

(1) The load on the tail plane at highest normal flight speed.
(2) The load on the tail plane in a terminal velocity nose-dive.
(3) The load on the tail plane suddenly applied, which just breaks the main planes, *i.e.* the case when pulling out suddenly from high-speed flight.

The greatest of these will determine the strength of the after-portion of the fuselage for downward loading.

C. Estimate the loads upon the forepart of the fuselage.

(1) The dead weight of engine, etc.
(2) The thrust of propeller (factor of 1 on static thrust only is required).
(3) The torque of engine (factor of 1 on maximum torque).

These three loads should be considered simultaneously, and the result will determine the strength required for the forepart of the fuselage. Certain members may be more highly stressed, due to (1) alone, when the engine is cut out.

D. Estimate the side loads upon the fin and rudder and combine with the torsion induced by them to determine the side strength of the after-part of the fuselage.

E. In an aeroplane capable of quick manœuvrability assume a 45° bank, and estimate the control loads on the rudder and elevators to counteract gyroscopic effect (see para. 36).

Check the strength of the fuselage already designed from other considerations for the combination of these loads.

The whole of the calculations for the fuselage resolve themselves into three types.

(1) The calculations for the structure under vertical loads either up or down.
(2) The calculation for the structure under side loads.
(3) The calculation for the structure under torsional loads.

The methods for dealing with each of these calculations will be considered in turn.

111. Vertical Loads on the After-part of the Fuselage.—Fig. 128 represents the after-part of a fuselage structure, with a down load P acting on the tail plane. This load is supposed to act at the extreme end of the fuselage, although, in most cases, it would be slightly further forward than this. The centre of pressure on a tail plane varies for different conditions from the leading edge to ·5 of the chord from the leading edge. For the sake of illustration, however, the assumption made above will be sufficiently accurate.

When the load is further forward the longerons are relieved, but the load in the vertical struts and wires may be increased.

It is necessary to find all the stresses in the fuselage structure arising from this tail load. It should be remembered that a fuselage consists of two girders, each of which may reasonably be assumed to take half of the load, so that P in the figure is only half the total calculated load for the condition of flight assumed.

The fuselage joins the centre section of the aeroplane at the line A*a*, and the after-part may be supposed to be a cantilever supported at the centre section, since, at this section, the fuselage loads find their reactions from the wing loads. The quickest method of determining the loads in the members for use in a drawing office is probably to construct a stress diagram and to scale from it the required figures. This method is described in para. 59.

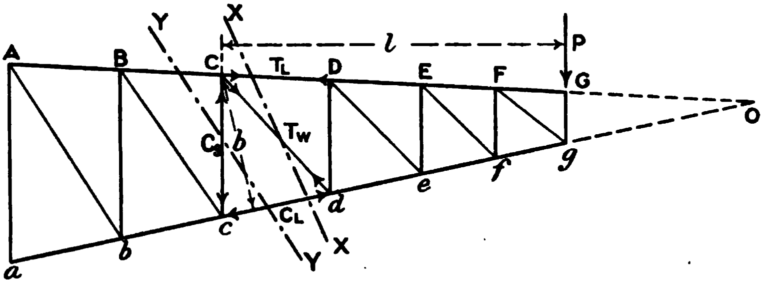

FIG. 128

Another useful way of attacking the problem is by Ritter's method of sections (para. 60) as follows: Assume the panel CD*dc* to be cut by the section line XX. The external forces on the fuselage are then held in balance by the forces in the three cut members CD, C*d*, *dc*. Denote these by T_L, T_W, and C_L respectively. Since the external and internal forces are in equilibrium the total moment about any point may be taken and equated to zero. By choosing suitable points two of the forces will have no moment, and the moments of the others may then be equated.

First, then, take moments about the point C, thus eliminating the moments due to T_L and T_W. Let the perpendicular distance from C to the bottom longeron be *b*, and the distance from C to the line of action of P be *l*.

Then $$C_L . b = Pl$$

or $$C_L = \frac{Pl}{b}$$

giving C_L directly.

Secondly, take moments about the point d. Representing the perpendicular distance from d to the top longeron by b', and the distance from d to the line of action of P by l', the result is obtained

$$T_L b' = Pl'$$

or

$$T_L = \frac{Pl'}{b'}$$

giving T_L directly.

Thirdly, produce the lines of CD and cd to meet at O, and from O drop a perpendicular on the line of Cd produced. Let the length of this perpendicular be b'', and the distance of O from the line of action of P be l''.

Then

$$T_W b'' = Pl''$$

or

$$T_W = \frac{Pl''}{b''}$$

Fourthly, to obtain the load in the vertical strut Cc assume another section line YY, as shown, and produce the lines of BC and cd to meet (since the top longeron is shown horizontal they meet at the point O previously found). Let the perpendicular distance from O to Cc $=b'''$, and from O to line of action of P $=l'''$ (in this special case $l'' = l'''$).

Then by taking moments about O,

$$\text{the load in } Cc = C_s = \frac{Pl'''}{b'''}$$

This method may appear rather laborious, but in practice it is not so, since it lends itself readily to tabulation. At the same time, most designers will probably prefer the graphical method of treatment, in which case it is wise to check the stress diagram loads by making calculations, as shown above, for one or two members of the truss. In the foregoing example it has been assumed that only a tail load is being carried. In addition to this, however, there may be various concentrated weights, such as the pilot and his instruments, mails, etc. The way in which these should be allowed for depends entirely on the condition of flight. If the load P represents the normal flight tail control load for the aeroplane at its horizontal maximum speed, the dead weights must be taken into account at their appropriate points. All loads should have the factor included before commencing calculation, the dead loads requiring the same factor as the tail load, *i.e.* that demanded for the main planes with the C.P. in the most aft condition consistent with horizontal flight.

If P represents the tail load in a vertical nose-dive, however, the weights of pilot, passenger, etc., may be considered as acting along the fuselage, *i.e.* in a direction perpendicular to the tail load, and in this case simply add direct compressive forces to those induced in the longerons by the load P. The factor here must also be consistent with the one required for this condition of flight (usually 1¼ to 1½).

It may be necessary once more to issue a word of caution—the dead loads must be divided between the two trusses in the same way as P.

These extra loads do not complicate the calculations to any appreciable extent. In the stress diagram they simply mean a slightly different load line at the beginning, and after that they work themselves out automatically.

In the method of sections it must be remembered that external forces on one side only of the section line must be considered in estimating moments, and when this point is understood, no further difficulty should be met.

Occasionally it will be found that loads do not come on to a joint of the fuselage directly, but in the middle of a bay. This is a point in design which should be avoided; but, if necessary, the method of treatment is to find the proportions of the load coming as reactions to the adjoining strut positions, and continue the work as before. In this case, however, special attention must be paid to the member which transmits the load in the first place, to ensure that it is of sufficient strength in bending.

112. Upward Loads on Fuselage.—The tail skid load, when the aeroplane is carrying its full weight, must first be calculated. This is an upward force on the rear end of the fuselage at the attachment point of the tail skid. At approximately this point, however, there is also a down load due to the weight of the complete tail unit, which relieves the upward load due to the tail skid reaction.

The nett upward load at the rear end is therefore

tail skid load — weight of tail unit

Any dead loads concentrated along the fuselage also relieve the forces, as they act downwards, and must, therefore, be taken into account.

The external loads having been thus determined, the forces in the members of the structure are found in exactly the same way as described for downward loads.

113. Forepart of Fuselage.—The loads are calculated from the weights carried and a knowledge of their disposition. These act vertically downward, as shown at *w*, Fig. 129, and should include the load factor required.

The torque is then calculated, and this puts a down load on one side of the fuselage and an up load on the other. The worst case is the down load, since this adds to the dead weights already considered. It is transmitted through the engine feet to the fuselage, and is represented by *t*, *t* in Fig. 129. A load factor is not required on this, but the maximum torque value should be used.

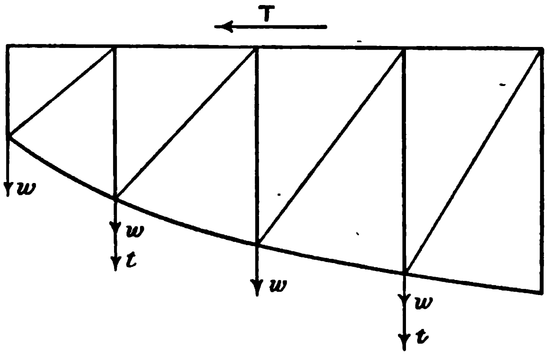

FIG. 129.

The thrust of the propeller acts as a direct load T in the longerons from the point of attachment of the cables and the other members, such as engine bearers transferring it to the structure. This also does not require a factor if the maximum or static thrust is taken. These loads, having been determined, the forces in the structure are found in precisely the same manner as for the rear portion of the fuselage by graphic methods or by direct calculation.

114. Horizontal Loads on After-Part of Fuselage.—In dealing with the horizontal forces in a fuselage arising from fin and rudder control loads the procedure is exactly the same as for the vertical loads just described, but owing to the curvature of the fuselage in plan, which is a common feature to most aeroplanes, it is quicker to draw a stress diagram than to make calculations by the method of sections, and to use this latter method only as a check on a few members.

115. Torsional Loads in a Fuselage.—The exact solution of the problems connected with the torsion of a fuselage is almost impossible. It is possible however, by making certain assumptions, to obtain results which are sufficiently accurate to afford a basis of design.

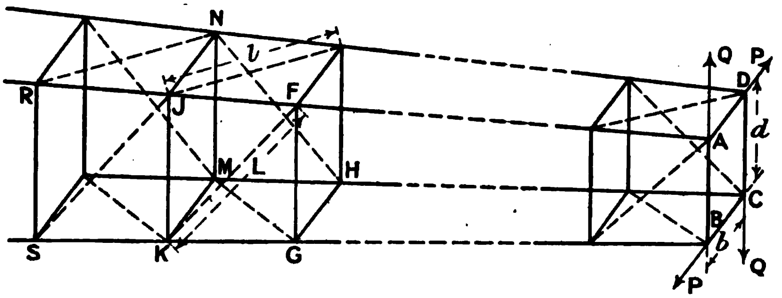

FIG. 130.

The assumptions made are :—

(1) The whole of the deformation is due to the stretch of the vertical and horizontal panel wires.

(2) The bulkhead bracing wires take such a small amount of load that their effect can be neglected. This assumption is fully justified by tests.

(3) The cross section of the fuselage is everywhere rectangular.

(4) The curvatures of the longerons are negligible.

Fig. 130 shows a fuselage supported at the panel RS with a couple μ applied at the end.

This couple may be replaced by forces P, P and Q, Q acting in the horizontal and vertical planes respectively, so that

$$\mathrm{P}d + \mathrm{Q}b = \mu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where d and b are the depth and breadth of the fuselage at the end panel.

Consider the top face of the fuselage.

The wires are stressed by the force P and the loads in them can be expressed as functions of P.

Let the load in any wire be represented by kP.

Let the length of this wire be l.

Let the cross sectional area of this wire be a.

Let $x = \frac{k^2 l}{a}$.

Let the modulus of elasticity for the wire be E.

Then the work done in stretching the wire is

$$u_t = \frac{1}{2} \cdot \frac{k^2 P^2 l}{aE} = \frac{1}{2} \cdot \frac{P^2}{E} x$$

And the total work done in stretching the whole of the top panel wires is

$$U_T = \frac{P^2}{2E} \Sigma x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In the bottom panels the opposite diagonals are stretched and if k_1, l_1, a_1, x_1 represent the corresponding values for these wires that k, l, a and x did for the top panels, then

$$U_B = \frac{P^2}{2E} \Sigma x_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For the side panels let KQ and $K_1 Q$ represent the loads in a wire on the port and starboard sides respectively. Also let A and A_1 be the cross sectional areas of these wires, L and L_1 the lengths of these wires, and X and X_1 defined as for x and x_1.

Then $$U_P = \frac{Q^2}{2E} \Sigma X \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and $$U_S = \frac{Q^2}{2E} \Sigma X_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Then the total work done on the panel wires of the whole fuselage is

$$U = U_P + U_S + U_B + U_T$$

Therefore $$U = \frac{Q^2}{2E}[\Sigma X + \Sigma X_1] + \frac{P^2}{2E}[\Sigma x + \Sigma x_1] \quad . \quad . \quad . \quad (6)$$

From equation (1) $$P = \frac{\mu - Qb}{d}$$

Substituting this value in equation (6)

$$U = \frac{Q^2}{2E}[\Sigma X + \Sigma X_1] + \frac{(\mu - Qb)^2}{2Ed^2}[\Sigma x + \Sigma x_1] \quad . \quad . \quad . \quad (7)$$

This value of U must be a minimum.

Hence $\frac{dU}{dQ} = 0$.

$$\frac{dU}{dQ}=\frac{Q}{E}[\Sigma X+\Sigma X_1]+\frac{b^2Q-\mu b}{Ed^2}[\Sigma x+\Sigma x_1]$$

Therefore $Q[\Sigma X+\Sigma X_1+\frac{b^2}{d^2}\Sigma x+\frac{b^2}{d^2}\Sigma x_1]=\frac{\mu b}{d^2}[\Sigma x+\Sigma x_1]$

or

$$Q=\frac{\mu b[\Sigma x+\Sigma x_1]}{d^2[\Sigma X+\Sigma X_1]+b^2[\Sigma x+\Sigma x_1]} \quad . \quad . \quad . \quad . \quad (8)$$

Similarly

$$P=\frac{\mu d[\Sigma X+\Sigma X_1]}{d^2[\Sigma X+\Sigma X_1]+b^2[\Sigma x+\Sigma x_1]} \quad . \quad . \quad . \quad . \quad (9)$$

If each face of the fuselage is symmetrical about a central longitudinal axis so that $x=x_1$ and $X=X_1$, then equations (8) and (9) above reduce to

$$Q=\frac{\mu b\Sigma x}{d^2\Sigma X+b^2\Sigma x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$P=\frac{\mu d\Sigma X}{d^2\Sigma X+b^2\Sigma x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Let the angle through which the fuselage is turned by the couple μ be θ. Then the work done by the couple $=\frac{1}{2}\mu\theta$.
Therefore

$$\tfrac{1}{2}\mu\theta=U$$

or

$$\theta=\frac{2U}{\mu} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Method of applying the Above Results.—Stress diagrams should be drawn for the horizontal and vertical bracing of the fuselage under consideration with a unit load applied in the appropriate direction.

If the fuselage is symmetrical as above, two stress diagrams only are needed; but if it is not, four will be required, giving results for loads in both directions on each set of bracing.

The loads in the wires under this unit loading are the values of K, K_1, k and k_1 required.

The lengths of the wires (L, L_1, l and l_1) are scaled direct from the frame diagram and the areas (A, A_1, a and a_1) are known from the size of wires employed.

X, X_1, etc., can now be obtained, and hence the values of P and Q.

The loads in the fuselage are obtained by multiplying the unit loads obtained from the stress diagrams referred to above by the values of P and Q thus found, so that a second set of stress diagrams is unnecessary.

P and Q now being known, the value of U can be determined from equation (7), and from this θ can be found by substitution in equation (12).

The work is best done in tabular form, and a suggested scheme is indicated below. In this it has been assumed that the fuselage is symmetrical. If this is not so, extra columns must be added for the other terms introduced.

1	2	3	4	5	6	7	8	9	10	11	
Bay	K	K^2	L	A	X	k	k^2	l	a	x	
					Σ					Σ	

It should be noticed also that the angle θ will be required in calculations dealing with the torsional oscillation of a fuselage, and columns may be conveniently added to the above table, showing the angle turned through by each bay. These being summed give the complete angular distortion of the structure. It is useful to keep the bays separate in calculating θ, as if trouble is experienced in a fuselage due to torsional oscillations, a table such as the above enables the weak bay to be detected.

Another solution of this problem has been obtained by Mr. Arthur Berry and Sub-Lieut. Garner, R.N.V.R.

The mathematical work is very complex, and is not quoted here, but the formulæ obtained, although differing in form from those above, give identical results.

The case considered is that of a fuselage with a straight taper.

Each bay of each of the four sides of the fuselage is supposed to be braced with a pair of diagonal wires in the usual way, but there are no wires in the bulkheads, with the exception of a pair in the bulkhead at the free end. It is assumed that when the couple is applied, one wire of each pair is always slack. The fuselage is supposed to be pin-jointed at all the joints, and the longerons and struts to be incompressible.

The applied couple μ can be resolved into two pairs of forces P, P; Q, Q, the first pair acting along the horizontal, and the second along the vertical struts of the end bulkhead. The forces P, Q must be so connected as to be equivalent to the given couple, but they are not assumed to be known separately. The points of the struts at which these forces act are unimportant, any alteration in them only causing an alteration in the stresses in the end bulkhead, and not altering the stresses in the rest of the fuselage.

The problem is indeterminate by the methods of pure statics, there being two arbitrary quantities; there is a further indeterminacy in the reactions at the fixed end, which are not discussed. Assuming that the wires, supposed just slack before the couple is applied, stretch in accordance with Hooke's Law, there is a further equation, which reduces the number of arbitrary elements to one. From experiment it is known that the tensions in the bulkhead wires are always small, so that it is assumed that the extension of a diagonal of the end bulkhead can be neglected, compared with those of the other wires. This makes the problem determinate.

The fuselage consists of n bays. One end is held fixed, and the couple is applied to the other end ABCD, as shown in Fig. 130. Only one wire of each pair stretches, and from the sense of the couple it follows that the wires shown in the figure stretch, the opposite wires remaining slack.

The notation is as used throughout this paragraph. Further, let

$$V = \Sigma \frac{l^3}{FG^2 . JK^2}, \quad v = \Sigma \frac{L^3}{GH^2 . KM^2}$$

the summation extending to all the n bays, of which FJ is a specimen.

Solution.

Tension in wire $\quad FK = \dfrac{dL}{FG . JK} Q$

Tension in wire $\quad GM = \dfrac{bl}{GH . KM} P$

Compression in longeron $\quad FJ = \dfrac{AJ}{JK} Q - \dfrac{AF}{GH} P$

Compression in longeron $\quad GK = \dfrac{AJ}{KM} P - \dfrac{AF}{Fb} Q$

Compression in strut $\quad FG = \dfrac{d}{FG} Q$

Compression in strut $\quad GH = \dfrac{b}{GH} P$

$$P = \frac{ad^3 v\mu}{Ab^4V + ad^4v}, \quad Q = \frac{Ab^3V\mu}{Ab^4V + ad^4v}$$

and the angle of torsion

$$\theta = \frac{2Vvb^2d^2\mu}{\Sigma(Ab^4V + ad^4v)}$$

Having determined all the forces in the members of the fuselage for the worst condition of each type of loading upward and downward on the tail structure, side loads from fin and rudder, torque from unequal loads on tail plane or from offset fin and rudder for engine torque, thrust, etc., the results should be tabulated and the members designed for the maximum load which each can be called upon to carry.

The longerons may be either struts or tension members, and must be considered from each standpoint, although it is usually only one part of the member which needs consideration under tension, and that is where it joins the main structure if it does not happen to be a continuous member, or at any other point where there is a joint in the timber.

The other wooden members are always struts, the tension being taken up by the bracing wires. The methods of calculating the sizes necessary for these members, having found the loads in them, will be dealt with in Chapter XVIII. on detailed design.

CHAPTER XIII

FORCES AND LOADS UPON CONTROL SURFACES

116. Insufficient Data.—The problem of the magnitude and distribution of the forces occurring for all flight conditions upon the control surfaces of an aeroplane is, unfortunately, one upon which detailed knowledge is lacking, owing to the insufficiency of tests both upon models and full-scale aeroplanes. The designer is, therefore, reduced to a method of calculation in which, by taking extreme cases, he is enabled to build a structure which will at least be safe.

If definite wind tunnel experiments are available for an aeroplane or its units under design, these should be taken as giving the distribution of loads required; but if, as is more often the case, such tests do not exist, recourse must be made to the cruder methods of estimation.

117. The Tail Plane and Elevators.—The loads which can occur upon the tail plane and elevator of an aeroplane vary considerably, both in magnitude and distribution, according to the condition of flight. The usual cases requiring consideration in design are as follows.

(1) At very slow speeds the centre of pressure upon the wings may be forward of the centre of gravity of the aeroplane, which will necessitate an upward tail load to keep it in balance.

(2) In high-speed horizontal flight the centre of pressure upon the wings is aft of the centre of gravity of the aeroplane, which will render a down load on the tail necessary for balance.

(3) In a dive the conditions are practically similar to those of high-speed horizontal flight—the centre of pressure is a long way behind the centre of gravity of the aeroplane, and a downward tail load is the result. The extreme case of this occurs if the aeroplane is diving at its terminal velocity, when the tail control load becomes very large.

(4) When an aeroplane's attitude is changed suddenly from a fine angle of incidence to a coarse one, a down load upon the tail plane is required to provide the moment turning the aeroplane. This load will, of course, be in excess of the steady control load required when flying in equilibrium at the finer angle of attack.

(5) The slip stream from the propeller has a rotating motion, and if the tail plane of the aeroplane is in this slip stream, the effect is an added upward load on one side of the tail and an added downward load on the other, giving an unsymmetrical loading to the surface.

Further, the tail plane may be so situated that only one portion of it is in the slip stream, and this may lead to an unsymmetrical load similar to the spin effect above. The magnitudes of the loads occurring due to the causes mentioned under cases (1) to (4) above, have already been dealt with under the discussion on fuselage loads in Chapter XII. The tail loads due to the propeller slip stream spin will be covered by the other cases, and if the strength is sufficient under these conditions, it will be ample for the unsymmetrical load.

The difficulty in making a correct estimate of the strength of a tail plane and elevators lies in the impossibility of determining the distribution of the load over the surface without carrying out actual tests—either model or full scale.

The centre of pressure shift on an average tail plane is very large, and it is, moreover, not uniform over the whole span. This was shown by full scale tests upon an aeroplane which had given trouble in service owing to the failure of the tail plane during flight. Pressure tubes were arranged over the whole surface, and these were connected to the measuring scale situated in the observer's cockpit. Photographic records of the pressure distribution were obtained at varying speeds up to 150 m.p.h., and the curve shown in Fig. 131 gives the centre of pressure plotting for the highest speed attained. It should be noted in this connection that the total load upon the tail and elevators was in close agreement with calculated figures, but it would be an impossibility to forecast the way in which experiment showed it to be distributed. Close to the fuselage the centre of pressure was well on the tail plane, but it rapidly came forward as sections nearer the tips were considered, crossed the leading edge and went well forward of the leading edge, coming back to the tail plane as the tip was reached.

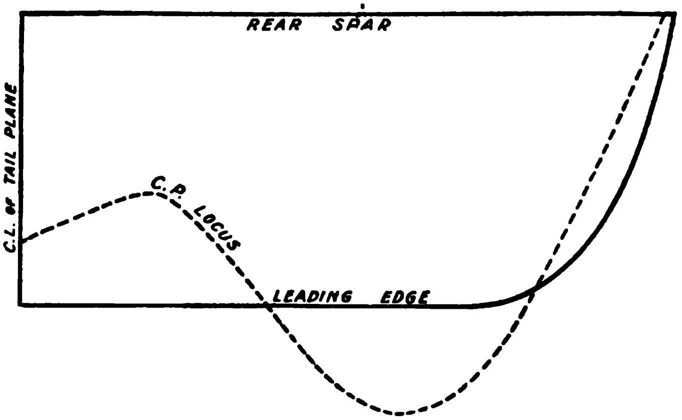

FIG. 131.

This result emphasizes the necessity wherever possible of obtaining actual experimental data relating to the pressure distribution, and where this is unobtainable of assuming extreme conditions for strength calculations. It is wise, therefore, to design a tail plane and elevator so that it has the required factor when the whole of the calculated control loads are carried on the leading edge in one case, and on a line ·5 of the combined chord from the leading edge in a second case.

If the centre of pressure is taken on the leading edge it is clear that this will imply a reversal of loading towards the trailing edge, since without such reversal the centre of pressure cannot reach the position stated. Fig. 132 shows diagrammatically a typical distribution which brings out the point mentioned.

For a centre of pressure of ·5 of the chord from the leading edge the simplest load diagram is a rectangle, also shown in Fig. 132. This, it may be mentioned, is not a form of distribution which actually does occur, but is an approximation which will be on the safe side and will simplify calculations.

The size of the front spar of the tail plane will be determined by the loads in it when the centre of pressure is forward, and the rear spar when the centre of pressure is back. The elevators will be most heavily loaded when the centre of pressure is back, and should be designed for this case.

The loading per foot run upon the front spar may be found in the following way.

The equivalent span for the spar should be determined in the same way as for the main planes, using the same wing tip corrections in the absence of more complete tests.

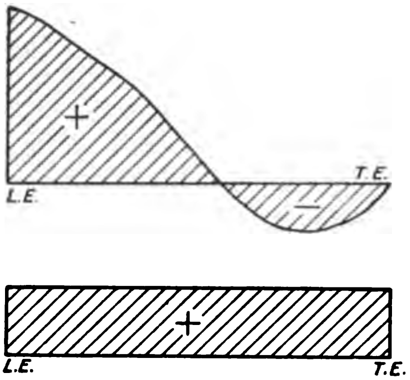

Fig. 132.

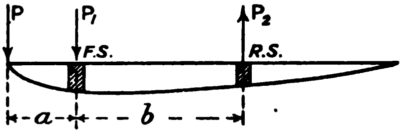

Fig. 133.

Let the total load on the tail be P, concentrated on the leading edge.

In Fig. 133 let the distance from the leading edge to the front spar be a, and from the front to the rear spar be b.

Then taking moments about the rear spar

$$P_1 b = P(a+b)$$

where P_1 is the total load on the front spar,

and so $$P_1 = \frac{(a+b)}{b}P$$

This is assumed distributed along the equivalent spar length uniformly.

There will be an upward load on the rear spar at the same time equal to P_2, where

$$P_2 = \frac{aP}{b}$$

This load will be partly distributed over the rear spar, similarly to the front spar load, and will partly be transmitted from the elevators through the hinges as a series of concentrated loads. As a general rule, however, this is not a very serious condition for the rear truss of the tail, the strength of which will be determined for the case of the centre of pressure in its most backward position.

Having found the loading upon the front spar, the method of calculation is straightforward, since a tail plane usually consists of two trusses similar to the main planes. The loads in these trusses are found by the usual methods, and the strengths of their members estimated as for corresponding members in the main plane structure.

The method of dealing with the rear truss is somewhat different, as follows. In the figure, A represents the leading edge, F.S. the front spar, and C the trailing edge.

At B is the rear spar and the elevator spar connected by a series of hinges.

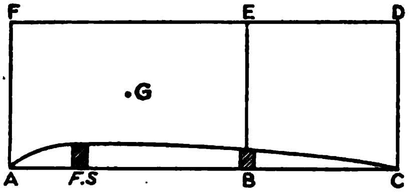

FIG. 134.

The load diagram may be represented by the rectangle ACDF, of which the portion ABEF is carried by the front and rear trusses of the tail plane, and the portion BEDC by the elevator. As a general rule, the elevator is so designed that the whole load on it is transmitted to the elevator spar, and this will be assumed in what follows.

In the first place, the evenly distributed load arising on the rear spar from the portion FEBA should be calculated by assuming it acting through its centre of gravity G.

Next, the distributed load upon the elevator spar must be calculated. This will be the total load represented by the area EDCB divided by the equivalent length of the elevator spar.

This spar should then be considered as a continuous beam supported at the hinges and the bending moments, and the reactions at these supports calculated by the theorem of three moments.

These reactions are transmitted to the rear spar of the tail plane as concentrated loads, so that the whole condition of loading of this member is now determined.

Having found the concentrated and distributed loads, the generalized theorem of three moments will enable the reactions at the support points of the truss and the bending moments in the spar to be calculated, and once this has been done, the strength of the truss is estimated by the usual methods.

Different types of tail planes present slightly different problems, but all of them are easily designed on the principles outlined above.

The calculation of the strength of the elevator itself is a somewhat more complicated problem, and generally introduces the necessity for making many assumptions. Consider the elevator flap shown in Fig. 135. This consists of an elevator spar E.S., and a trailing edge T.E., connected by ribs *rr*. The control is effected by means of a kingpost or control lever K.P., which should be wired back to a point *p* on the stringer *s*. In the first place, it is clear that the elevator may be turned through an angle by two means according to the method of attachment of the control lever. If this lever is rigidly attached to the elevator spar the control is effected by torsion of this spar; but if the lever is not rigidly attached the control is effected through the back wire *b.w.*

As a general rule, the first of these methods is adopted, but it is well in design to allow for either, in case of partial failure. The rule to be borne in mind is to commence with the design of the weaker member and follow up to the main structural parts.

The trailing edge may be considered purely as a boundary, *i.e.* it need not be assumed to carry any load—in many aeroplanes the trailing edge is formed of kite cord with very satisfactory results. The stringer *s* may be either a structural member or simply a stabilizing member in order to prevent lateral failure of the ribs. The latter is its usual function, and the loads in it then are negligible.

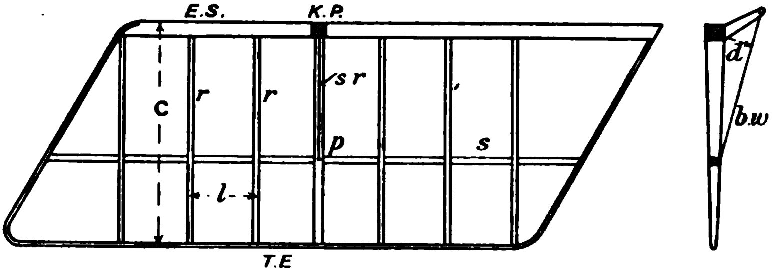

FIG. 135.

The ribs are the next members in importance. Their function is to transmit the load from the surface to the elevator spar, and to do this efficiently they should be well connected to this member. The loads transmitted to the elevator spar are in turn transmitted to the rear spar of the tail plane through the hinges, and thence through the main trusses of the tail structure to the fuselage.

In the figure let the chord of the elevator be denoted by *c* inches, and the space between ribs by *l* inches. Then, since the loading is assumed to be evenly distributed over the surface, the load carried by one rib is

$$w \,.\, c \,.\, l \text{ lbs.}$$

where *w* is the loading per square inch.

The maximum bending moment on the rib is at the point of attachment to the elevator spars, and is

$$\frac{wc^2l}{2} \text{ inch-lbs.}$$

The rib must, therefore, be designed to carry this moment, and the attachment must be designed to transmit it to the spar.

If the kingpost is attached to the spar, so that it can operate by torsion, all the ribs will be much the same. In this case special attention should be paid to the fitting of the control kingpost.

If A is the total area of the elevator in square inches, then the total moment to be transmitted is

$$\frac{A\,.\,w\,.\,c}{2}\text{ inch-lbs.}$$

and this gives the required strength of the fitting.

If no reliance is to be placed on this fitting, and control is to be exercised through the back wire *bw*, the rib *sr* will require to be specially designed, as it will have a compressive load in it arising from the wire.

Let the perpendicular distance from the elevator spar to the back wire be d.

Then the moment of the control force F in this wire about the elevator spar is dF.

This must be equal to the moment of the elevator load about the same point, so that

$$dF = \frac{A\,.\,w\,.\,c}{2}$$

or

$$F = \frac{A\,.\,w\,.\,c}{2d}\text{ lbs.}$$

The horizontal component of this force F will be taken on the special rib *sr*, so that this member must be designed to carry its normal distributed load, plus the thrust due to F.

The ribs having been designed the next member is the elevator spar.

This will have a series of concentrated loads each equal to wcl at the rib attachment points, and a torque of $\frac{wc^2l}{2}$ inch-lbs. in addition at each of these points.

This spar, then, is stressed under the system of loads and moments shown in Fig. 136, the reactions being at the kingpost for the torsion and at the hinges for the direct loads.

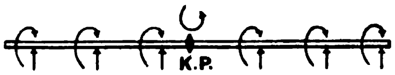

FIG. 136.

Since the stress is the principal concern in the first place, the procedure will be as follows.

The distribution of the torque only affects the angle through which the spar is turned, the maximum stress being unaffected, so that, in the first place, the whole torque may be considered concentrated at one section.

The amount of the torque is $\frac{A\,.\,w\,.\,c}{2}$ inch-lbs. $= T$. The stress in the spar due to this torque will depend upon the shape of the spar section, and is only directly calculable for simple shapes. If the section is solid circular

$$f = \frac{2T}{\pi R^3}$$

where R is the radius.

For a hollow circular section it is

$$f = \frac{2TR}{\pi(R^4 - r^4)}$$

where R and r are the external and internal radii respectively.

If the section is of very thin material and circular

$$f = \frac{T}{2\pi R^2 \theta}$$

where θ is the thickness of the material.

For a solid ellipse if $2a$ = major axis and $2b$ = minor axis

$$f = \frac{2T}{\pi a^2 b}$$

at the end of the minor axis; and

$$f = \frac{2T}{\pi a b^2}$$

at the end of the major axis.

For a solid rectangular section the maximum stress is at the centre of the longest side, and is

$$f = \frac{T}{ab^3}\left(3 + 1{\cdot}8\frac{b}{a}\right)$$

where a is the length of the long side and b that of the short side.

An approximate formula for any hollow section in which the thickness is small compared with the smallest outer dimension is

$$f = \frac{T}{t(A + A')}$$

where A is the area of the outside boundary,
A′ ,, ,, inside boundary, and
t is the thickness at the narrowest width of the figure.

The above formulæ will enable a figure for the shear stress due to the torque to be arrived at, and with this must be combined the stress due to bending, which is found in the usual way.

If f_1 is the shear stress due to the torsion and f is the direct stress; then the maximum shear stress is given by

$$\sqrt{\tfrac{1}{4}f_1^2 + f^2}$$

and the maximum direct stresses are given by

$$\tfrac{1}{2}f_1 + \sqrt{\tfrac{1}{4}f_1^2 + f^2}$$

Neither of these values should exceed the allowable limit for the material of which the spar is designed.

In some cases it may be necessary to calculate the angle through which the spar is turned under the torsion from the ribs, and then it will be necessary to consider the effect of each rib torque acting at its point of application separately.

If l is the distance of any rib from the control kingpost, N the modulus of rigidity for the material used, and θ the total angle turned through in radians. Then for a solid circular section

$$\theta = \Sigma \frac{2\mathrm{T}l}{\pi \mathrm{R}^4 \mathrm{N}}$$

For a hollow circular section

$$\theta = \Sigma \frac{2\mathrm{T}l}{\pi(\mathrm{R}^4 - r^4)\mathrm{N}}$$

For a solid ellipse

$$\theta = \Sigma \frac{(a^2 + b^2)\mathrm{T}l}{\pi a^3 b^3 \mathrm{N}}$$

For any symmetrical section—including a rectangle—the approximate formula is

$$\theta = \Sigma \frac{40\mathrm{J}\mathrm{T}l}{\mathrm{A}^4 \mathrm{N}}$$

where J is the polar moment of inertia and A is the cross sectional area.

The summation sign in the above expressions denotes that F must be calculated for every rib on one side of the kingpost, and the sum of these gives the total angle of twist upon the spar.

118. Fin and Rudder.—The loads which can occur upon the fins and rudders of an aeroplane in flight have already been dealt with in discussing the strength of the fuselage to resist side loads.

The same difficulty as regards the distribution over the surface is experienced here as with the tail plane and elevators, and, in consequence, the designer is forced to resort to assumptions.

Tests upon models show that when the heaviest loads occur upon fins and rudders, *i.e.* when the rudder is hard over, the centre of pressure upon the surface is well forward.

It is therefore safe—in the absence of exact data—to assume that the load on the rudder is distributed over the surface in such a way that its centre of pressure is at one-third of the chord from the leading edge.

In the case of a combined fin and rudder, judgment must be used as to how much of the fin should be assumed operative, *e.g.* in the design shown in Fig. 137, it would probably be underestimating the loads on the rudder if the centre of pressure on the line AB were assumed to be at $\frac{1}{3}c$ from A, since the interference effect of the fuselage would probably be such as to render a large part of the fin inoperative.

In this case, therefore, it would be wise to take out the strength of the rudder as if there were no fin attached to it, *i.e.* take the whole control load required distributed over the rudder surface so that its centre of pressure is at one-third of the rudder chord.

The assumptions required in any particular case must be left to the individual designer, since it is impossible to lay down general rules. It cannot be too firmly insisted upon, however, that the rudder is a very important unit, and its strength must be well on the safe side.

The actual procedure in estimating the loads in the various members is very similar to that used for the elevators. Fig. 138 shows a balanced rudder of a usual type built up of rudder post R.P., ribs *r*, *r*, leading edge *le*, trailing edge *te*, stringer *s*, control kingpost K.P., and special strut rib *sr*.

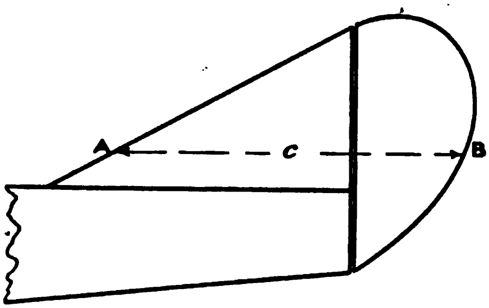

FIG. 137.

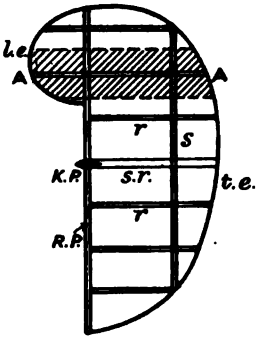

FIG. 138.

The total load on the rudder is P. The total area of the rudder is A. Assuming that the trailing and leading edges are not designed to take any load, the function of the ribs is, as in the case of the elevator, to transmit the loads and torques to the rudder post, which in turn transmits them to the stern post of the fuselage.

Consider the rib AA. This member carries the load upon the area which is shaded. Let this area be *a*. Then the load on rib AA $= \frac{a}{A}$ P.

The simplest distribution curve giving a centre of pressure coefficient of ·33 is a triangle, with its maximum ordinate at the leading edge. Hence Fig. 139 can represent the loading upon the rib AA.

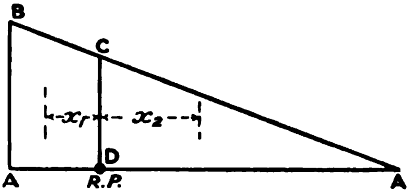

FIG. 139.

From this it is clear that the front or balance part of the rib carries the load represented by the area BCDA, and the rear part of the rib the load represented by DCA.

The centres of gravity of these areas should be found, and calling the distances of these from the post x_1 and x_2, the bending moment at D from the balanced portion is

$$\text{(Load corresponding to area BCDA)} \times x_1$$

And at D from the rear portion is

$$(\text{Load corresponding to area CDA}) \times x_2$$

Every rib is treated in this way, and the calculations then follow exactly the lines laid down for the elevator flaps.

The fin itself should be made strong enough to carry its own portion of the total load on the fin and rudder surfaces, assuming a distribution to give a centre of pressure at one-third of the chord, even in the case quoted previously, where it is possible that it is so shielded that heavy loads cannot come upon it.

The difficulty in constructing a strong and rigid fin is so small that very stringent assumptions may be made as to the loading upon it without increasing the weight of the resultant structure beyond a negligible amount.

119. Ailerons.—As regards the actual calculations for the strength of an aileron, the methods given for an elevator may be applied directly.

The loads upon an aileron, however, are not very large, and vary according to the position of the centre of pressure upon the aerofoil.

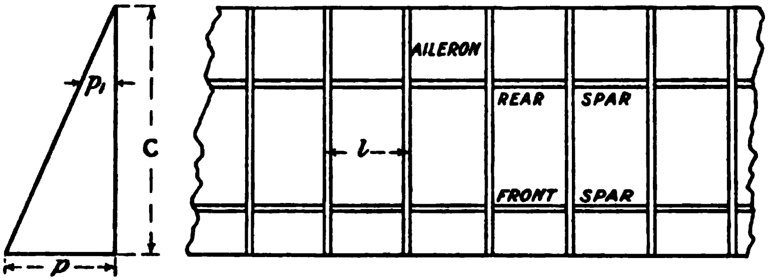

FIG. 140.

Here, again, when tests are available they should be used; but in their absence the triangular distribution of load may be safely assumed with the maximum ordinate upon the leading edge.

In Fig. 140 is shown a portion of a wing and the attached aileron. If w is the load per square inch upon the planes, C the chord, and l the distance between ribs in inches: then the load on one rib $= Clw$ lbs.

If p is the intensity of loading on the rib at the leading edge of the plane
and p_1 ,, ,, ,, ,, ,, ,, ,, aileron,

Then $\quad p = 2wl$ lbs. per inch of chord

and $\quad p_1 = \frac{C_1}{C} \cdot p$

where C_1 is the aileron chord.

Therefore the total load on one aileron rib is

$$\frac{C_1}{C} p \times \frac{C_1}{2} \text{ lbs.}$$

or

$$\frac{C_1^2}{C} wl \text{ lbs.}$$

This is distributed over the rib, so that its centre of pressure is $\frac{1}{3}C_1$ from the leading edge of the aileron.

Therefore the moment about the aileron spar is

$$M = \frac{C_1^3 wl}{3C} \text{ inch-lbs.}$$

These ribs are designed in the same way as rudder and elevator ribs, and the loads are transmitted from the aileron spar to the main rear spar exactly as described for the elevators.

It should be noticed in connection with ailerons, that when extra loads are thrown upon them, due to the manipulation of the controls, the aeroplane immediately commences to bank, and the load is automatically relieved, so that such a margin of strength as is required on elevators is not so essential in these flaps.

120. Variable Flaps.—Some aeroplanes have been designed with adjustable wing flaps which could be elevated or depressed on both sides simultaneously, differing in this respect from the usual wing flaps or ailerons which are raised on one side and lowered on the other.

The object of these adjustable flaps was to alter the effective camber of the wings, and so obtain either a high lift or a high speed section at will. Some amount of success attended the experiments, and it is probable that the idea will be revived.

The loads upon these surfaces are heavier than upon ailerons, since the aeroplane does not give to them as it does in a bank.

For this reason they must be stronger in construction than ailerons usually are, and the control cables working them must also be heavier, as well as fittings, pulleys, etc.

It would be wise, therefore, in designing flaps for this purpose to assume both a triangular loading as above, and also an evenly distributed load, as suggested for the elevator, and to make all members strong enough to resist the forces induced by such loading.

Apart from this, there is no more difficulty in designing such members than in designing an ordinary aileron.

121. Wind Brakes.—On some fast landing aeroplanes flaps are introduced which can be lowered to act as brakes in order to slow up the aeroplane in a shorter distance than would otherwise be possible.

These may be lowered to meet the air, practically normally, and so should be designed for this condition.

The normal pressure of air at 60 m.p.h. is about 10½ lbs. per square foot, and this figure may be designed to. If the landing speed is V m.p.h., the loading will be $\left(\frac{V}{60}\right)^2 \times 10{\cdot}5$ lbs. per square foot, and a factor of safety is required on this. These surfaces are not used in the air, but only to decelerate a fast landing, and they are not, therefore, so important from the view of safety in flight as the control surfaces. At the same time, their failure might occasion a mishap on landing, and they should, therefore, receive careful consideration.

CHAPTER XIV

LANDING LOADS

122. General Considerations.—The methods outlined in preceding chapters have only considered the aeroplane under various conditions of flight. It is proposed in this chapter to examine what parts of the structure are stressed when the aeroplane lands, and to give methods of calculation for finding the strengths of these parts.

Owing to the uncertainty with regard to the conditions at the instant of alighting, it is usual to investigate the aeroplane structure not for this condition, but for standing loads when the aeroplane is stationary upon the ground, and to allow a factor of safety which will cover the shocks due to the actual landing. A factor of from four to six upon these standing loads has been found satisfactory for all modern types of aeroplanes in service.

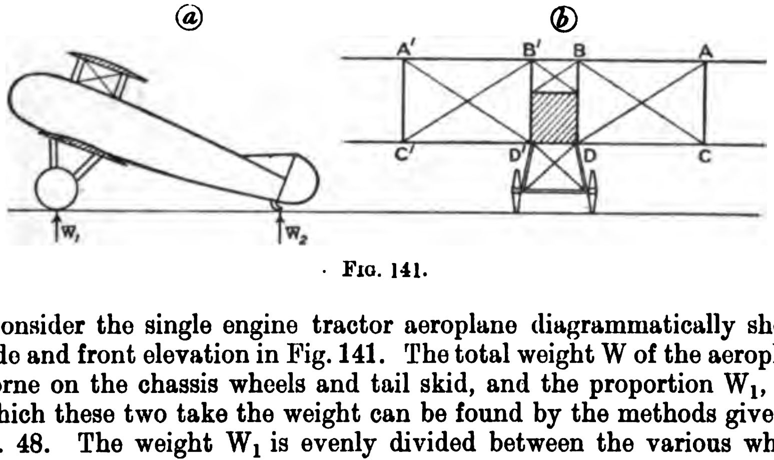

Fig. 141.

Consider the single engine tractor aeroplane diagrammatically shown in side and front elevation in Fig. 141. The total weight W of the aeroplane is borne on the chassis wheels and tail skid, and the proportion W_1, W_2, in which these two take the weight can be found by the methods given in para. 48. The weight W_1 is evenly divided between the various wheels in the chassis. In paras. 124 and 125 it is explained how these forces W_1 and W_2 affect the landing carriage, tail skid and body of the aeroplane.

In the front elevation shown in (*b*) Fig. 141, BC, B′C′ are the landing or anti-flying wires of the aeroplane, and these wires, in the front and rear trusses, carry the weight of the wing structure when the aeroplane is on the ground. These wires put compressive end loads into the bottom spars of the aeroplane, and it is important that these spars should be examined from this point of view.

In the case of a twin engine aeroplane there is usually an under-carriage under each engine, so that when the aeroplane is resting upon the ground there are three points of support, namely, the two under-carriages and the tail skid. The reactions at these can be found by balancing out the aeroplane in the following way :—

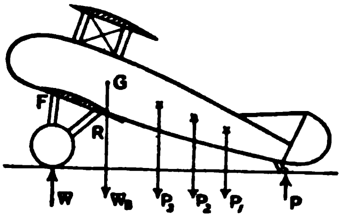

FIG. 142.

In Fig. 142 W represents the wheel base, P the tail skid, P_1, P_2, P_3, and so on, concentrated loads in the fuselage, as passengers, tanks, etc.

By taking moments about P we obtain at once the total reaction at the wheel base, and by taking moments about W the reaction on the tail skid is found.

The fuselage itself is supported at three points : the tail, and the front and rear trusses.

Let W_B = the weight of the fuselage and the weights carried in it, acting at G its centre of gravity.

By taking moments about F and R, the front and rear trusses, we obtain the proportion of the weight of the fuselage carried on each truss.

This weight is carried by the front and rear flying wires from the fuselage to the engine, and puts tensions in the bottom centre section spars and compressions in the top centre section and inner bays, part of these being relieved by the end loads set up by the antiflying wires in the outer bays. The weight of the wing structure outside the engine nacelles is borne by these anti-flying wires, as in the single engine aeroplane.

The weight of the wing structure may be taken as evenly distributed in the same way as explained in para. 97. The centre of gravity of a wing may usually be taken with sufficient accuracy as coming midway between the spars so that each truss carries one half of the weight.

123. Effect of Engine Pull.—Before commencing a flight it is usual to test the engines while the aeroplane is held on the ground, and during this test heavy loads may be thrown on the structural members. As a general rule the aeroplane is prevented from moving forward by chocks under the wheels.

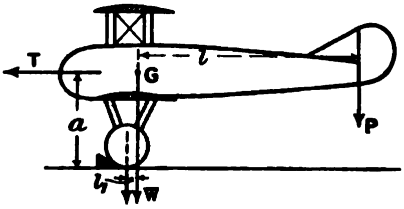

FIG. 143.

In the case of a single engine tractor the loads coming upon the aeroplane when the engine is full out under these conditions are confined to the centre section bracing and fuselage bracing. If the engine is placed high in the wings it will generally be necessary to hold the tail down.

In Fig. 143 let T = the thrust of the propeller.
P = the force required to hold down the tail.
W = the total weight of the aeroplane.

Then if G is the centre of gravity, and measurements are as in the figure, we have, taking moments about the wheel base,

$$Ta - Wl_1 - P(l + l_1) = 0$$

or

$$P = \frac{Ta - Wl_1}{(l + l_1)}$$

This load on the tail will be up or down according as $Wl_1 >$ or $< Ta$.

The centre section incidence wires finally take the loads due to the engine pull to the landing gear, where they balance the resistance due to the chocks.

It may happen that the aeroplane is held back by the tail while the engine is running, and in that case the pull of the engine must be taken along the fuselage structure. The static pull of a modern aeroplane engine may be taken to lie between 6½ and 7 lbs. per horse-power.

In a twin engine aeroplane the pull of each engine comes directly on to the incidence wires in the engine nacelles, and so down to the chassis. If, as would be unusual, the chassis is placed under the fuselage, the engine pull travels along the anti-drag wires of each plane to the centre section, and balances out as before. This pull is distributed between the wings in the inverse proportion of the distances of the line of thrust from the two planes.

124. Skid Loads.—As explained in para. 48, there is, when the aeroplane is standing, a load W on the tail skid which has to be transmitted through the fuselage structure to the centre of the aeroplane. The examination of the members of the fuselage under the load follows precisely the lines laid down in Chapter XII. It should be remembered, however, when calculating the strength of the fuselage members for the skid loads, that the weights of the tail unit, of the fuselage structure itself, and concentrated weights such as pilot, passengers, and goods carried, are all acting as relieving loads. It is necessary to look into the local attachment of the tail skid to the fuselage.

Fig. 144 shows a diagrammatic side elevation of the tail skid structure of an aeroplane. AA′, BB′, are the top and bottom longerons. AC is a shock absorber, either rubber or a steel spring CBD the actual tail skid. The load W is taken at the point D, and is resisted first of all by bending and compression in BD; B acts as a fulcrum, and the load W puts a tension on the spring AC, which in turn has a component in AA′. There is a tension in BB′ and a compression in AA′, each equal to the horizontal component in AC, and forming a couple whose moment is equal to the moment of W about A or B. The tension in AC is found directly by taking moments about B. Let the vertical component of this tension be W′. Then the load in AB is W + W′, This extra component W′ washes out at A, and the nett load coming

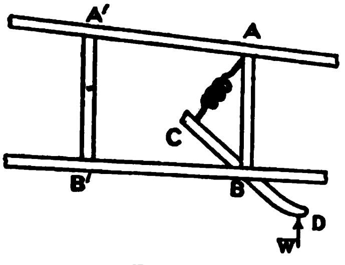

FIG. 144.

upon the remainder of the fuselage is W, less the weights of the tail unit, and so on.

125. Chassis Loads.—Fig. 145 shows the front and side elevations of a typical chassis. ABB′A′ is the axle; BC, B′C′, BD, B′D′ the struts; B′C, BC′ the bracing wires.

If W is the total load coming on the chassis, found as explained in para. 48, then in a level landing the weight coming on each wheel is $\frac{W}{2}$.

These loads must first of all be transmitted by bending on the axle to B and B′. At some points shock absorbers should be placed in order to prevent too sudden a load coming on the structure. Such absorbers are usually carried at B and B′, or in the struts BC, B′C′, BD and B′D′. BB′ is subjected to a constant bending moment equal to $\frac{W}{2} \times a$, and an end load which is usually tensile.

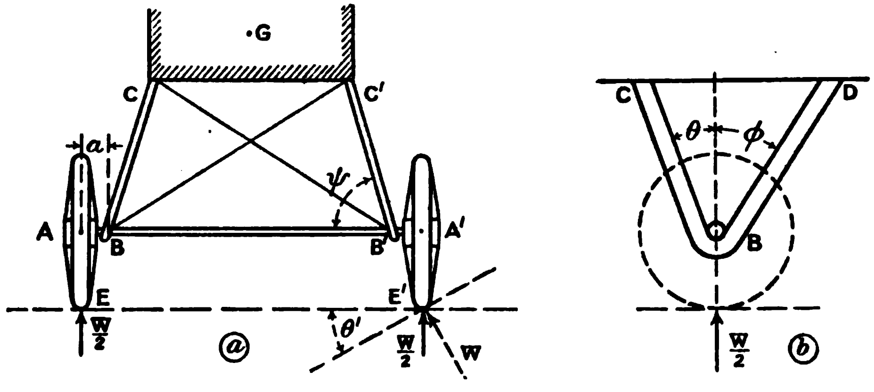

FIG. 145.

The bending moment at B is $\frac{W}{2} \times a$, neglecting the small effect due to the distribution of the load each side of B by the shock absorber.

BC, B′C′, BD and B′D′ are in compression under the load W, and their loads may be calculated by resolving along their directions. In Fig. 145 (*b*) if θ and ϕ are the angles BC and BD make with the line of action of W/2, and ψ is the angle the plane BCD makes with BB′, the loads in BC and BD are, respectively, $\frac{W \sin \phi}{2 \sin (\theta + \phi)} \cdot \frac{1}{\sin \psi}$ and $\frac{W \sin \theta}{2 \sin (\theta + \phi)} \cdot \frac{1}{\sin \psi}$. The end load in the axle BB′ is $\frac{W}{2} \cot \psi$.

The correct amount of rubber required in the shock absorbers can be found in the following way :—

Let l = the limit of vertical travel, measured in inches, allowed on the axle before failure of the under-carriage occurs.

N = static load factor at which the under-carriage fails.

Then assuming two wheels, the static load on each wheel at failure $=\frac{NW}{2}$, and the work done in stretching the rubber in the absorber

$$=\frac{l}{2}\cdot\left(\frac{NW}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let A = the cross sectional area of the rubber cord in square inches before stretching.
L = the total length of cord used in one under-carriage in inches.
E = the modulus of elasticity of the rubber in lbs. per square inch.
e = the total extension of the cord in inches.
f = the ultimate *allowable* stress per square inch of rubber.
P = the load required to stress the rubber to f lbs. per square inch.

Then $$e=\frac{fL}{E}=\frac{PL}{AE}$$

and the work done on the rubber due to the load P is

$$e\times\frac{P}{2}=\frac{P^2L}{2AE}=\left(\frac{P}{A}\right)^2\times\frac{AL}{2E}=\frac{f^2V}{2E} \quad . \quad . \quad . \quad . \quad (2)$$

where V = the volume of rubber used.

The work absorbed by the rubber must be equal to the work done on it by the load $\frac{NW}{2}$.

Therefore equating (1) and (2)

$$\frac{f^2V}{2E}=\frac{NWl}{4}$$

or $$V=\frac{NWlE}{2f^2}$$

giving the volume of rubber required for each shock absorber.

From this result a selection of the size cord can be made, as any reasonable combination of A and L giving the above value of V will be satisfactory.

The possibility of an aeroplane landing on one wheel must be considered, because in so doing it may throw a heavy compressive load into the axle. The maximum angle at which safe landing on one wheel is possible is limited in one of two ways, either—

(1) When the centre of gravity, G, of the aeroplane is directly over the base of one wheel, or

(2) When the wing tip touches the ground.

As a general rule the latter condition settles the maximum angle of landing. This angle is, in most aeroplanes, between 15° and 20°.

In Fig. 145 let θ' be the angle between the horizontal and the axle when the aeroplane is landing on one wheel.

In such a landing it must be remembered that the load on the wheel is W, and not $\frac{W}{2}$. We will consider this from the wheel base up to the axle and chassis structure generally, and so to the fuselage.

First of all, there is a bending on the wheel itself, taken up by bending on the axle at A′. The amount of this bending moment is $W \sin\theta' \times d$, where d is half the diameter of the wheel.

The total bending moment on the axle is $W \times a'$, where a' is the perpendicular distance from B′ on to the line of action of W.

The compression in BB′ is $W \cos\theta' \cot\psi - W \sin\theta'$ by resolution. This is taken up by the wire BC′, and the struts BC, BD, the loads in which are found in the usual way.

The loads in the chassis structure being known, the strength of the various members may be calculated by the methods to be outlined in the following chapters.

The following table gives the size of tyre required for two-wheeled chassis for aeroplanes weighing up to 12,800 lbs.

The sizes quoted are for Michelin standard tyres.

TABLE.

Size of tyre.	Contents of inner tubes, cu. ins.	Suitable for aeroplane weighing.
26″ × 2½″	400	2,000 lbs.
700 × 85 mm.	757	3,780 ,,
750 × 85 ,,	812	4,000 ,,
800 × 85 ,,	867	4,400 ,,
710 × 90 ,,	861	4,300 ,,
760 × 90 ,,	923	4,610 ,,
810 × 90 ,,	985	4,900 ,,
870 × 90 ,,	1,055	5,250 ,,
910 × 90 ,,	1,100	5,500 ,,
815 × 105 ,,	1,350	6,750 ,,
875 × 105 ,,	1,450	7,250 ,,
915 × 105 ,,	1,520	7,600 ,,
820 × 120 ,,	1,776	8,900 ,,
850 × 120 ,,	1,845	9,240 ,,
880 × 120 ,,	1,900	9,500 ,,
920 × 120 ,,	2,000	10,000 ,,
895 × 135 ,,	2,480	12,400 ,,
935 × 135 ,,	2,560	12,800 ,,

CHAPTER XV

THE PARTIALLY DISABLED AEROPLANE

126. The General Problem.—The problem of the disabled aeroplane is one which must be faced by the designer. Any aeroplane should be capable of reaching the ground safely even after certain parts of it have failed in the air. It is impossible to guard completely against the effect of breakage of any part, for that would mean either complete duplication of all parts, or such an increase in structure weight to give the necessary load factor, that the aeroplane would be too heavy for any useful purpose. The designer, therefore, can only duplicate or strengthen such parts as are liable to accidental breakage, or to see that, should any particular part of the structure fail, the remainder will be able to carry the extra loads thrown upon it until the aeroplane has landed. The main flying wires should always be duplicated, either directly or by means of incidence bracing. There are certain parts of an aeroplane which must definitely be ruled out from a duplication point of view. It would, for example, be absurd to have duplicate interplane struts, duplicate fuselage rails, and so on, for these can easily be designed with any reasonable margin of safety thought fit, without adding too greatly to the percentage structure weight of the aeroplane. It occasionally happens that certain drag struts of an aeroplane are duplicated, as, for example, those at the part of the planes where these are attached to the centre section. In that case either might fail under the total load coming upon them.

127. Breakage of a Flying Wire.—In this chapter certain breakages which may happen to an aeroplane will be considered, and the effect of these on the rest of the structure. The first and most important of these is the breaking of a flying wire, an accident frequent in aerial fighting, where the wires or fittings were often damaged by shrapnel. In ordinary straightforward flying, however, a flying wire may break, and unless the design has been carefully considered from this point of view, the remainder of the structure may fail to carry the extra loads suddenly thrown upon it. In all the work which follows, the aeroplane is assumed to be flying steadily and not manœuvring, the assumption being made that the pilot will land as quickly as possible once his aeroplane is damaged.

Suppose the flying wire AD of the single bay unstaggered aeroplane shown in Fig. 146 breaks. The total reaction at A has then to be taken up by the incidence wire A*c*. The effect of this is to transfer the total lift at A from the front truss to the rear truss, which now has to carry this extra load. That is the first result of the wire breaking. The second is that since the incidence wire A*c* is in play, an extra load is thrown upon the top and bottom internal drag bracing of the aeroplane. This load is the resolved part of the tension in A*c*, in the planes A*ab*B and C*cd*D. The third result of the snapping of AD is to eliminate the end thrust in AB due to this flying wire. On the other wing of the aeroplane, however, there is still a thrust in A′B′. The thrusts in AB, A′B′ under ordinary conditions, were equal and opposite, and so balanced one another. The moment, however, that AD ceases to act, and the consequent thrust due to it in AB becomes zero, the thrust in A′B′ has to be taken up partly by the internal drag bracing, partly by the flying wire BE′, and partly by relief of A′D′. In the rear spars a similar thing is happening. The transference of load from A to *a* results in an increased tension in *ad*, and a corresponding increased thrust in *ab*, and this extra thrust again has to be taken up partly by the internal bracing, partly by the wire *b′e*, and partly by the flying wire *a′d′*. As in the top plane, there are unbalanced loads in the bottom plane. There is, for example, a tension in D′D due to A′D′ which normally is balanced by the tension due to AD. When AD fails, this tension must be taken up, as in the top plane, by the internal bracing of the bottom plane, and by the bracing of the centre section.

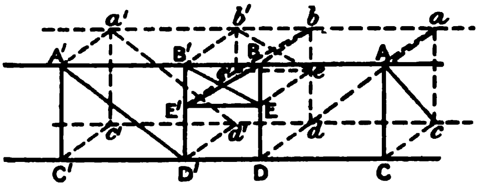

FIG. 146.

The general effects, therefore, of the failure of the wire AD are—

(1) To cause the spar A′A to tend to move in the direction A′A. This is resisted by the internal bracing wires and BE′.

(2) To cause the spar *a′a* to tend to move in the direction *aa′*. This is resisted by the internal bracing wires *b′e* and *a′d′*.

(3) To cause the spar C′C to tend to move in the direction CC′. This is resisted by the internal bracing wires and the centre section bracing.

(4) To cause the spar *c′c* to tend to move in the direction *c′c*. This is resisted by the internal bracing wires, the centre section bracing, and the wire *a′d′*.

(5) To cause the top plane A*ab*B to tend to rotate backwards, and the bottom plane C*cd*D to tend to rotate forwards. These movements are resisted by the incidence bracing wires acting in the *opposite* direction to AC, *i.e.* wires as *b*E.

(6) To increase the tension in *ad*, and the compression in *ca*.

Similar effects follow if the wire *ad* fails, though, of course, the tendency of movements would be in the reverse directions.

The actual problem of balancing out the loads which come upon the structure when a flying wire has failed, is not one which can be solved statically. It can be reasonably approximated to by the principle of least work; but this method has to be applied for every single case separately, and in a big aeroplane it is one which is far too laborious for any practical purpose. The engineer, therefore, must fall back upon certain static approximations and this method the experience of the authors has shown to be sound. These assumptions, naturally, must vary with the actual problem under consideration. In the particular case we have been discussing, for example, it would be reasonable to suppose that the extra unbalanced end thrust in A′B′ is taken up equally by all the anti-drag wires between B′ and A, provided, as is usually the case, these anti-drag wires are all of the same strength and are at about the same angle to the spars. If one anti-drag wire, say the centre section, were of a much greater strength than the others, and at a better angle, then it could safely be assumed that the greater part of the extra end thrust is taken by it. This extra end thrust, too, may be taken partly on the centre section wires, and it provides some criterion for the strength of these wires, which have very little in them in normal flight.

Alternatively, the whole of the unbalanced loads, on the front and rear trusses, and on the top and bottom planes, may be balanced out on the centre section bracing. This is, as a rule, too heavy on the centre section, but the method has been in use in the Air Ministry, and has been found satisfactory. When the centre section is calculated under these conditions, it is unnecessary to design for the full factor. This latter method has the advantage over other approximations in that it does ensure the centre section being strong enough to take the loads if necessary, and though no account may be taken of the relief due to the flying wires and other internal bracing wires, the strength of these is sufficiently examined, as a rule, under other conditions of flight.

It will be found in actual practice that considerable judgment is required in making the necessary assumptions for the distribution of the loads coming on the centre section. The following rough rules will be found useful, and they have the merit of having been used over a considerable period during the war, and being found successful.

Take the extra, *i.e.* unbalanced, end thrusts in the spars on the centre drag bracing and centre section flying wires. If there are two drag bracing wires, let them take this end thrust equally, since these wires are invariably equal in strength and at the same angles to the spar. If there is one drag bracing wire and one flying wire, divide the load between them in the proportions $\frac{A_1 \cos \theta_1}{l_1} : \frac{A_2 \cos \theta_2}{l_2}$; where A_1, A_2 are the areas, l_1, l_2 the lengths, and θ_1, θ_2 the angles the wires make with the spar.

The extra drag forces due to the transference of lift reactions from one truss to the other should be taken on the centre incidence wires, as well as the normal drag loads. If the loads are divided up in the way suggested it will be found to simplify the work considerably.

The tension in the incidence wire which takes the flying load has a component along the drag bracing, and this load in the first instance is not relieved at all. The strut A*a*, for example, must take the whole of this component, though the strut B*b* may not have to do so on account of the relief afforded by the incidence wire *b*E. Such struts, therefore, as A*a* must be carefully designed to insure that they are capable of taking the extra load thrown on them. It is usual to design these struts, under such conditions, for half the factor that is required for the particular condition of flight being investigated.

There is, of course, an extra load in the interplane strut *ac*, due to the transference of load from one truss to the other, and as in the case of the drag strut a suitable factor is half the normal factor. The same factor is required on the flying wire, which now takes the whole lift of the wing at the strut.

Up to the present the effect of the snapping of a flying wire has only been considered on an unstaggered aeroplane. The staggered aeroplane, however, presents no additional difficulties, but care must be taken when finding the loads that the right loads are calculated due to the stagger of the wires and struts.

The biplane with two or more bays presents far greater difficulties for accurate calculation of the effect of the breaking of a flying wire, and the problem is incapable of full solution without recourse to the methods outlined in the chapter on least work. One or two suggestions, however, may be made for simplifying the work involved, and at the same time making close enough approximations for all practical purposes.

Consider the three-bay single engine biplane shown in Fig. 147. Let the wire AF be broken. Then, as already shown, there is a transference of load from the front to the rear truss, and extra loads are thrown upon the drag bracing by the incidence wire A*e*. In the case of the multiple bay aeroplane, however, these extra loads may easily wash themselves out before reaching the centre section bracing, for any tendency of the two wings to rotate will be resisted by the wires *b*F and *c*G. Moreover, the unbalanced end thrust in the front and rear spars may be taken up by one, two or three flying wires, as well as the drag bracing. Thus the actual additional loads coming on the centre section of the aeroplane when the wire AF breaks may be negligible.

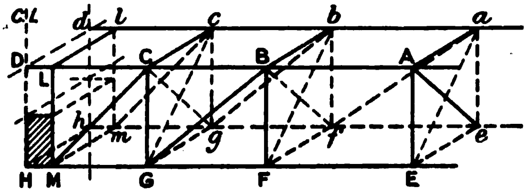

FIG. 147.

The nearer the centre line of the aeroplane is the broken wire, however, the more heavily stressed is the centre section. If the inner flying wire, CM say, breaks, then the differences of the unbalanced loads of the two wings must, in the first instance, begin to adjust themselves at the centre section bracing, and this generally gives the worst loads which can occur in the centre section of a multiple bay biplane.

It is unfortunately impossible to say which wire breaking will give the heaviest compression or worst case for a spar, though it is possible to settle the question for flying wires, interplane struts, and drag bracing. The worst cases for these are as follows :—

1. For the *front* interplane struts the worst case is the breaking of the corresponding *rear* flying wire in the adjacent panel with the centre of pressure back.

   The worst case for BF, for example, is when the wire *bg* breaks and the C.P. is in its rearmost position.
2. For the *rear* interplane struts the worst case is the breaking of the corresponding *front* flying wire in the adjacent panel, with the centre of pressure forward.

   The worst case for *bf*, then, is when the wire BG breaks and the C.P. is in its most forward position.
3. The worst case for any flying wire is found by taking the *total* lift up to that point on the wire.

   For example, if the wire *bg* breaks, then BG must take the whole lift of the wings up to B.
4. The worst case for the drag bracing can be settled panel by panel. For the drag bracing in A*ab*B, for example, cut AF with the C.P. forward, or *af* with the C.P. back, and one of these must give the worst case for the drag bracing in this particular panel.

   The worst case for any other panel is found in the same way. The effect of the breaking of the wire AF on the inner drag bays may be neglected. The worst case for C*cl*L will be when CM or *cm* breaks.

The authors have found that if the spars are satisfactory for normal flight and nose-diving conditions, that they are so for any wire broken, and so their experience leads them to believe that this case need not be investigated. It is important, however, to know which will give the worst maximum loads when designing fittings, and the following investigation shows which wires it is necessary to break to obtain the worst case for the inner bays of the aeroplane. As will be seen, it is not necessary to consider the breaking of every wire.

The assumptions made in this investigation are that the bays do not get longer as the centre line of the aeroplane is approached, that is to say, the length of each bay is either equal to or less than the length of the bay immediately outside it; and that there is the same stagger front and rear.

In Fig. 148, l_0 = the length of the effective overhang.

l = the length of the remaining half span of the aeroplane.

Let the lengths of any two consecutive bays be x and y, and let s be the length from x to the centre line of the aeroplane as in the figure.

Then the *extra* load in the innermost bay when the flying wire in x is broken is proportional to

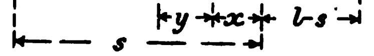

Fig. 148.

$$X = s\{x + 2(l - s) + 2l_0\}$$

and the *extra* load in the innermost bay when the flying wire in y is broken is proportional to

$$\mathrm{Y} = (s - x)\{y + 2(l - s - x) + 2l_0\}$$

The worst load in the innermost bay is greatest when the flying wire in x is broken, provided that X—Y is positive.

$$\text{Now}\quad \mathrm{X} - \mathrm{Y} = s\{x + 2(l - s) + 2l_0\} - (s - x)\{y + 2(l - s + x) + 2l_0\}$$
$$= x(2l - 4s) + s(x - y) + xy + 2x^2 + 2l_0x$$

and this is certainly positive provided that

$$s \leqq \tfrac{1}{2}l$$

Hence it is never necessary to go more than half-way along the half span when investigating the maximum loads in the spars due to the breaking of a wire. This is a useful rule to remember, and will often save a considerable amount of laborious work.

It should be noted that if the flying wires are offset, and one is broken on the front truss, say, the fixing moments on the rear truss may be very considerably increased, and the stresses at the points of support of these spars should therefore be investigated carefully. The incidence wire taking the load, too, may be offset in such a way as to put a twist on the spar.

128. Breakage of a Wire in a Twin Engine Aeroplane.—The problems to be solved when a wire is broken in a twin or multiple engine aeroplane are very similar to those in a single engine aeroplane. Any flying wire broken outside the engine, indeed, may have its results investigated in precisely the way already outlined. Between the engines, however, the problem becomes complicated, for in this section either a flying wire or landing wire may be in play in normal flight.

Consider the semi-elevation of the twin engine aeroplane shown in Fig. 149. AC is the centre line of the engine nacelle, B*bd*D the attachments of the wings to the centre section. Then it may easily happen, since the C.G. of the engine is normally close to the front spars, that when the centre of pressure is back, the weight wire BC and the flying wire *ad* are in play. When the centre of pressure is forward the flying wires AD and *ad* will both probably be in play, and the results of the failure of one of these may be followed up as already shown. We will consider the case, however, when the weight wire BC and the flying wire *ad* are in play. If the wire BC breaks, the proportion of the weight of the engine, less the total lift on the front spars up to C,

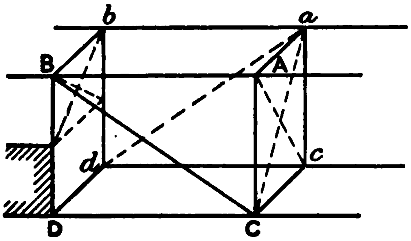

Fig. 149

must come on the incidence wire *a*C, the reaction at A being taken up either in tension on the engine strut AC, or on the flying wire AD. The effect of the incidence wire *a*C coming into play is to increase the loads on the drag bracing of the bottom plane, and to relieve them on the top plane. An additional effect is to relieve the flying wire *ad*, and so relieve the back spar.

Suppose, on the other hand, the wire *ad* breaks. Heavy loads are immediately transferred to the front truss by the incidence wire *a*C, loads which will most certainly reverse the load in BC, *i.e.* bring AD into play, and the problem becomes analogous to the single engine aeroplane with a broken flying wire.

129. Engine Cutting Out.—In a twin engine or multiple engine aeroplane, one of the engines may cut out in flight, and it is necessary to investigate this case.

Consider the twin-engine aeroplane shown in Fig. 150. Suppose the port engine fails. The speed of the aeroplane will begin to drop, as well as the total resistance, but there will be on the wings an unbalanced thrust T as shown. This must be taken up by a force P on the fin and rudder, and a force equal and parallel to P on the wing. The latter force is small, and may be neglected as regards its effect on the wing structure. If l_1 is the distance of the line of thrust from the C.G. of the aeroplane, and l the distance of the centre of pressure of the fin and rudder from the C.G., we get the force P from the equation

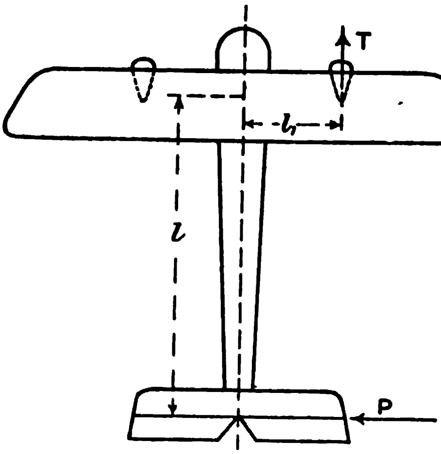

FIG. 150.

$$Pl = Tl_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This force P will usually be offset on the fuselage, and the latter must be examined from this point of view.

130. Damaged Triplane.—The problem of the damaged aeroplane becomes an extremely difficult one to solve in the case of a triplane, on account of the redundancy of the structure. At the outset it may be remarked that the only accurate methods of finding the loads in any part of the structure when one part has given way are: (1) careful approximations on the method of least work; (2) loading tests on the complete aeroplane. The first method, at the best, is laborious, and has the disadvantage that it is difficult to make close approximations to the effects of fittings of various kinds. Wherever there is any doubt, and it is possible to test, then undoubtedly the better method is the second, and by the use of suitable extensometers on all the wires of the aeroplane, a close approximation of the effect of a broken wire may be obtained without the actual destruction of any part of the aeroplane.

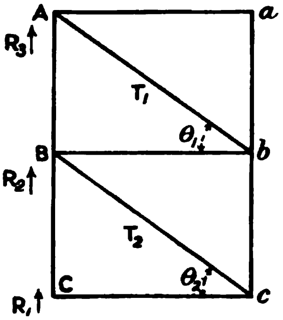

FIG. 151.

It will generally be found near enough in actual practice to take the loads on the incidence wiring, in such a way that the horizontal components of these wires balance out in the middle plane.

For example, in Fig. 151, if a front wire is broken, the loads R_1, R_2, R_3 must be taken up by A*b* and B*c*.

If θ_1, θ_2 are the inclinations of the wires to the chords of the planes, assumed parallel, and T_1, T_2 are the tensions in them,

$$T_1 \sin \theta_1 + T_2 \sin \theta_2 = R_1 + R_2 + R_3 \quad . \; (1)$$

$$T_1 \cos \theta_1 = T_2 \cos \theta_2 \; . \quad . \quad . \; (2)$$

The component of T_1 along the top plane chord is assumed to be taken up by the top plane drag bracing, and the similar component of T_2 along the bottom plane drag bracing.

131. Fuselage Bracing.—If one of the side bracing wires of the fuselage is broken, then the whole load which may come on the tail plane is transferred at this panel to the other side of the structure by means of the bulkhead bracing wire. The wires and struts in the intact side of the fuselage, forward of the broken wire, now carry double load, and hence the structure, as a whole, can only stand up to half the load it would carry when intact. Hence the rule that the bulkhead wires should be made strong enough to carry one-half of the worst tail load occurring.

In conclusion, it should be pointed out that duplication should be resorted to wherever possible; and the effect of any one of the duplicating parts failing thoroughly examined, as far as practicable. The designer is warned, however, to avoid such duplication which, though apparently strengthening his structure, may, as a matter of fact, weaken it. A duplicating member should be put in with the view that the original might fail, and not to relieve the loads the original member has to carry.

CHAPTER XVI

SOME GENERAL CONSIDERATIONS IN AEROPLANE DESIGN

132. Necessity of working to a Specification.—An aeroplane, like any other structure, will be designed for a definite purpose, and this purpose must be kept steadily to the fore in all details of the design.

The first important point, then, is that the designer should have a very clear conception of the duties which his aeroplane has to perform, and for this purpose a specification is absolutely essential. He will be wise, therefore, unless he is provided with a specification by the authority for whom the aeroplane is to be constructed, to draw one up for himself before he commences his layout.

The principal points to be laid down should be—

(1) The engine power available for the design.
(2) The amount and nature of the load to be carried.
(3) The top speed which is required from the aeroplane.
(4) The landing speed required.
(5) The climb which is necessary.
(6) The range of the aeroplane.
(7) Restrictions as to overall dimensions.
(8) Factors of safety required.

Once these are settled the general arrangement of the aeroplane may be considered. As a general rule, the choice of engines will be limited, and this item may therefore be considered one of the known factors in the design.

133. Weight Items.—The total weight of an aeroplane may be divided up into a series of items as under.

(1) The weight of the power unit including the engine or engines, radiators and propellers.

(2) The structure weight, which includes the whole of the main plane structure, fuselage and tail unit structure and the chassis.

(3) The fuel and oil and tanks.

(4) Necessary loads—including pilot or pilots, instruments, etc.

(5) Useful load—including all the load which the aeroplane is primarily designed to carry, such as mails, passengers, or war load.

Of these the only one left unsettled by the specification conditions is the structure weight. Since the structure cannot be designed until the total weight of the aeroplane is known, it is necessary to make an approximate calculation as to the weight. This is best done by comparison with other aeroplanes of similar type. It is found, however, that in well-designed aeroplanes the structure weight bears a fairly constant ratio to the whole, varying from about 28 per cent. in the case of small single- and two-seater aeroplanes to 31 or 32 per cent. for the heavier class of bombers. Float seaplanes are 4 or 5 per cent. heavier than these figures. The following table will show the variation in the structure percentage weight for several typical aeroplanes :—

STRUCTURE PERCENTAGE WEIGHTS.

Aeroplane.	Total weight (lbs.)	Structure weight (lbs.)	Percentage.
Sopwith Camel	1,454	451	31·0
S.E. 5 A.	2,038	604	29·7
Sopwith Dolphin	2,068	602	29·1
R.E. 8	2,592	758	29·25
Bristol Fighter	2,783	794	28·5
F.E. 2 B.	3,150	1,090	35·1
D.H. 4	3,313	1,048	31·6
Fairey Campania	5,416	2,123	39·2
H. 12 Flying boat	9,092	3,720	41·0
Vickers Vimy	9,571	2,928	30·6
Handley Page 0–400 . . .	12,187	3,859	31·7
F. 3 Flying boat	13,400	4,942	36·9

134. Choice of Aerofoil.—Having made an estimate of the total weight of the aeroplane, the next step is to determine the wing area required. Before this can be done, however, it will be necessary to decide what aerofoil section is to be used for the wings, and this will be settled by the general purpose for which the aeroplane is required. If speed is an essential quality as, for example, in a small racing craft, a low camber section would be chosen giving a comparatively low value of k_L with a high value for the lift-drag ratio. On the other hand, if the principal function of the aeroplane is that of a weight carrier, *e.g.* in a mail aeroplane or bomber, necessitating getting off the ground rapidly with a heavy load, an aerofoil would be chosen which would give a high value of the lift coefficient irrespective—in reason—of the ratio of lift to drag. When the qualities required combine both speed and climb, as in a fighting scout, it is of course necessary to consider very carefully both the conditions mentioned above, viz. high lift and maximum lift-drag ratio, and for this purpose curves of various wing sections plotted for k_L against k_L/k_D as shown in para. 12, are necessary. The choice of a suitable wing section is one which needs considerable care, and in most cases it will pay to map out approximate performances for the aeroplane, using different wing sections, and then selecting the most satisfactory.

The aerofoil having been decided upon, the area of the supporting surface required can be estimated from a consideration of the maximum landing speed allowable. The aeroplane will land at approximately its stalling angle for which the lift coefficient k_L is known. Then the area of the main planes in square feet is given by

$$S = \frac{W}{k_L \rho V^2}$$

where V is the specified landing speed in feet per second.

The loading of the main planes and the aerofoil section are now known, and a rough performance for the aeroplane can be worked out. If it is found that the conditions of top speed and climb can be fulfilled, attention may be turned to the general layout. If, on the other hand, it is found that the specification is not met in either of these particulars, another attempt must be made, either varying the loading or the aerofoil section, or both.

135. Arrangements of Planes.—Assuming that the performance required is considered attainable, however, the next problem becomes one of arrangement. The decision as to whether the main planes shall be of monoplane, biplane, or multiplane design is one very largely of overall dimensions. As regards efficiency from an aerodynamic standpoint, the monoplane is best, and the efficiency drops off as the number of planes is increased; but the other consideration of compactness must be of paramount importance in the case of the larger types.

The span of the planes will probably be fixed by considerations of this nature, and as a good aspect ratio will be one of the aims of the designer, the problem will more or less solve itself. An aspect ratio of less than six is rarely advisable, and the higher it is the better from an efficiency point of view.

The question of the gap requires attention next, and here, unless overall height becomes of primary importance, the designer will have to use his judgment, as no definite rule can be laid down. The general practice is to make the gap about equal to the chord, but aeroplanes have been flown and given good performance where the gap was only about two-thirds of the chord. In the case of a small gap-chord ratio the wings should be staggered for aerodynamical efficiency. It is a safe rule, however, to keep this ratio in the neighbourhood of ·9 to 1·1.

136. Overall Length.—The length of the aeroplane is a question which must now be settled, and here practice varies quite considerably. In some types the tail plane is four to five chord lengths behind the trailing edge of the main planes, and in others it is barely two. An intermediate figure is probably the best to use. It should be remembered that the shorter the fuselage the greater the loads required on the tail and rudder to control the aeroplane, and the greater the areas required for stability. Further, the effect of downwash from the main planes and propeller slip stream are more pronounced in the case of a short body, which may be a disadvantage. On the other hand, it is an advantage to have a short aeroplane for storing purposes, and this must be borne in mind when settling the final design.

In this rough layout it is important to consider if the aeroplane has to fold. If so, care should be taken to arrange span and length so that the folded size is as small as possible. There are, of course, other problems connected with the folding, but these are of a more detailed nature and connected with the design of the fittings.

137. Disposition of Weights.—A matter which has not hitherto been considered, but which now requires attention, is the disposition of the weights to be carried by the aeroplane. These must be so arranged that the position of the centre of gravity of the aeroplane lies in a suitable position with regard to the main planes, and, if necessary, the latter must be moved to enable this to be done.

Such movements of the main planes may take one of three forms—

1. A bodily movement of the whole wing structure relative to the fuselage.

2. A movement of one plane only, giving a stagger to the planes. The effect of a forward stagger is to move the equivalent plane forward relative to the C.G. of the aeroplane, while a backward stagger gives the reverse effect.

3. The wings may be swept back. The effect of this is to move the centre of pressure on the planes aft relative to the centre of gravity. If an aeroplane has its C.G. too far aft, therefore, such a sweep back will improve the balance.

No definite position can be said to be the best, as the aerofoil used determines this to a large extent, but it may be safely laid down that the centre of gravity should be between ·28 and ·4 of the chord of the main planes, measured from the leading edge of the equivalent plane.

138. Position of Chassis.—The chassis or under-carriage may now be considered in relation to its position on the aeroplane.

If GB, Fig. 152, represents the vertical through the centre of gravity with the aeroplane at rest on the ground, the line GA through the axle of the under-carriage should make an angle a with BG of about 20°.

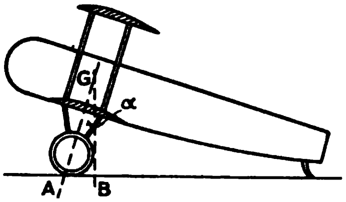

FIG. 152.

If a is too small there is a danger of the aeroplane falling forward on its nose when landing or getting off; while if it is too large a very heavy load is thrown upon the tail skid when the aeroplane is on the ground, which necessitates a heavier fuselage than might otherwise be necessary, and also renders the aeroplane difficult to control on the ground.

So far nothing has been said about the engine, but the position of this and the size of mounting, etc., required are more or less fixed, and allow of very little scope for altering the general layout of the aeroplane.

The foregoing deals with the more general problems connected with the preliminary design of an aeroplane. Some of the more detailed questions and considerations involved must now be studied.

139. Spacing of Bays in Main Planes.—In a multiple panel design of main plane structure the distributed load along any spar may be considered constant.

As the centre of the aeroplane is approached, however, the end compression in the spars increases. If the bays were made of equal length, therefore, the bending due to the distributed load would be the same in each bay, but the effect of end compression would be very much greater in the inner bays than the outer, and the spar sections would have to be correspondingly increased. To such an extent is this effect marked that in a large aeroplane the outer bay spar sizes are determined chiefly by the distributed load forces, and the inner ones chiefly by the end compression. On account of this it is economical design to arrange the interplane struts so that the bays decrease in length from the wing tips to the centre of the aeroplane. By this means it is possible to retain the same section spar throughout, and keep the stresses in it fairly uniform.

The general case of the best spacing for a multiple bay aeroplane is not amenable to a simple mathematical treatment, but in the case of a single bay aeroplane the correct position for the interplane strut for economy in design may be determined in the following manner.

In Fig. 153 let the effective span of the plane be L. Let the bay length be kL. Let the loading per unit length be w. Assume a pin-joint at A.

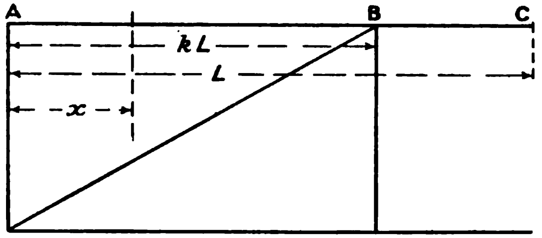

FIG. 153.

Consider the top spar and take moments about A to find the reaction at B.

$$R = \frac{wL^2}{2kL} = \frac{wL}{2k}$$

At a distance x from A the bending moment

$$M = \frac{w(L-x)^2}{2} - \frac{(kL-x)wL}{2k}$$

The point of inflection is given when $M = 0$;

i.e. when $$\frac{w(L-x)^2}{2} = \frac{(kL-x)wL}{2k}$$

or when $$x = \frac{L(2k-1)}{k}$$

and this value of x is also the distance between the points of inflection in the spar.

The greatest bending moment between the points of inflexion is

$$M_0 = \frac{wL^2(2k-1)^2}{8k^2}$$

And the bending moment at B is

$$M_B = \frac{wL^2(1-k)^2}{2}$$

The maximum stress in the bay will be

$$f = \frac{wL^2(2k-1)^2}{8k^2 I} y\left(\frac{Q}{Q-P}\right) + \frac{P}{A}$$

where Q is the Euler crippling load of the spar on the length between the points of inflection; P is the direct end thrust in the spar; I, y and A have their usual significance. The stress at B just inside the joint will be given by

$$f = \frac{wL^2(1-k)^2 y}{2I} + \frac{P}{A}$$

Assuming the same size spar is to be used throughout these values of f should be equal, so that

$$\frac{wL^2(2k-1)^2 y}{8Ik^2}\left(\frac{Q}{Q-P}\right) + \frac{P}{A} = \frac{wL^2(1-k)^2 y}{2I} + \frac{P}{A}$$

or

$$\frac{(2k-1)^2}{4k^2}\left(\frac{Q}{Q-P}\right) = (1-k)^2$$

when

$$\frac{Q}{Q-P} = 1\text{; } i.e. \text{ when } P = 0$$

this reduces to the equation

$$4k^4 - 8k^3 + 4k - 1 = 0$$

the solution being $k = \cdot 7$.

When $\frac{Q}{Q-P} = 2$ the equation becomes

$$2k^4 - 4k^3 - 2k^2 + 4k - 1 = 0$$

the solution of which is $k = \cdot 67$.

From these results it appears that the interplane strut in a single bay aeroplane, pin-jointed to the centre section, should be placed about two-thirds of the length of the spar from the hinge position.

This result is of interest, as it agrees remarkably well with the requirements to avoid vibration of a spar.

This question of vibration was investigated experimentally by Lieut. A. H. Stuart,[1] R.N.V.R., and he found that the overhang portion of a spar should not exceed 30·5 per cent. of its whole length, or otherwise trouble was experienced.

On all grounds, therefore, it is advisable to make the bay about 70 per cent. of the whole length, and rather more than less to diminish the possibility of vibration trouble.

140. Spacing of Spars in the Aerofoil.—It will be seen on reference to the drawing of any aerofoil that while the front part is comparatively deep, admitting a spar of good depth, the rear portion tapers rapidly, and the strength of the spar may have to be obtained by an increase in relative width. This is, of course, uneconomical from the weight standpoint, but it is unfortunately unavoidable.

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[1] R. & M. of A.C.A., No 386.

In spacing the spars there are two points which must be borne in mind—

(1) Owing to the heavy intensity of loading on the nose of an aerofoil the leading edge from the front spar forward should be as short as possible.

(2) The spacing between the spars should be as large as can be arranged, since in a nose dive a couple has to be taken upon the trusses, and by separating these as widely as possible the loads are reduced with a corresponding saving in material and weight.

The shape of the aerofoil used, of course, sets limits for both these points, and the only way to determine satisfactorily the possible spacing is to draw out the wing section to a large scale, and try various spar sections in it. It should be remembered when doing this to draw the inside boundary of the rib flanges, and not the outside, or otherwise the spar designed will be found too deep to fit the aerofoil by an amount equal to twice this flange depth.

141. Rib Spacing.—The design of suitable ribs will be dealt with in a later chapter, but before this important point can be considered, it is necessary to settle how far apart these members shall be spaced. In practice it is customary to make the spacing from 12 to 18 inches, and if these limits are kept no difficulty need be anticipated. In large aeroplanes, of course, it may be advisable to increase this spacing, and the question then becomes one which the designer has to tackle seriously. Again, in seaplanes it is advisable to reduce the space between ribs on the bottom planes when these are close to the water, since they may receive very heavy loads when landing in a rough sea, which would seriously damage a normal plane. For this reason, it has been found satisfactory to reduce the spacing to about 6½ inches, and although the effect is, of course, an increase in weight the results obtained appear to justify the design. The principal point which will determine the limit to which rib spacing may be carried is the length over which fabric may be left unsupported. It is obvious that the greater the distance between the ribs the greater will be the sag of the fabric covering, and consequently the greater the loss of efficiency due to alteration of the wing section. This, then, is the practical question to be faced, and the figures previously quoted from standard practice are those which have been found to give very satisfactory behaviour as regards the fabric remaining taut.

It is possible, however, to examine this question of rib spacing a little more closely from the structural standpoint. Every rib is composed of members acting purely as ties, and of struts or members under combined compression and bending. As the spacing between the ribs is increased so the load on each one grows, and hence the forces in the members composing it.

Consider how the strength and weight of these members vary for different loads.

First a simple tension member.

The load carried is

$$P = fA$$

where f is the stress permissible and A the cross sectional area.

i.e. $$P \propto A$$
$$\propto S^2$$

where S is a linear dimension of the section.

Also $$W = lA$$

where l is the length of the member which will be constant.

Therefore $$W \propto A$$
$$\propto S^2$$

or the strength and the weight go up in the same ratio. Hence, as far as the weight of the tension members of the rib structure are concerned, it is immaterial what the spacing of the ribs is.

For a strut member

$$P = \frac{\pi^2 EI}{l^2}$$

or $$P \propto I$$
$$\propto S^4$$

and $$W \propto S^2$$

Hence the weight goes up as the square of the dimensions, and the strength as the fourth power, showing that it is economical to allow the ribs to be spaced as far apart as possible.

Similarly for a beam member

$$M = \frac{fI}{y}$$

or $$M \propto \frac{I}{y}$$
$$\propto S^3$$

while $$W \propto S^2$$

so that here again the strength goes up faster than the weight for an increase in dimensions, showing that a wide spacing is economical.

A member with combined lateral load and end thrust will be intermediate in its weight for strength ratio between a pure strut and a pure beam, and therefore its strength will increase more rapidly than the weight in this case also. It is clear, therefore, that considering the question from a structural standpoint only, the ribs should be spaced as widely apart as possible, and so the only limitation is the one previously mentioned, viz. the permissible distance over which fabric may be stretched.

142. Effect of Distributed Weight.—In the smaller types of aeroplanes the whole of the load to be carried is concentrated in the fuselage. As the size increases, however, it is usual to divide the weights and carry some portion in the wings, *e.g.* in the case of a multiple engine aeroplane some of the engines are mounted in nacelles placed some distance from the centre line of the aeroplane. This method of division has an appreciable effect upon the main plane structure weight, as a little consideration will show.

Consider two extreme cases of an aeroplane carrying its whole load concentrated at the centre, as is customary in most small types, and the same aeroplane having its whole load evenly distributed along the spars. In the first place the wings in steady normal flight carry an upward load of $(W - w)$, where W is the total weight of the aeroplane, and w is the weight of the wing structure. This is evenly distributed over the whole surface and is the usual case taken in design.

Consider next the second case mentioned above. The upward load in steady normal flight due to the air reaction is W, but acting against this is the distributed load, also W, so that in normal flight the trusses would not be stressed at all. The second case is, of course, a practical impossibility, but it leads to the conclusion that some distribution of load is a good method of reducing structure weight.

Take a more practical case as shown in Fig. 154. A shows the half-elevation of an aeroplane with the whole of its load concentrated at the centre. The load carried upon one half of the wings is $\frac{W - w}{2}$.

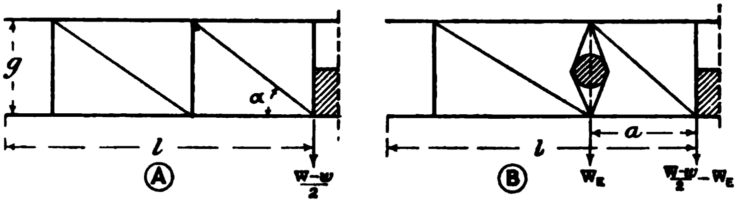

FIG. 154.

The total end load in the inner bay top spars is found, by taking moments, to be

$$P_s = \frac{(W - w)}{4g} l$$

and in the wire it is the load due to the shear across that bay, *i.e.* $\frac{S}{\sin \alpha}$.

In case B the same arrangement is shown with the exception that the weight W_E is taken from the centre and placed in the wings.

The total load as before is W, and the distributed load on one half of the wings is $\frac{W - w}{2}$.

The load in the inner bay is now, however,

$$P_s = \frac{W - w}{4g} l - \frac{aW_E}{g}$$

and in the wire it is

$$\frac{1}{\sin a}(S - W_E)$$

Comparing these figures with those for case A above, it is clear that a saving can be effected in the weight of the spars and wires. Fig. 155, Case I., shows the outline arrangement of an aeroplane with engines carried in the wings as indicated. Cases II. and III. are modifications of this arrangement made purely to determine the effect upon structure weight.

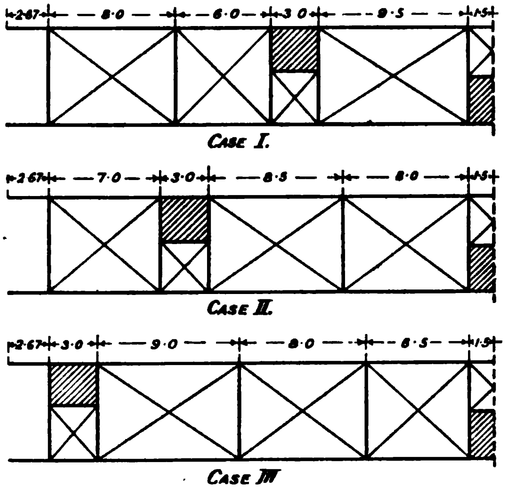

FIG. 155.

Case I. showed the heaviest structure weight, and Case II. the lightest. The arrangement in Case II. saves 2½ per cent. of the total weight of the aeroplane in structure as compared with Case I., and Case III. saves about 1 per cent. compared with Case I.

If weights are to be divided, therefore, it appears to be advisable to locate them somewhat as shown in Case II. This, of course, only considers the problem from the structural standpoint, but there are other points which must be considered. The principal of these is the fact that placing weights far away from the centre of the aeroplane increases the rolling and yawing moments of inertia very considerably, and in consequence the stability and control may be seriously affected.

Another objection to placing engines far from the centre line is that if by any chance those on one side of the aeroplane are damaged or deliberately cut out, the thrust of the remaining ones gives a large turning moment about the axis of the aeroplane, which has to be balanced by a rudder load. This may necessitate very large rudders, which will be more difficult to control, and the efficiency of the aeroplane will be impaired.

In landing an aeroplane with divided weights there are, of course, heavier loads thrown on the wing structure than in one with all the weight central, and so this condition becomes one of increasing importance. This question, therefore, is one which must be carefully considered for every aeroplane, and all the possibilities weighed before a decision is made upon the design.

143. Proportions of a Fuselage.—The proportions of a fuselage are largely determined by the loads which have to be carried, and their disposition, and also by the necessity of maintaining a good streamline body shape. These considerations will usually fix the outline of this structure fairly rigidly, and the question of bay spacing is one largely of taste. It will be found, if a number of different spacings are adopted, and the weights of the resultant structures calculated, that there is very little to be saved by the use of one rather than the other. If the fuselage be divided into a large number of small bays the longerons being of shorter lengths can be made of small dimensions, but any saving is swallowed up by the extra number of struts, wires, and fittings required. If a small number of bays is adopted as a basis, the weight of struts and fittings will diminish, but the longerons will be proportionately heavier, so, as stated before, this part of the design is a matter which the designer must judge by eye.

The cross section of a fuselage should be so designed that its depth and width are as nearly equal as possible, consistent with other conditions which arise, as by this means torsional stiffness is kept up to a high value.

Another point which is of fundamental importance is the section at the rear portion where the tail plane is attached. This should be kept as large as possible in order to give a good support to the tail spars. Failure to do this may result in the tail twisting badly with a possibility of complete failure. This has happened to one or two aeroplanes during trial flights, and may cause failure in an otherwise good design.

144. The Choice of Load Factors.—The selection of suitable load factor and factor of safety for an aeroplane depends primarily upon the consideration of two points.

First, the purpose for which the aeroplane is intended; and secondly, the ease with which it is controlled. These two are somewhat intermixed and the size of the aeroplane is closely associated with them both. It is clear from what has been said in previous chapters upon the evolutions through which an aeroplane can be put, and the loads coming upon the structure during these manœuvres, that the principal use of the factor is to guard against failure during such manœuvres. If an aeroplane is intended for fighting or for sporting purposes it is therefore essential that the load factor should be kept as high as possible so that there is a margin of safety under all conditions of flight likely to occur. The possible conditions of flight depend, of course, upon the ease with which such an aeroplane can be controlled; but in any aeroplane intended for the purposes mentioned above, controllability is one of the first essentials. At the other end of the range is the case of the aeroplane intended purely for commercial purposes or for heavy bombing in war. The function of such an aeroplane is simply that of transporting as much load as possible from one point to another. While it will, of course, need to be readily controllable, the fact of its heavy weight and large dimensions will prohibit the possibility of any but the most straightforward types of flying as well as the fact that no quick manœuvres will be necessary. The factor required for an aeroplane of this sort is, therefore, only one which will cover the ordinary risks of flying, such as bumpy weather or wind gusts.

It should be borne in mind that an increase in the factor does not mean anything like the same proportion of increase in the structural weight, so that it is often very false economy to cut the factor to the very lowest figure admissible. Any increase over the minimum is an added guarantee of safety, and provides a reserve against the use of inferior materials and workmanship which may at some time or another creep into any aeroplane, however careful the inspection.

In addition to the strength of the structure it is also necessary to consider the rigidity. A structure may be amply strong but at the same time very flexible, and this will occasion endless trouble in erection and may throw the aeroplane out of truth during flight to such an extent that its performance is seriously impaired. Such trouble may quite easily be remedied by using a larger factor in the design of the structure, and although it will, of course, necessitate some increase in weight, the advantages gained will more than compensate for this.

Factors have already been suggested for certain conditions in the chapter dealing with the fuselage. Experience during the war showed that for main planes, reasonable load factors ranged from seven and five on the front and rear trusses respectively in the case of fast scouts and fighters to four all over for heavy bombers.

Experience will alone show whether these figures are the most suitable for commercial conditions.

CHAPTER XVII

DETAILED DESIGN

It has been shown in the previous chapters how an aeroplane in flight is loaded and the methods which may be employed to estimate the amount of such loads and the strength of the members resisting them. In the following chapters the various structural members will be examined in some detail and the special points arising in their design considered.

145. Fabric.—The outer covering of the main plane structure, control surfaces and fuselage should be made of the best quality linen, although during the war it has been found necessary to sanction the use of cotton fabric owing to the immense amount of material required. This fabric is stretched over the framework and sewn to it. It is then doped in order to cause it to shrink and form a good taut covering. From the structural point of view the fabric serves purely as a covering and is not considered much in estimating the strength of the aeroplane. It does, however, exercise a useful function in this respect, as it stabilizes the ribs against lateral failure when it is properly attached to them and also helps the main plane spars in their resistance to secondary failure about the major axis by forming a bracing between the front and rear spars. It must be borne in mind, however, that the tightening of the fabric puts a heavy load in some members, and the tendency is for the structure which is being covered to distort when doping is done.

Most parts of the structure are sufficiently strong in themselves to resist this distortion, *e.g.* the fuselage and usually the fin, rudder and tail plane; but in the main planes it is necessary to make special provision to prevent serious distortion and collapse. This is done by the use of specially stiff end ribs at the inner end of the wing and also at the edge adjoining the inner side of the aileron. The leading and trailing edges are of sufficient strength in themselves to resist distortion, and these end ribs stiffen the boundaries of the plane, which would collapse without some special attention of this sort.

The fabric itself receives the air load, which it transmits to the former ribs in a similar manner to that in which the road covering on a bridge transmits the traffic loads to the cross girders. In addition to this load it has, of course, to withstand the internal stresses induced by the tightening action of the dope.

The Air Ministry specification strength for the best linen fabric is 95 and 92 lbs. per inch width respectively on the warp and weft when tested in tension at a rate of loading of 150 lbs. per inch per minute. A lower strength than this might be used quite safely on certain types of slow and heavy aeroplanes, but such reduction is not recommended for any type which is intended for quick manœuvring work.

148. Ribs.—The function of the ribs or formers is to receive the loads from the fabric and transmit them to the spars. These members, therefore, are beams supported at the spars with overhanging portions beyond these supports. The load carried is a distributed one partly on the lower surface and partly on the upper, arising from the air loads transmitted by the fabric. In addition to this load there is the fabric tension to be taken into account. This gives a load at the leading and trailing edges as shown in Fig. 156.

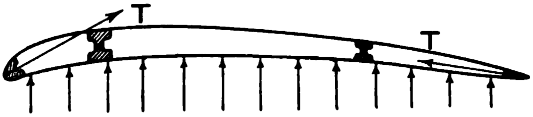

Fig. 156.

A rib is a complete engineering structure, and as such requires careful design if the best and most economical result is to be obtained. It is possible to obtain very good results by calculation, but it is always advisable to check these results, especially for an aeroplane which is out of the ordinary in size or loading, by making destruction tests of actual specimens. Methods of carrying out these tests are given in Chapter XX.

The first point to settle in the design of a rib is, of course, the loads which it will have to support.

If $w =$ the loading per square foot on the planes of the aeroplane ;
$c =$ the chord of the main planes in feet ;
$s =$ the spacing of the ribs in feet ;
and $p =$ the load on one rib ;
then $p = wcs$.

This load is distributed along the rib in a manner which varies for every point and position of the centre of pressure, and it is therefore necessary to design for at least the two cases when the centre of pressure is in its most forward position, and also when it is in its most rear position. Curves giving the distribution of pressure may be found in Reports of the Advisory Committee for Aeronautics, and these curves should be used when possible. In the absence of these, however, it is sufficient to design for the two cases as shown in Fig. 157.

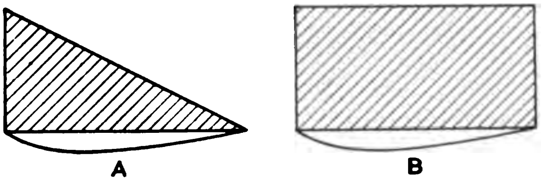

Fig. 157.

A shows a triangular distribution corresponding roughly to the C.P. at ·33 of the chord, and B shows a uniform loading corresponding to a C.P. at ·5 of the chord.

The tension due to the fabric may be considered as follows.

Fig. 158 shows a sketch of the leading edge of a rib with fabric upon it. T represents the total pull on top and bottom surface. If t be the tension due to doping per inch, then $T = st$.

The force T acts tangentially to the curves of the leading edge, top and bottom, and is resisted by internal forces in the rib. If θ is the angle between the tangents, then, as shown in the diagram of forces in the figure, the resultant R bisects θ and its value is

$$R = 2T \cos \frac{\theta}{2}$$

$$= 2st \cos \frac{\theta}{2}$$

The value of t may be taken as 3 to 4 lbs. per inch. The same method is applied to the trailing edge. The forces upon the rib are now all determined. The next step is to decide upon the type of rib, and various possibilities are open.

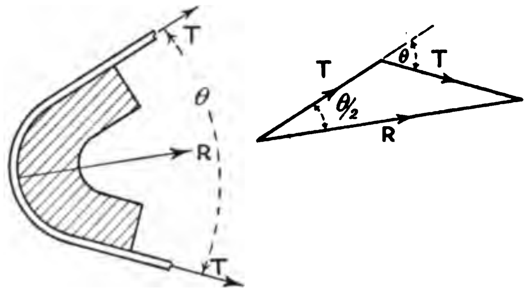

FIG. 158.

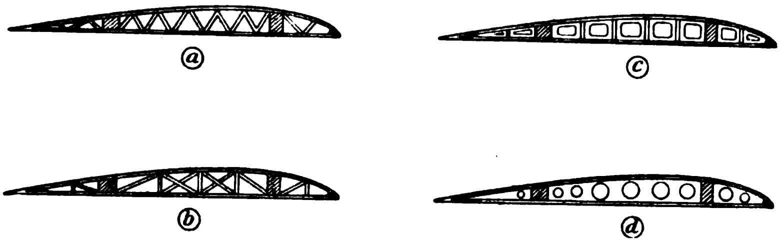

FIG. 159.

Fig. 159 shows a few varieties of design in use.

(*a*) is of warren girder construction.

(*b*) is an N girder construction.

(*c*) is an " open window " construction.

(*d*) is a solid web with lightening holes cut away.

For any but the smallest aeroplanes the first two are to be recommended, and (*b*) is the better of these for the reason that the compression members are always the shortest possible, whereas in (*a*) these members are as long as the tension strips.

Since the strength of a strut is inversely proportional to the square of its length, it follows that there is a great waste in weight when these members are longer than necessary.

(*c*) and (*d*) are practically the same type except that (*c*) is shown with stiffening strips for the strut members. These may or may not be required, and can be used in (*d*) as well as in (*c*) if necessary.

Consider first the design of a rib of the type (*b*). The same method of design is of course applicable to (*a*).

The outer profile is determined already by the section of aerofoil to be used, and this should be drawn out carefully to a large scale and inked in, as it will be required for several attempts. A first arrangement of the internal bracing can then be sketched in in pencil. The load curve ABC should now be set up to scale and the forces R_1 and R_2 obtained as previously explained (Fig. 160).

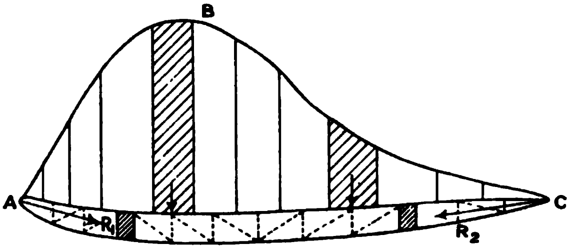

FIG. 160.

The load coming upon each node of the girder is found by measurement of areas from ABC, which should be divided up as indicated in the figure.

The spar reactions are now found by means of the polygon of forces in the usual way and a stress diagram for the structure drawn.

The procedure is repeated for the case when the centre of pressure is in its most backward position, the corresponding pressure distribution curve being taken. The loads obtained in the various members of the truss are tabulated for both conditions, and the design of each member carried out to withstand the maximum.

Tension members usually consist of thin strips of spruce, and their size is generally fixed by conditions of end attachment to the flanges of the rib and not by stress considerations.

The strut members may be made of spruce strips with stiffening tacked and glued to them (Fig. 161) or may be made of solid spruce.

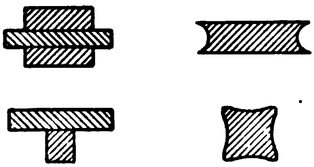

FIG. 161.

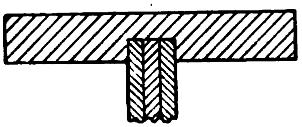

FIG 162.

Various other sections may be used as indicated in Fig. 161, but this is largely a question of taste to be settled by the individual designer.

The flanges are almost always of T section, and their design is a comparatively simple matter, as they consist of laterally loaded struts, and may be tested by standard methods.

In this type of design it is usual to make the various members of separate pieces of wood fastened to the flanges; but it is possible to cut the whole of the web portion out of three-ply, and to glue on local stiffening pieces for strut members where required. In this case the flange is formed of a lath of spruce grooved to take the central web portion to which it is glued (Fig. 162).

Some bays of a rib of the type under consideration will need to be counterbraced, and a method of determining which will require this needs some explanation.

The function of the diagonal cross strips is to take the shear load across the section at which they occur, and the direction of this shear is the factor determining which diagonal in the bay is required.

Fig. 163 (A) shows a rib with the load curve ABE for the most forward position of the centre of pressure plotted upon the chord AE as a base line. The reactions at C and D, the spar positions, are found by one of the usual methods, and then by an integration of the load curve the shearing force diagram AFGKHJE is determined. This shearing force curve cuts the base line at K, indicating that at this point the shear force changes sign. The same work is done for the rib when the centre of pressure is in its most aft position, and a new shearing force diagram A′F′G′K′H′J′E′ (Fig. 163 (B)) is obtained.

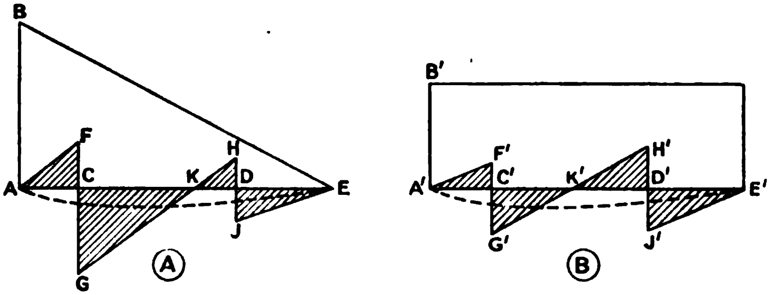

FIG. 163.

The shear force in this case changes sign at K′, and so it is clear that between the two points K and K′ on the rib, the shear may be either positive or negative, and hence any bays occurring between these points require counterbracing.

The cases considered enable the rib to be designed for normal flight conditions, but there is a special case which requires emphasis. When the aeroplane is flying at a high speed, *e.g.* in a dive, a very heavy down load is thrown on the front part of the aerofoil, and the strength of the ribs for this condition must be very carefully considered. Pressure distribution curves will show this loading for fine angles of incidence, and the treatment is exactly similar to the cases described above.

In view of the concentration of load upon the front part of the rib it is always advisable to make this portion as short as possible, and as rigid as it can be obtained.

Many fatal accidents have occurred through the neglect of this principle in rib design.

Another point requiring attention is the attachment of the ribs to the spars. As already pointed out, the function of the ribs is to transmit the load upon the fabric to the spars, and to enable them to do this work efficiently a good attachment is essential. The load is transmitted by shear across the web portion of the rib, and so this portion must be very strongly attached to the web of the spar.

Fig. 164 shows a type of construction which was the partial cause of a fatal accident. The web C stopped short against the web of the spar, and was not attached to it in any way. Consequently the shear on the rib had to be transmitted to the spar through the small rib flanges. These were not sufficiently strong to do this, and the leading edge failed under the heavy local loads in high-speed flight. The rib should have been attached by means of fillets, as indicated at B, which allow the whole of the cross section of the rib to come into play and not the flanges only.

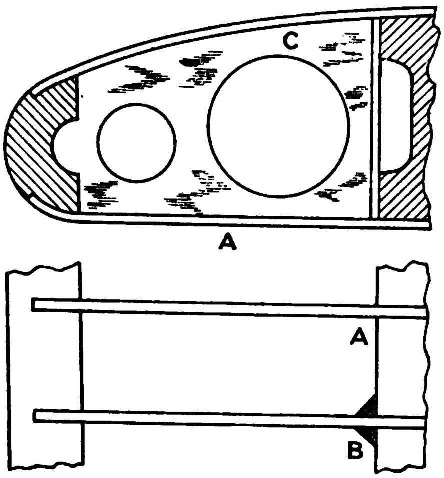

Fig. 164.

So far the case of the trussed rib only has been considered, and no mention has been made of the open window type of construction. As stated at the commencement of this chapter, this type should only be used on comparatively small aeroplanes.

The loads coming upon a rib are, of course, independent of the internal design of the member, and so much of the preceding work stands good for any construction. It is, however, unfortunately almost impossible to make calculations upon a structure of this type, since the whole strength is dependent on the rigidity of the joints of the vertical struts to the flanges. The flanges are in pure bending under the air load, and have also bending moments in them due to the loads transmitted through the vertical members, but these latter moments are not determinate with any degree of accuracy. If this type is contemplated, therefore, it is safest to rely on test figures alone for the strength.

The special case of the compression ribs will be dealt with in the next chapter, under struts.

147. Leading and Trailing Edges.—Owing to the fact that there is heavy local loading on the front of the ribs, the leading edge of the plane should be of a rigid and strong construction. The selection of a section is, however, a matter of taste, as the loads upon it are very uncertain.

In order to strengthen the portion of the rib between the leading edge and the front spar, and also to retain the correct shape of the aerofoil, it is usual to insert small nose ribs or riblets between the main ribs. These are either stopped on the spar or preferably carried over it (p. 19).

The trailing edge is not important as a load carrying member, since the greatest load upon it is the tension due to fabric.

It is quite common practice to make this member of kite cord, and this

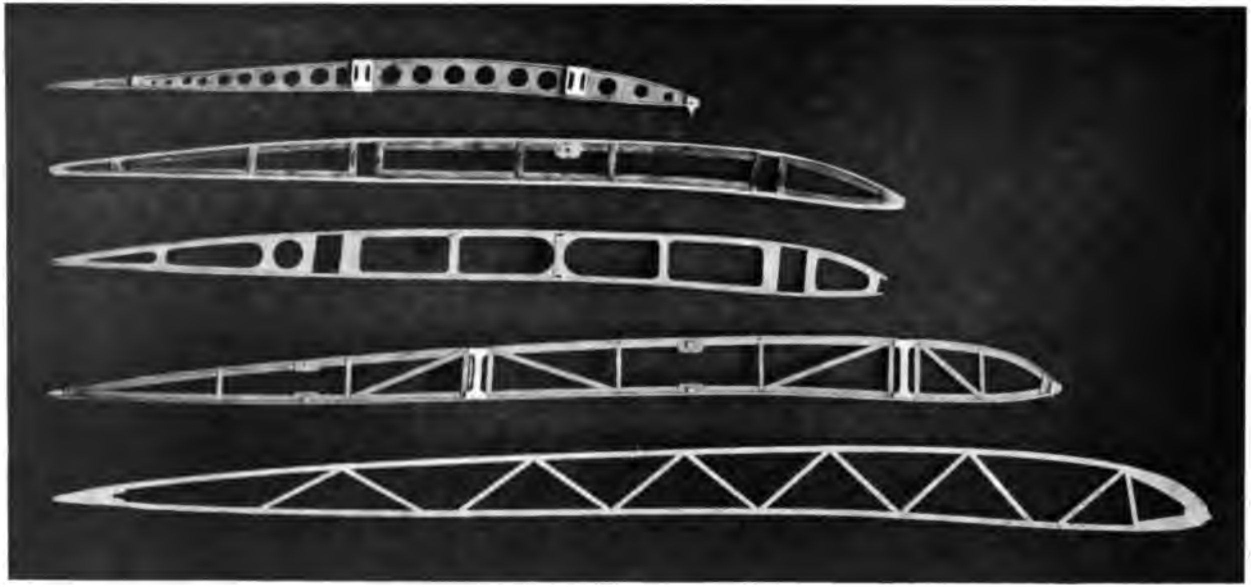

Plate XVI.—Typical Main Plane Ribs.

answers very well. Spruce may be and often is used, but it is preferable to employ small metal sections. The advantage of metal over wood is that it does not warp, and also for the small size required the latter material is very liable to fracture in service, whereas metal may bend a little but will not break. The difference in weight is very little and is compensated for by the longer life and freedom from accidental fracture when metal is used.

148. Main Spars.—The loads upon the main spars and the methods of estimating the strength of these members have already been dealt with, but there are various points in connection with design which require some attention. By far the commonest form of spar is the timber one, usually spruce or an equally good wood, cut out of the solid and spindled to an I section for lightness.

There is very little to be said on the main question of the design of these members. The overall dimensions are practically fixed by the shape of the aerofoil to be used, and questions of strength determine the amount which can safely be spindled out. On detail points, however, there are important questions to be considered.

At the interplane struts and drag strut positions the spar should be left solid, partly to give the fittings a flat bed and partly to obtain the necessary strength for the spar at these points. It is the usual practice to attach interplane fittings to the spars with vertical bolts, and so it is necessary to keep a solid section at all such points.

Spindling should not start rapidly, but should be gradual from the solid to the fully lightened section, since a sudden change of section leads to a concentration of stress, and is often dangerous even when the strength on paper appears satisfactory. Fig. 165, A, shows the wrong, and Fig. 165, B, the correct method of doing the spindling. Questions as to the positions of bolt holes in the spars will be dealt with when discussing the design of main plane fittings.

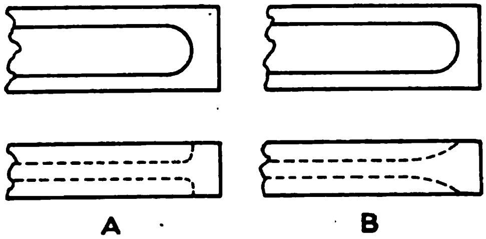

Fig. 165.

149. Built-up and Box Spars.—The serious shortage of suitable timber for spar construction during the war has led to the development of various forms of construction by which economy in this material could be effected. Many forms of built-up and box spars of spruce and similar timber have been used with great success. The one objection to all forms of this construction is that they depend so largely upon glue for their strength; but given good quality glue, which is not affected by exposure, the built-up members are as strong as the solid spindled section of the same dimensions.

The first type of built-up spar is the laminated.

These are constructed of several laminæ of timber glued together as shown in Fig. 166. These members were found on test to be just as strong as solid members of the same overall dimensions made from the same material. After gluing, these spars can be spindled just like solid ones (B); but it should be noted that where a lamination is spindled right through as at (C), the radius of spindling should be kept as large as possible in order that the glued area may be as big as can be obtained. For this reason the form shown at (D) or (E) is the best, and should be adopted wherever possible, as it gives a gluing depth equal to the whole depth of the spar. The section shown at (F) is built up of horizontal laminæ. The thickness of the laminations should be so arranged that the flange is divided into two equal portions by the joint.

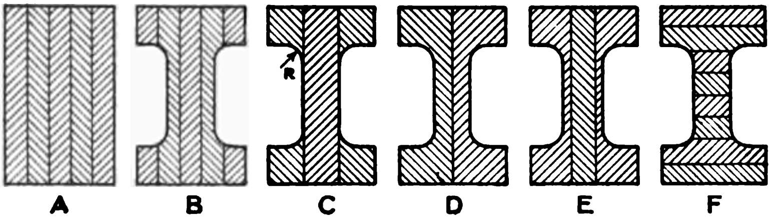

Fig. 166.

It should be noted in connection with this form of construction that no bolts or pegs are required. Tests upon specimens with and without bolts have shown that not only is there no increase of strength obtained by their use, but actually a decrease was noted.

Box spars are of several slightly differing forms, but all contain the same principle of separate pieces of timber for the webs and flanges.

Fig. 167 shows various forms of this type of spar. A is a plain box with flanges and webs of spruce glued and bradded or screwed together. The fillets *f* at the corners may or may not be present, but their principal function is to increase the glued surface between flanges and webs, and as this is one of the most important points in the construction of such members they will always appear in good design.

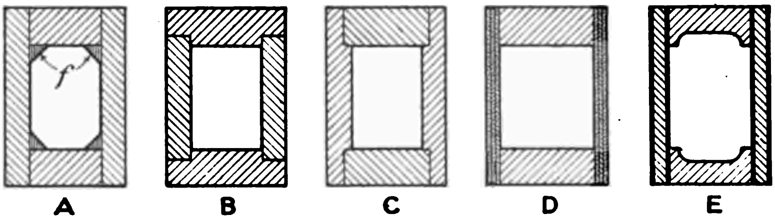

Fig. 167.

B shows a similar form, but the flanges are recessed for the webs, and since this decreases the vertical glue area it is not to be recommended. C shows the webs recessed for the flanges. This is a sound form of construction, but it involves rather more work than that shown at A, and has no compensating advantages over the latter. D shows a box spar with spruce flanges and three-ply webs. At first sight this would appear to be a very good form, but owing to the fact that three-ply is not as good as spruce for resisting compressive stresses the composite flange is slightly weaker than a solid spruce one would be. Three-ply, however, is excellent material for resisting shearing stresses, and its use for webs is therefore sound. E shows a section taken from a German aeroplane, and this is probably the most scientific of all the sections shown. The webs are made of spruce sandwiched between two strips of thin three-ply and the flange is of spindled spruce. These webs combine the advantages of three-ply and spruce, and so the objection raised to type D does not exist. The spindled flanges permit the glueing surface to be very large and do away with the necessity of fillets as shown in A.

From the foregoing remarks, then, it should be clear that the criteria for a good box spar are—

(1) Correct materials for flanges and webs.

(2) Large gluing surfaces.

The main attachment of the members is provided by glue, but in order to add to the strength screwing and bradding are resorted to. As long as the glue holds, this is not an added source of strength but rather a weakness, but if the glue fails somewhat it does give greater security.

In bradding and screwing, great care should be exercised to ensure that the holes are staggered. Not more than one hole should come on any cross section, and the form of staggering giving the best results is shown in Fig. 168, where the full lines show the brads on the front and the dotted lines those on the rear face of the spar.

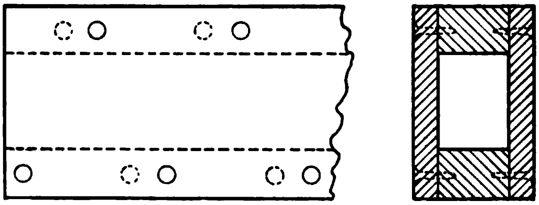

FIG. 168.

At any place in a box spar where the load is applied, or where a joint in the spar occurs, it is necessary to fit a packing piece. These packing pieces should be tapered at the ends in order to avoid a sudden change of cross section in the spar, and should be well glued into position.

Fabric wrapping is often placed on spars of this type, but it adds nothing to their strength as long as the glue holds. It does, however, offer some protection against weathering, and in this respect is useful, but the extra weight and work involved probably do not justify its use.

150. Splices in Spars.—It is often necessary, for the sake of economy in material, to splice the main spars. During the war the extreme scarcity of timber of suitable quality for aeroplane construction led to the practice of splicing spars in practically every aeroplane.

Owing to the importance of the members involved it is of first rate moment that the type of splice and its manufacture should receive every attention. The splice which is used must be able to withstand a combined bending and axial load, either tensile or compressive, although the latter is the one which causes most difficulty. Further, it should have a very high efficiency when compared with the strength of the unspliced spar which it is to replace, and this efficiency must be obtained by the simplest methods possible. The question of simplicity is important for two reasons: first, the speed of production; and secondly, that of reliability.

It should be remembered that every splice has to be made and fitted by a workman, and the more intricacies there are in the type selected the greater is the risk of incorrect manufacture, and in most splices the correct facings of the two parts is all-important. It is proposed to discuss some of the varieties of splices which have been tested, some with good and others with ill success. The authors must acknowledge their indebtedness in this matter to a Report issued by the A.I.D. giving the results of an investigation into the whole question and containing many valuable suggestions and test results. All splices which are of any use rely largely for their strength on the presence of glue. It is, however, inadvisable to depend entirely on glue, and bolts or pegs are often inserted so that should the glue entirely fail, the joint still has a fair margin of strength. Since the glue plays such an important part in the strength of the splice it would be reasonable to assume that the larger the areas of glued surface, the greater would be the strength, and within limits this has been shown by test to be true.

The report previously referred to states that the result of tests showed that short splices always gave a very low efficiency, which could be greatly increased for the same type by increasing the length. This is ascribed to the fact that the glued surface in a short splice is largely confined to the end grain of the wood, which absorbs the glue and renders it only partially effective.

The splices tested may be divided into types as follows:—

(1) Plain scarfed splices, in which the pieces are cut on the bevel (Fig. 169).

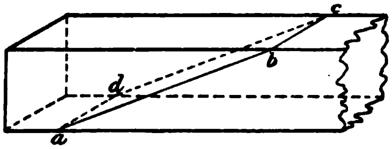

Fig. 169.

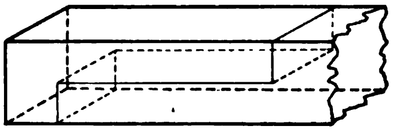

Fig. 170

(2) Tabled splices where the glued surfaces are horizontal (Fig. 170).

(3) Fingered splices (Fig. 171).

All these types have modifications in the way of steps, keys, pegs, and so on; and there exist combinations of two types. Some few joints do not come into any of the categories strictly, but form isolated groups, which are, however, not very important.

151. Plain Scarfed Splices.—Fig. 169 shows the simplest form of this splice, in which two pieces of timber are cut on the bevel and glued together along the face *abcd*. Fig. 172 shows a similar joint, but with the glued face in the vertical plane. This form of splice is usually pegged or screwed and fabric wrapped.

A test upon that shown in Fig. 169 screwed and taped, the slope of the splice being 4 to 1, gave an efficiency of 53 per cent. When the length was increased to give a slope of 8 to 1, and wooden pegs were provided along the centre line of the spar, the tape being retained, the efficiency of the joint rose to 94 per cent., failure starting by slipping, followed by the breaking of the pegs.

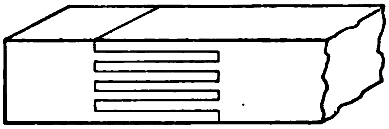
FIG. 171.

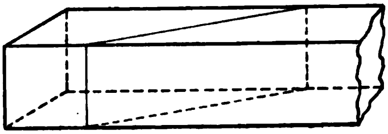
FIG. 172.

The same form of splice, but with the glued face vertical, gave an efficiency of 98 per cent. (Fig. 173). In this case the slope was nearly 14 to 1. This was the best of the simplest splices tested, and is one which has, with slight modifications, been adopted by the Air Ministry as the standard. The standard is shown in Fig. 181.

As showing the effect of length on the efficiency of a splice another test is quoted. This was done on an almost similar specimen to the last except that the pegs were staggered about the central line of the spar. The length of the splice, however, was reduced to half, *i.e.* the slope was about 7 to 1. The joint failed at 68 per cent. efficiency, a loss of 30 per cent. Various modifications of this type were tried, but only one needs any mention. This is the glued, wedged, pegged and taped splice shown in Fig. 174.

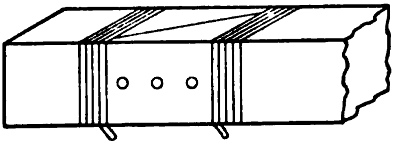
FIG. 173.

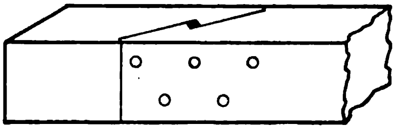
FIG. 174.

With a slope of 14 to 1 and six pegs, this gave 73 per cent. efficiency, failure occurring by shearing of the steps at the wedges and transverse fracturing at the pegs. This form of joint without glue and pegs has a very low efficiency.

An interesting form of the straight spliced joint has been adopted by the American engineers. Instead of a flush gluing face the surface is serrated, thus giving a larger glueing area (Fig. 175). Other details are as in British practice. Tests show that this joint gives about the same efficiency as the standard English splice when glued, but should the glue fail it has a higher strength.

152. Tabled Joints.—Only one form of tabled joint was tested, which is shown in Fig. 176. This consists of a splice of length four times the depth of the specimen and is glued, wedged and fitted with keys. The efficiency of this joint was only 24 per cent. of the solid spar, failure occurring at the hardwood key. There are very many variations of the tabled joint, but the test quoted shows the unsuitability of the type for use in spars.

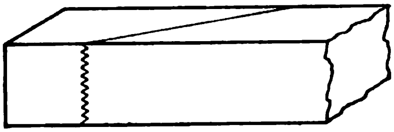

FIG. 175.

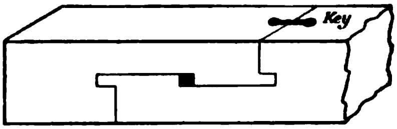

FIG. 176.

153. Fingered Joints.—As a general rule this type of joint has a very high efficiency due to the large areas of glued surface, and the fact that it approximates to the interleaving of the fibres in the natural wood. It is, however, an extremely difficult type to make soundly, and this renders it impractical as a commercial proposition.

Fig. 177 shows a joint in which there are alternating fingers in the two halves to be spliced. These are glued, and the whole is fabric wrapped and taped. The joint is not symmetrical above and below. The efficiency of this splice is about 70 per cent. of the solid. Fig. 178 shows a similar type in which the fingers are wedge-shaped, and are horizontal instead of vertical. The efficiency of this was 96 per cent., probably largely due to the greater length of the splice and consequent increased glued area.

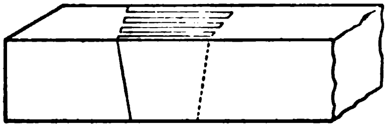

FIG. 177.

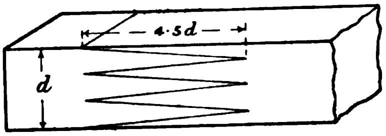

FIG. 178

By arranging the wedge fingers vertically, and adding two pegs in addition to gluing and taping (Fig. 179) the joint was made to show an efficiency of 108 per cent.

Besides the true fingered joints mentioned there is a type which is really in the same class. These are butt joints with connecting pieces inserted. One of these is shown in Fig. 180, where three connecting pieces of three-ply are used. This joint, glued and screwed but not taped, failed across the butt with an efficiency of 50 per cent.

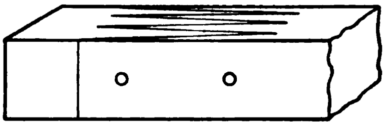

FIG. 179.

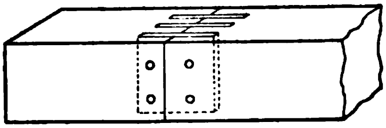

FIG. 180.

When an extra three-ply connection was inserted screws omitted and taping added, the efficiency rose to 60 per cent.

The best joint of this butt type tested was 8½ inches long instead of

8 inches, and had three vertical ash tongues ¼ of an inch thick. It was glued and taped, but had neither screws nor pegs. The efficiency was 89 per cent.

From the foregoing it will be seen that neglecting the fingered type, since this is so complicated in manufacture, the best form of splice is the straight scarf, glued, pegged or bolted and wrapped, and this has been adopted as the Air Ministry standard. The standard splice is shown in Fig. 181.

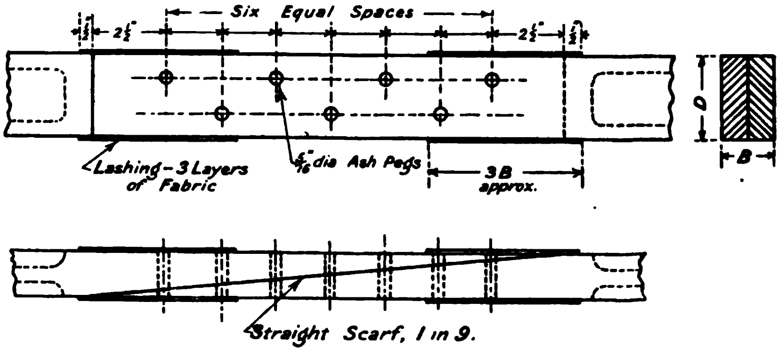

FIG. 181.

154. Position of Splices.—All the splices mentioned were tested under a combined bending moment and direct thrust, but the smaller the moment on them the greater is the strength. All splices in solid spars should, therefore, be placed as near a point of inflection as can be arranged, and in no case at a point where the bending moment exceeds one-half the maximum in the bay. Also no splice should be put at a point where there is a concentrated load from a drag strut, wire or interplane strut.

When splicing box spars the straight scarf joint should also be used both for flanges and webs. In this case the flanges should be jointed at a point where the bending moment does not exceed one-half of the maximum and the webs as near as possible to points of minimum shear. In splicing laminated spars the various laminæ should be scarfed separately, and the scarfs arranged so as to break joint.

CHAPTER XVIII

DETAILED DESIGN—STRUTS

155. Long and Short Struts.—The importance of struts in the design of an aeroplane renders it necessary to go rather fully into this side of the subject. It will be most convenient to deal with the various sections of the aeroplane by themselves, discussing the strut members occurring in each in relation to the special problems they present. Before this can be done, however, some general remarks on this kind of member are necessary.

If a very short strut or pillar be subjected to a compressive load a shortening takes place before ultimate failure occurs. When this failure happens it does so by a direct crushing of the specimen, the actual form of crushing depending on the kind of material under test. A brittle material, *e.g.* stone, fails by an actual partition of material along lines approximately at 45 degrees to the load, due to a shearing stress across these planes, whereas a material like mild steel would fail by actual flattening or flowing of the material. The two cases are indicated in Fig. 182. If a very long strut be tested to destruction, however, failure occurs in a very different manner. Assume the strut to be quite straight at the commencement of the test and the load applied absolutely axially. Up to a certain point nothing happens, the strut keeps perfectly straight and carries the load without any trouble. Moreover, if the strut be deflected slightly, it will return to its straight position, showing that it is in a state of stable equilibrium. When a certain load is reached, however, the

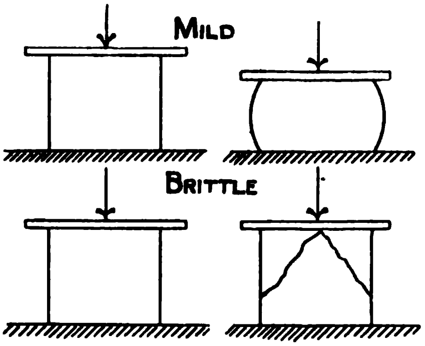

FIG. 182.

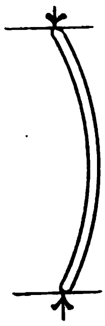

FIG. 183.

strut, on being deflected slightly, will remain in the deflected position, and is now in a state of neutral equilibrium. Any increase in this load, however slight, causes the deflection to increase until it reaches such an amount that the ultimate fibre stress of the material is reached and failure occurs. This failure is known as buckling, and the load just causing buckling to occur is called the critical load of the strut.

It will be understood from the foregoing that there are two distinct types of failure of a strut, very short members failing by the stress exceeding the ultimate compressive stress of the material, and very long members failing by buckling. Between the two extremes of very short and very long struts failure occurs by a combination of direct compressive stress and buckling.

156. Euler's Theory of Long Struts.—The long strut, as pointed out, fails by buckling, and hence the problem of the determination of its crippling load becomes really one of stability. Euler's solution of this problem is the one generally used in calculations of strength for long struts. The Euler strut is homogeneous, of uniform cross section, and very long in relation to its cross sectional dimensions. The load is supposed to be applied perfectly axially.

Suppose AB to be such a strut with a load P applied, and having pin-jointed ends. Let the deflection at the centre from the line of action of P be a.

Take co-ordinate axes tangential and normal to the strut at its centre.

At a distance x from the origin let the deflection be $a - y$.

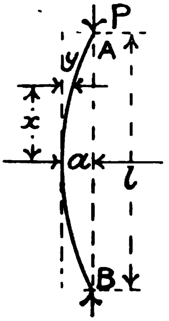

FIG. 184.

Then
$$\frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{1}{EI}\{P(a-y)\}$$

where I is the least moment of inertia of the cross section of the strut.

Therefore
$$\frac{d^2y}{dx^2} + \frac{P}{EI}\cdot y = \frac{P}{EI}a$$

The solution of this equation is

$$y = A \sin\left(\sqrt{\frac{P}{EI}}\cdot x\right) + B \cos\left(\sqrt{\frac{P}{EI}}\cdot x\right) + a \quad . \quad . \quad (1)$$

where A and B are constants to be determined.

When $x = 0$, $\quad y = 0$

Therefore $\quad B + a = 0 \quad$ or $\quad B = -a$

Also when $x = 0$, $\quad \frac{dy}{dx} = 0$

Hence, differentiating

$$\frac{dy}{dx} = A\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}\cdot x\right) - B\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}\cdot x\right)$$

and substituting, $0 = \mathrm{A}\sqrt{\frac{\mathrm{P}}{\mathrm{EI}}}$, whence $\mathrm{A} = 0$, and equation (1) becomes

$$y = -a \cos\left(\sqrt{\frac{\mathrm{P}}{\mathrm{EI}}} \cdot x\right) + a$$

At the end A in Fig. 184, $y = a$, $x = \frac{l}{2}$

Therefore
$$a = a\left\{1 - \cos\left(\sqrt{\frac{\mathrm{P}}{\mathrm{EI}}} \cdot \frac{l}{2}\right)\right\}$$

or
$$a \cos\left(\sqrt{\frac{\mathrm{P}}{\mathrm{EI}}} \cdot \frac{l}{2}\right) = 0$$

For this equation to hold, either a or $\cos\left(\sqrt{\frac{\mathrm{P}}{\mathrm{EI}}} \cdot \frac{l}{2}\right)$ must be equal to zero. If a is zero there is no bending, and the critical load has not been reached.

Hence, for the critical value of P

$$\cos\left(\sqrt{\frac{\mathrm{P}}{\mathrm{EI}}} \cdot \frac{l}{2}\right) = 0$$

The smallest value of $\sqrt{\frac{\mathrm{P}}{\mathrm{EI}}} \cdot \frac{l}{2}$ giving this condition is $\frac{\pi}{2}$

Hence
$$\sqrt{\frac{\mathrm{P}}{\mathrm{EI}}} \cdot \frac{l}{2} = \frac{\pi}{2}$$

or
$$\frac{\mathrm{P}}{\mathrm{EI}} \cdot \frac{l^2}{4} = \frac{\pi^2}{4}$$

Therefore
$$\mathrm{P} = \frac{\pi^2 \mathrm{EI}}{l^2}$$

This gives the critical load for a long column. If, instead of being pin-jointed, the strut is rigidly fixed at the ends (Fig. 185), it is clear that the slope at the ends and at the middle being zero, there are points of inflection at the quarter points of the strut D and E.

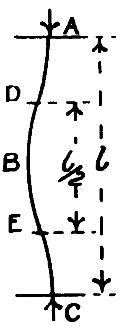

Fig. 185.

The part DE can now be considered the pin-jointed strut, and its length is $l/2$. This is known as the equivalent strut length of AC. Hence for this method of fixing

$$\mathrm{P} = \frac{\pi^2 \mathrm{EI}}{(l/2)^2} = \frac{4\pi^2 \mathrm{EI}}{l^2}$$

Any other particular cases may either be deduced as above or solved by the method shown for the original result.

157. Short Columns.—For a very short column,

if f = the ultimate compressive stress;
A = the cross sectional area;
P = the failing load;
$\mathrm{P} = f\mathrm{A}$

For struts which are sufficiently long to have some tendency to buckle, and yet are not long enough for the direct compressive stress to be negligible, it is clear that the ultimate failing load must lie between $P=\frac{\pi^2EI}{l^2}$ and $P=fA$. Many formulæ have been proposed for this case, but probably the most used is that known as the Rankine formula. This is

$$P=\frac{fA}{1+a\left(\frac{l}{k}\right)^2}$$

where P, f, A, l have the values previously assigned them, k is the least radius of gyration of the cross section, and a is a constant for the material of the strut, which should be determined by experiment.

Consider this formula. It is clear that when l becomes very small the value of P approaches the figure fA, which is the correct value for the ultimate strength of short pillars.

By a proper adjustment of the value of a the formula can be made to reduce to the Euler equation at the other end of the scale.

Thus
$$P=\frac{fA}{1+a\left(\frac{l}{k}\right)^2}$$

or
$$a=\frac{fAk^2}{Pl^2}-\frac{k^2}{l^2}$$

When l is very large compared with k,

$$\frac{k}{l}=0 \quad\text{and}\quad P=\frac{\pi^2EI}{l^2}$$

hence
$$a=\frac{fI}{\pi^2EI}=\frac{f}{\pi^2E}$$

This theoretical value of a, although not as good as one derived from experimental results, is, nevertheless, useful in cases where such tests cannot be readily obtained. If the Rankine formula quoted is required for struts with fixed ends, the value of the equivalent strut length is $\frac{1}{2}l$, which, being substituted for l in the result gives the required value of P.

158. Eccentrically Loaded Strut.—So far only the case of a strut with an axial load has been considered, but there is practically always an eccentricity of loading due either to the form of the end fitting or to variation in the shape of the strut due to inaccurate workmanship, or to both of these causes. It is therefore necessary to know what effect an eccentric load may have upon the ultimate strength of a member. Consider the case of an Euler strut with a load P, having the line of action at a distance e from the vertical axis of the strut before bending (Fig. 186).

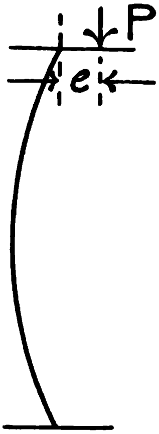

FIG. 186.

Adopting the same notation as in paragraph 156,

$$\frac{d^2y}{dx^2}=\frac{P}{EI}(a+e-y)$$

$$\frac{d^2y}{dx^2}+\frac{P}{EI}\cdot y=\frac{P}{EI}(a+e)$$

The solution of this is

$$y=(a+e)\left\{1-\cos\left(x\sqrt{\frac{P}{EI}}\right)\right\}$$

when $x=0$, $y=0$ and $\frac{dy}{dx}=0$, and when $x=\frac{l}{2}$, $y=a$, and therefore

$$a=(a+e)\left\{1-\cos\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)\right\}$$

or

$$a\cos\frac{l}{2}\sqrt{\frac{P}{EI}}=e\left\{1-\cos\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)\right\}$$

$$a=e\left\{\sec\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)-1\right\}$$

The bending moment at the origin, that is, the centre of the strut, is $P(a+e)$

$$=Pe\sec\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)$$

If I is the minimum moment of inertia, and h is the distance from the most stressed fibre to the neutral axis of the section, then the maximum compressive stress in the strut is

$$f=\frac{Peh}{I}\sec\left(\frac{l}{2}\sqrt{\frac{P}{EI}}\right)+\frac{P}{A}$$

where A is the cross sectional area of the strut.

159. Eccentric Load on Short Column.—Consider the case now of a short column under an eccentric load P (Fig. 187). It is assumed that the column is sufficiently short to fail by compression only, and that there is no buckling effect.

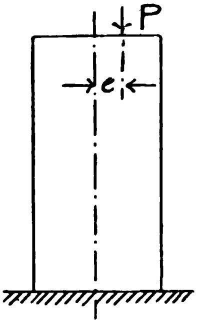

FIG. 187.

The stress on the material is made up of two parts—

(1) The direct stress $=P/A$ distributed equally over the area.

(2) The bending stress $=Peh/I$, where I and h are used as in the previous paragraph.

This bending stress is the maximum value, and is compressive on the inner and tensile on the outer fibre, varying uniformly between.

Hence the maximum stress in the material is

$$f_m = \frac{P}{A}\left(1 + \frac{eh}{k^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and the minimum

$$f_0 = \frac{P}{A}\left(1 - \frac{eh}{k^2}\right)$$

In Rankine's formula, already quoted,

$$f_m = \frac{f}{1 + a\left(\frac{l}{k}\right)^2}$$

where f is the ultimate compressive stress of the material considered.

The effect of an eccentricity e is in effect to reduce this value of f in the ratio of $\dfrac{1}{1 + \frac{eh}{k^2}}$, as shown in equation (1) above. Hence, for the case of a strut which is of sufficient length to be taken out on Rankine's formula, and which has an eccentricity e, the form of the equation becomes

$$P = \frac{fA}{\left\{1 + a\left(\frac{l}{k}\right)^2\right\}\left\{1 + \frac{eh}{k^2}\right\}}$$

It should be noted that this eccentricity may be due to a curvature of the strut as in a bent longeron. A variation in the elastic qualities of a member is equivalent to an eccentricity of loading, and hence practically every strut is of this type.

160. Southwell's Modification of Smith's Formula for Pin-Jointed Struts.—If a long strut were of absolutely uniform section and perfectly straight it would give an ultimate crippling load very near to that calculated by Euler's formula; but in practice there is always an eccentricity in the strut due to deviation from straightness and to slight variation in thickness.

In view of this fact, the figures given on test are not quite so high as the Euler load allows. Further, struts are often required which are not sufficiently long for their strength to be governed purely by their resistance to buckling. It has been found by a large number of experiments that a formula devised by Professor Smith and modified by Major R. V. Southwell, R.A.F., gives a very good figure for the crippling load of tubular struts of normal straightness. The following statement of the formula and method of constructing a curve representing it is taken from a pamphlet issued by the Admiralty Air Department, edited by Professor C. F. Jenkin, and the curves in Appendix II. are also reproduced from the same source.

This formula for the limiting load of a round-ended or pin-jointed strut is

$$P = pA = \frac{p_y A}{1 + \frac{h\delta}{k^2} \sec\left(\sqrt{\frac{p}{Ek^2}} \times \frac{l}{2}\right)}$$

Where P = The limiting load.[1]
p = The limiting average stress.
A = The area of the cross section of the strut.
p_y = The yield point of the material.[2]
h = The greatest distance of any point on the section from the centre line (= the outside radius for a circular tube).
δ = The equivalent eccentricity of loading.
k = The radius of gyration of the cross section.
E = Young's Modulus of the material.
l = Length of the strut.

The equivalent eccentricity of loading δ is the sum of three terms—

(1) The crookedness of the tube, *i.e.* the maximum deflection of any point on the centre line of the tube from the line through the centres of the pin-joints (due to crookedness of the tube).

In commercial tubes the crookedness need not exceed 0·02 inch per foot run of tube. With careful manufacture it may be made much less.

(2) A quantity representing the effect of the eccentricity of the bore of the tube. This may be taken as 0·025 times the bore of the tube, if the maximum eccentricity is due to a variation of thickness of the wall of the tube of ± 10 per cent. (Total difference between thickest and thinnest side = 20 per cent. of mean thickness.)

(3) The radius of the friction circle of the pin-joint (*i.e.* the radius of the pin × coefficient of friction). This term may be omitted if the tube is pinned to attachments which do not rotate so as to assist the bending of the strut.

This formula cannot be used directly, because p appears on both sides; but curves may be drawn and the results read from them.

161. To construct a Curve representing Smith's Modified Formula.—The co-ordinates chosen are the limiting average stress p and l/k the ratio of the length to the least radius of gyration of the cross section of the strut.

First, draw Euler's Curve given by the equation

$$p = \frac{\pi^2 E k^2}{l^2}$$

also the horizontal line p = elastic limit or yield point (LS, in Fig. 188).

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[1] The limiting load is the load which makes the maximum stress in the tube equal to the yield point or elastic limit, whichever is inserted in the formula. The formula is true for struts of any section, and of any elastic material, if p be used instead of p_y.

[2] The strictly accurate formula should have p_e, the elastic limit, instead of p_y, the yield point.

Next choose any value of $\lambda = \sec\left(\sqrt{\frac{P}{Ek^2}} \times \frac{l}{2}\right)$ with its corresponding value of l/l' $\left(l' \text{ given by } p = \frac{\pi^2 E k^2}{l'^2}\right)$ from the following table, and mark off M so that

$$\frac{LM}{MO} = \lambda \frac{h\delta}{k^2}$$

and on the horizontal MD mark off Q, so that

$$\frac{MQ}{MD} = \frac{l}{l'}$$

Then Q is a point on the required curve. Any number of points may be found by selecting a series of values of λ and l/l' from the table.

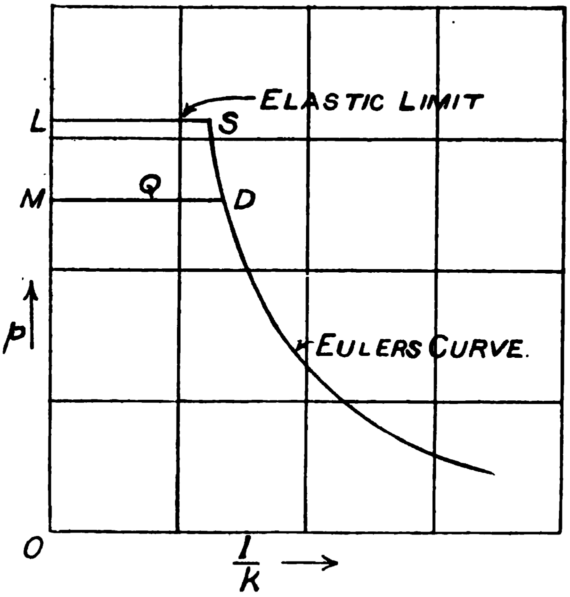

Fig. 188.

Table of Values of λ corresponding to Different Ratios l/l'.

$\frac{l}{l'}$	λ	$\frac{l}{l'}$	λ	$\frac{l}{l'}$	λ	$\frac{l}{l'}$	λ
0·0	1·000	0·25	1·082	0·50	1·414	0·75	2·613
0·05	1·003	0·30	1·122	0·55	1·540	0·80	3·236
0·10	1·012	0·35	1·173	0·60	1·701	0·85	4·284
0·15	1·028	0·40	1·236	0·65	1·914	0·90	6·329
0·20	1·051	0·45	1·315	0·70	2·203	0·95	12·745

It will be noticed that the curve constructed in this way depends on the values chosen for Young's modulus E, the elastic limit or yield point, and the equivalent eccentricity δ. For a given quality of steel (or for tubes made in accordance with one specification) Young's Modulus and the elastic limit or yield point may be assumed to be constant; but the value of δ will generally depend on the length of the strut. The curve, as drawn above, is only correct for one value of δ.

If a series of curves be drawn for a series of values of δ, then the limiting stress may be read off one or other curve for any strut made of the steel (determined by E and the elastic limit or yield point) for which the curves are drawn.

If the value of δ be assumed to be a definite function of the dimensions of the tube, say, for commercial tubes—

$$\delta = \frac{\text{length}}{600} + \frac{\text{internal diameter}}{40}$$

then a new curve may be drawn for a reasonably straight tube of any selected size, giving the limiting stress for all lengths. The curves in Appendix II. have been drawn in this way, and give the limiting loads for struts of any length made of tubes of standard dimension and the quality of steel specified in Air Board Specification T. 6 ($E = 13{,}600$ tons per square inch; $p_y = 28$ tons per square inch).

The formula and curves may be used for struts of all lengths and all thicknesses of wall greater than $\frac{\text{diameter}}{100}$.

This method of obtaining curves for Smith's formula is due to Major R. V. Southwell.

162. Deflection of a Strut.—A strut whose length is very great compared with its diameter may give a large deflection under the limiting load; it is, therefore, desirable to calculate the deflection of such a strut.

The deflection due to the limiting load is given by the expression—

$$\text{Deflection due to limiting load} = \frac{2k^2}{d} \times \frac{\overline{\text{AB}}}{\overline{\text{BC}}} - \delta$$

If the strut is initially crooked, the total deflection will be the sum of the initial deflection and the deflection due to the limiting load.

In the above expression $\overline{\text{AB}}$ and $\overline{\text{BC}}$ are to be measured off the strength curve corresponding to the strut. For example, suppose that the curve in Fig. 189 corresponds with the tube in question, and let—

OC = The length of the strut (56 inches).
CA = The yield point (28 tons per square inch).
CB = The ordinate to the curve at point B, *i.e.* the limiting stress (13·1 tons per square inch).
δ = The equivalent eccentricity of loading (0·2 inch).
d = The outside diameter of the strut ($2\frac{1}{8}$ inches).
k = The radius of gyration of the cross section (0·73 inch).

Then in Fig. 189

$$\frac{AB}{BC} = 1{\cdot}13$$

$$\frac{2k^2}{d} = 0{\cdot}501 \text{ inch}$$

Therefore the deflection due to the limiting load $= 0{\cdot}367$ inch. The figure shows clearly how the deflection increases with the length, by the increase in the ratio $\frac{AB}{BC}$.

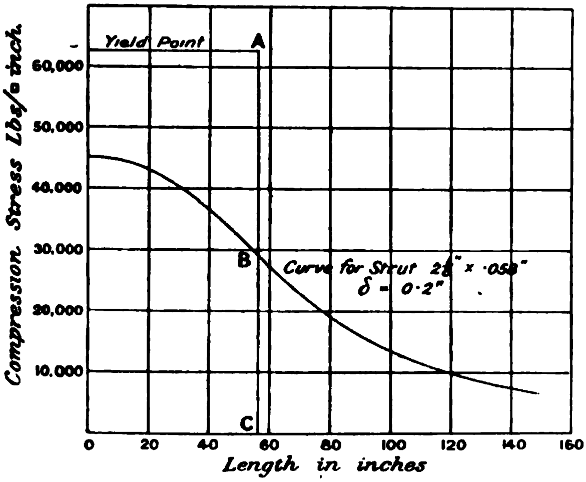

FIG. 189.

163. Modification of Perry's Approximate Formula.—For the limiting stress of a strut this is—

$$p = \frac{1}{2}Q - \sqrt{\frac{1}{4}Q^2 - p_y p_E}$$

where

$$Q = p_y + \left(1 + \frac{d\delta}{2k^2}\right)p_E$$

and

$$p_E = \frac{\pi^2 E}{\left(\frac{l}{k}\right)^2}$$

(This is the Eulerian value of the maximum stress.)

As before $p =$ The limiting average stress.
$p_y =$ The yield point of the steel.
$l =$ The length of the tube.
$k =$ The radius of gyration of the cross section of tube.
$\delta =$ The equivalent eccentricity of loading (defined above).
$d =$ The diameter of the tube.
$E =$ Young's Modulus.

This formula gives values of p about 6 per cent. below those given by Smith's formula; it thus errs on the safe side. It may be applied directly to calculate p for struts of any length, and any thickness of wall greater than $\frac{\text{diameter}}{100}$.

For use in conjunction with this formula, the expression for the deflection of a strut under the limiting load may be written—

$$\text{Deflection due to limiting load} = \frac{2k^2}{d} \times \frac{p_y - p}{p} - \delta$$

If the strut is initially crooked, the total deflection will be the sum of the initial deflection and the deflection due to the limiting load.

If, in any particular case, the tube to be used is not of a gauge or diameter covered by the curves on pp. 305, 306, and 307, the curve on p. 309 will enable its strength to be calculated without the need for working through the whole formula.

Streamline tubular struts are occasionally used, but as a general rule circular ones are preferable, owing to more certainty in manufacture and a greater degree of standardization, thus ensuring more uniformity in strength. If the streamline ones are used, however, the strength may be calculated on the Smith formula.

For oval tubes the results have been worked out, and Fig. 266 shows the form of section and the strength curve which enables any size strut to be dealt with if it is made to Air Board Specification T.6, giving a yield point of 28 tons per square inch and a value of E of 13,600 tons per square inch.

164. Robertson's Strut Formula for Pin-Jointed Struts.—A formula devised by Major Robertson, R.A.F., gives extremely good results when judged by test check figures. This formula is

$$f = \frac{f_c + (\eta + 1)f_e}{2} - \sqrt{\left(\frac{f_c + (\eta + 1)f_e}{2}\right)^2 - f_c f_e}$$

where f = the ultimate stress which may be allowed on the strut.
f_c = the ultimate compressive stress of the material of the strut.
f_e = the ultimate buckling stress which is the Euler crippling load for the strut divided by the cross-sectional area.
η = ·003 l/k, a figure obtained by Robertson from a careful analysis of many results;

where l is the length of the strut and k the least radius of gyration.

This formula takes account of the fact that there is always a certain amount of unavoidable eccentricity of loading, however carefully the end fittings are designed. This eccentricity is partly due to the fittings, and partly to the fact that material is never really homogeneous, which gives a virtual eccentricity or initial curvature.

165. Pin-Jointed Strut with a Concentrated Lateral Load.—Let AB be a pin-jointed strut under an end load P, and a concentrated side load W.

Let the dimensions be as in Fig. 190.

Take the origin at A.
For any point from $x = 0$ to $x = a$

$$\mathrm{EI}\,.\frac{d^2y}{dx^2} + \mathrm{P}y = \left(\frac{b}{a+b}\right)x\mathrm{W}$$

$$y = \mathrm{A}_1 \sin \mu x + \mathrm{B}_1 \cos \mu x + \frac{\mathrm{W}}{\mathrm{P}}\,.\left(\frac{b}{a+b}\right)x$$

where $$\mu^2 = \frac{\mathrm{P}}{\mathrm{EI}}$$

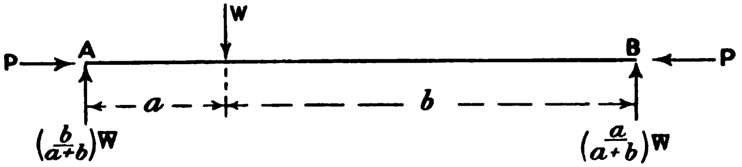

FIG. 190.

Since $y = 0$ when $x = 0$, $\mathrm{B}_1 = 0$

At $x = a$ we have

$$y_{\mathrm{W}} = \mathrm{A}_1 \sin \mu a + \frac{\mathrm{W}}{\mathrm{P}}\,.\frac{ab}{a+b}$$

So by taking the origin at B we have when $x' = b$, $y_{\mathrm{W}} =$ a similar expression with a and b interchanged derived from the second span.

$$\text{therefore}\quad \mathrm{A}_1 \sin \mu a + \frac{\mathrm{W}}{\mathrm{P}}\cdot\frac{b}{a+b}a = \mathrm{A}_2 \sin \mu b + \frac{\mathrm{W}}{\mathrm{P}}\cdot\frac{a}{a+b}b \quad . \quad . \quad . \quad (1)$$

Also the two values of $\frac{dy}{dx}$ are equal and opposite, since W is approached from opposite sides.

$$-\mu\mathrm{A}_1 \cos \mu a + \frac{\mathrm{W}}{\mathrm{P}}\cdot\frac{b}{a+b} = -\left\{-\mu\mathrm{A}_2 \cos \mu b + \frac{\mathrm{W}}{\mathrm{P}}\cdot\frac{a}{a+b}\right\} \quad . \quad (2)$$

Hence, from (1) and (2)

$$\frac{\mathrm{A}_1}{\sin \mu b} = \frac{\mathrm{A}_2}{\sin \mu a} = \frac{\mu\mathrm{A}_1 \cos \mu a + \mu\mathrm{A}_2 \cos \mu b}{\mu \sin \mu b \cos \mu a + \mu \sin \mu a \cos \mu b}$$

$$= \frac{\mathrm{W/P}}{\mu \sin \mu(a+b)}$$

from (2) from $x = 0$ to $x = a$

$$y = \frac{\mathrm{W}}{\mathrm{P}}\cdot\frac{\sin \mu b}{\sin \mu(a+b)}\cdot \sin \mu x + \frac{\mathrm{W}}{\mathrm{P}}\,.\left(\frac{b}{a+b}\right)x$$

$$\frac{d^2y}{dx^2} = -\mu^2\frac{\mathrm{W}}{\mathrm{P}}\cdot\frac{\sin \mu b}{\mu \sin \mu(a+b)}\cdot \sin \mu x$$

$$M = EI.\frac{d^2y}{dx^2} = -\frac{W \sin \mu b}{\mu \sin \mu(a+b)}.\sin \mu x$$

At $x = a$
$$M_{max} = -\frac{W \sin \mu a \sin \mu b}{\mu \sin \mu(a+b)}$$

166. The Crippling Load of a Strut Unsymmetrical about the Centre.—This method of solution is due to Mr. J. Case. In Fig. 191 A′*o*A is a strut unsymmetrical about its centre line *oy*.

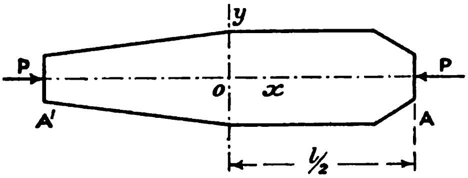

FIG. 191.

Let P_1 = the crippling load of a strut with its two halves like *o*A.
Let P_2 = the crippling load of a strut with its two halves like *o*A′.
With the usual notation we have for small deflections

$$-EI.\frac{d^2y}{dx^2} = Py$$

Therefore for *o*A′
$$\frac{dy}{dx} = \int_x^0 \frac{Py.dx}{EI} + \phi \quad . \quad . \quad . \quad . \quad (1)$$

and for *o*A
$$\frac{dy}{dx} = -\int_0^x \frac{P.y.dx}{EI} + \phi \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where ϕ is the slope at *o*.

Integrating (1) and (2) and putting $y = o$ when $x = \frac{l}{2}$ or $-\frac{l}{2}$

For *o*A′
$$y = \phi(x + \tfrac{1}{2}l) + \int_{-\frac{l}{2}}^x \int_x^0 \frac{Py}{EI}.dx.dx$$

and for *o*A
$$y = \phi(x - \tfrac{1}{2}l) + \int_x^{\frac{l}{2}} \int_0^x \frac{Py}{EI}.dx.dx.$$

If d = the deflection at the centre these give

$$d = \tfrac{1}{2}\phi l + \int_{-\frac{l}{2}}^0 \int_x^0 \frac{Py}{EI}.dx.dx. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and
$$d = -\tfrac{1}{2}\phi l + \int_0^{\frac{l}{2}} \int_0^x \frac{Py}{EI}.dx.dx \quad . \quad . \quad . \quad . \quad (4)$$

Now y is everywhere small and decreases continuously from d at the centre to zero at the ends of the strut. Assume then for a strut symmetrical about the centre section

$$y = d \cos \frac{\pi x}{l}$$

The difference between the ordinates will be small everywhere, and the difference between the second integrals of the curves will be still smaller.

Adding (3) and (4)

$$\frac{2d}{\text{P}} = \int_{-\frac{l}{2}}^{0}\int_{x}^{0} \cdot \frac{y}{\text{EI}} dx \,.\, dx + \int_{0}^{\frac{l}{2}}\int_{0}^{x} \frac{y}{\text{EI}} \,.\, dx \,.\, dx \,. \quad . \quad . \; (5)$$

For a symmetrical strut the same assumption leads to

$$\frac{2d}{\text{P}} = d\int_{-\frac{l}{2}}^{0}\int_{x}^{0} \frac{\cos \pi\left(\frac{x}{l}\right)}{\text{EI}} \,.\, dx \,.\, dx + d\int_{0}^{\frac{l}{2}}\int_{0}^{x} \frac{\cos \pi\left(\frac{x}{l}\right) \,.\, dx \,.\, dx}{\text{EI}}$$

$$= 2d\int_{-\frac{l}{2}}^{0}\int_{x}^{0} \frac{\cos\left(\frac{\pi x}{l}\right)}{\text{EI}} \,.\, dx \,.\, dx$$

$$= 2d\int_{0}^{\frac{l}{2}}\int_{0}^{x} \frac{\cos\left(\frac{\pi x}{l}\right)}{\text{EI}} \,.\, dx \,.\, dx$$

Applying this first to a strut whose two halves are like *o*A, and then to one whose two halves are like *o*A′, we have

$$\int_{-\frac{l}{2}}^{0}\int_{x}^{0} \frac{y}{\text{EI}} \,.\, dx \,.\, dx = \frac{d}{\text{P}_1}$$

and

$$\int_{0}^{\frac{l}{2}}\int_{0}^{x} \frac{y}{\text{EI}} \,.\, dx \,.\, dx = \frac{d'}{\text{P}_2}$$

Hence, from (5)

$$\frac{2}{\text{P}} = \frac{1}{\text{P}_1} + \frac{1}{\text{P}_2}$$

from which P is determined in terms of the calculable values P_1 and P_2.

167. Strut with Two Unequal Compressions in Two Portions.—The solution of this case, which occurs in the spars of an aeroplane due to the fact that each main bay may have several drag bays, is given by Miss Cave-Browne-Cave.

The strut AB has a load P in the portion AC, and a load *k*P in the portion CB. It is pin-jointed at A and B, and has the same EI throughout. The unequal compressions are balanced by reactions at the pin-joints, no external couple being applied by the junction of the bays.

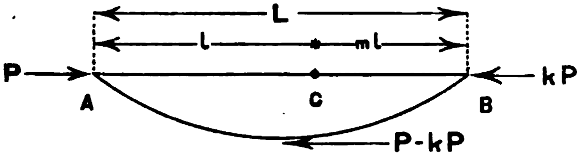

FIG. 192.

The ordinary Euler method then leads to the following results.

The failing load for the bay with the greater compression $= \frac{\pi^2 EI}{L^2 K}$

where $$K = \left\{\frac{\pi}{\theta(1+m)}\right\}^2$$

and θ is defined by

$$\theta\left(\cot\theta + \frac{\cot(m\sqrt{k}\,.\,\theta)}{\sqrt{k}}\right) = \frac{(1-k)^2}{k(m+k)}$$

where $$k = \frac{\text{smaller compression}}{\text{larger compression}}$$

and $$m = \frac{\text{length of bay with smaller compression}}{\text{length of bay with larger compression}}$$

If the arithmetic mean of the two compressions is taken as acting on the full length of the strut, the error is very small, as will be seen from the following table.

k	m	error.
$\frac{1}{4}$	1	2 %
$\frac{1}{4}$	>1	<2 %
·64	·7	$\frac{1}{2}$ %
·64	1	$\frac{1}{4}$ %

This error is so small as to be negligible, and the arithmetic mean may, therefore, be taken on the full length of the strut to obtain the required result.

168. Interplane Struts.—These members are always of sufficient length to allow of them being calculated on the Euler formula, but it must be carefully ensured that sufficient area is left at the ends to take the direct stress. They may be either parallel or tapered; the latter, when the tapering is done to a correct form, being more economical both in weight and head resistance.

The estimation of the strength of the parallel strut presents no difficulty, the ordinary Euler formula being directly applicable.

169. Barling and Webb Tapered Strut.—The tapered strut is not such a simple proposition as the above. The best form of taper to adopt is one which gives a strut failing simultaneously at every section, since this is obviously the most economical in material. The mathematical solution of this problem is extremely complicated, but it has been solved by Messrs. Webb and Barling,[1] and their work, having been verified by actual tests, enables the designer to set out his most economical strut. The mathematical work is not quoted here, but the results are given in such a form as to enable them to be applied quickly to any design.

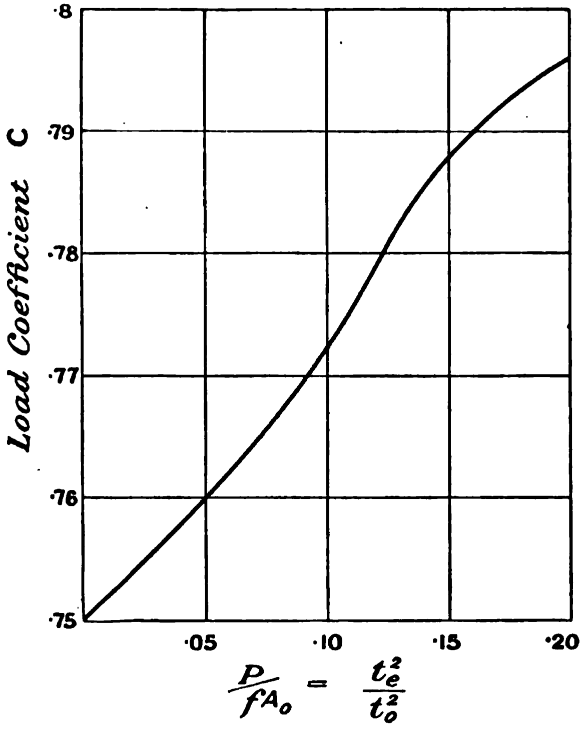

FIG. 193.

Every section normal to the axis is assumed to be geometrically similar to every other section parallel to it.

Let P = the ultimate load which the strut has to carry.

l = the length.

t_o = a dimension of the strut at the centre section.

A_o = the cross-sectional area of the strut at the centre section = at^2.

I_o = the moment of inertia of the centre section = βt_o^4.

$\left.\begin{matrix} t_e \\ A_e \\ I_e \end{matrix}\right\}$ are corresponding values for the end section.

C = a load coefficient defined as the ratio of P to the Euler failing load for a parallel strut of length l and moment of inertia I_o.

f = the ultimate stress of the material.

---

[1] Aeronautical Reprints, No. 9. Royal Aeronautical Society. "Aeroplane Struts," Barling and Webb.

The coefficient C varies with the value of $\frac{P}{fA_0}$, as shown in Fig. 193.

To design a solid tapered strut proceed as follows:—

Assume a value of C about ·8, and from the equation

$$P = \frac{C\pi^2 E\beta t_0^4}{l^2}$$

determine the first approximation to t_0.

Now find A_0 and calculate the value $\frac{P}{fA_0}$. From the curve determine the value of C corresponding to this, and repeat the calculation for t_0 using the new C.

Then $t_e = t_0\sqrt{\frac{P}{fA_0}}$ fixing the end dimensions of the strut.

The curves in Fig. 194 show the correct tapers for various values of the ratio $\frac{t_e}{t_0}$, and so the strut may be designed.

The saving in weight by the use of this design compared with the parallel strut may be as much as 13 per cent.

170. Elliptically Tapered Pin-Jointed Strut.—The following method of finding the strength of an elliptically tapered strut is due to Mr. Arthur Berry.

Consider a strut of length l.

The cross sections at all points are geometrically similar, and the taper along the length is elliptical.

Let I_0 = the moment of inertia of the centre cross section.
I = ,, ,, ,, of any section at a distance x from the centre.
A_0 = the area of the centre cross section.
A = ,, ,, ,, at a distance x from the centre.

Then
$$I = I_0\left(1 - \frac{x^2}{b^2}\right)^2$$

and
$$A = A_0\left(1 - \frac{x^2}{b^2}\right)$$

Then, as for the standard Euler strut,

$$EI.\frac{d^2y}{dx^2} + Py = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let mA_0 be the area of the cross section of a uniform strut which can just stand up to the load P, so that

$$\left(\frac{l}{2}\right)^2 P = \left(\frac{\pi}{2}\right)^2 . EI_0 . m^2$$

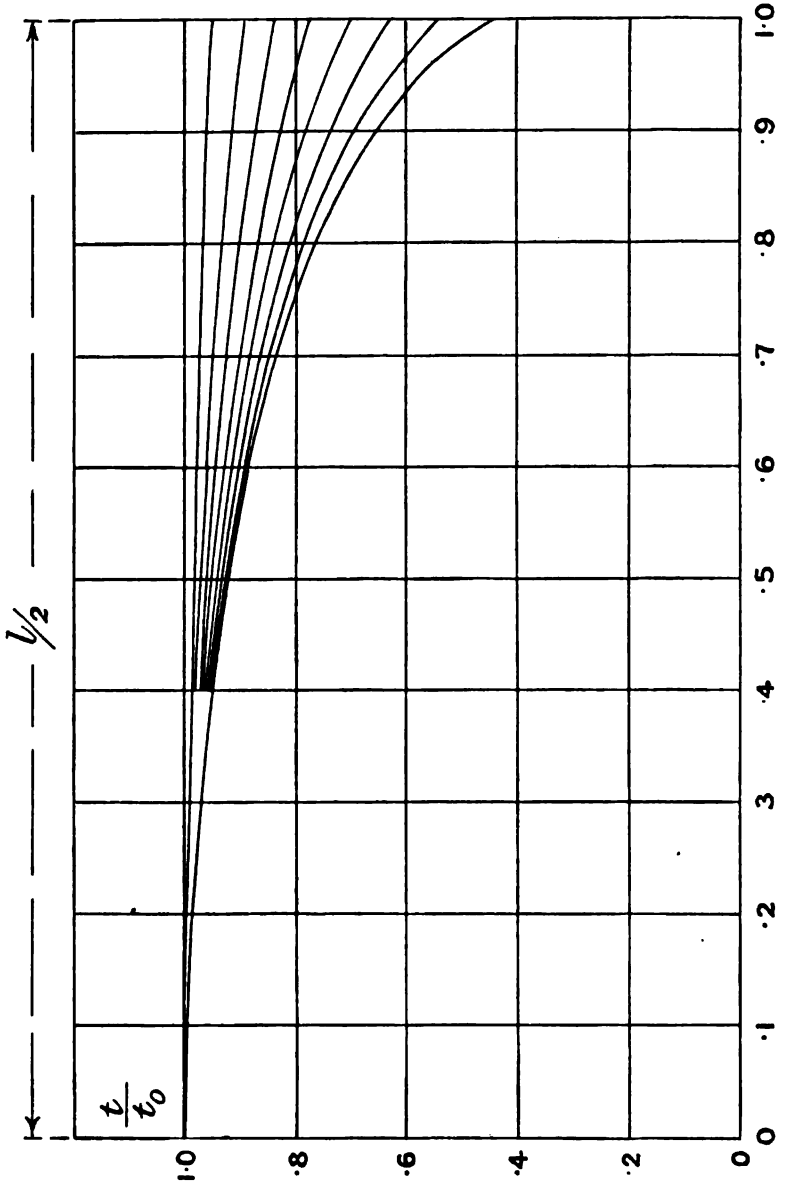

FIG. 194.

Substitute in (1) for I and P

$$EI_0\left(1-\frac{x^2}{b^2}\right)^2\cdot\frac{d^2y}{dx^2}+\frac{1}{\left(\frac{l}{2}\right)^2}\cdot\left(\frac{\pi}{2}\right)^2.EI_0m^2y=0$$

or

$$\left(1-\frac{x^2}{b^2}\right)^2\cdot\frac{d^2y}{dx^2}+\frac{m^2}{\left(\frac{l}{2}\right)^2}\cdot\left(\frac{\pi}{2}\right)^2.y=0$$

Put $x = b \tanh \theta$ and $\frac{l}{2} = b \tanh a$.

Then
$$\left(1 - \frac{x^2}{b^2}\right) = \frac{1}{\cosh^2 \theta}$$

and
$$\frac{dx}{d\theta} = \frac{b}{\cosh^2 \theta}$$

therefore
$$\left(\frac{1}{\cosh^2 \theta}\right)^2 \cdot \frac{\cosh^2 \theta}{b} \cdot \frac{d}{d\theta}\left(\frac{\cosh^2 \theta}{b} \cdot \frac{dy}{d\theta}\right) + \frac{m^2}{b^2 \tanh{}^2 a}\left(\frac{n}{2}\right)^2 y = 0$$

$$\frac{d^2y}{d\theta^2} + \left(\frac{1}{\cosh^2 \theta}\right) 2 \cosh \theta \sinh \theta \, . \frac{dy}{d\theta} + \frac{m^2}{\tanh^2 a}\left(\frac{\pi}{2}\right)^2 y = 0$$

$$\frac{d^2y}{d\theta^2} + 2 \tanh \theta \, . \frac{dy}{d\theta} + \frac{m^2}{\tanh^2 a}\left(\frac{\pi}{2}\right)^2 y = 0$$

$$\cosh \theta \, . \frac{d^2y}{d\theta^2} + \frac{2dy}{d\theta} \cdot \frac{d(\cosh \theta)}{d\theta} + y \frac{d^2(\cosh \theta)}{d\theta^2}$$
$$- y \cosh \theta + \frac{m^2}{\tanh^2 a}\left(\frac{\pi}{2}\right)^2 y \cosh \theta = 0$$

$$\frac{d^2}{d\theta^2}(y \cosh \theta) + \left\{\frac{m^2}{\tanh^2 a}\left(\frac{\pi}{2}\right)^2 - 1\right\} y \cosh \theta = 0$$

The solution is
$$y \cosh \theta = \mathrm{C} \cos \left\{\theta \sqrt{\frac{m^2}{\tanh^2 a} \cdot \left(\frac{\pi}{2}\right)^2 - 1}\right\}$$

(This is arranged to satisfy $\frac{dy}{d\theta} = 0$ when $\theta = 0$, so there is only one constant of integration.)

Since y vanishes at the end where $x = \frac{l}{2}$, $\theta = a$

$$\cos \left\{a \sqrt{\frac{m^2}{\tanh^2 a}\left(\frac{\pi}{2}\right)^2 - 1}\right\} = 0$$

therefore
$$a \sqrt{\frac{m^2}{\tanh^2 a}\left(\frac{\pi}{2}\right)^2 - 1} = \frac{\pi}{2}$$

or
$$m^2 = \tanh^2 a \left\{\frac{1}{a^2} + \left(\frac{2}{\pi}\right)^2\right\}$$

The curve in Fig. 195 has been plotted from this result, and from it the strength of an elliptically tapered strut can be obtained, as a percentage of the strength of the parallel strut of the same centre dimensions.

171. Pin-Jointed Strut with Straight Taper.—It is assumed that a beam with a linear taper, and loaded centrally, will bear the same relation to a parallel beam as regards stiffness that two struts, one linearly tapered and the other parallel, will do.

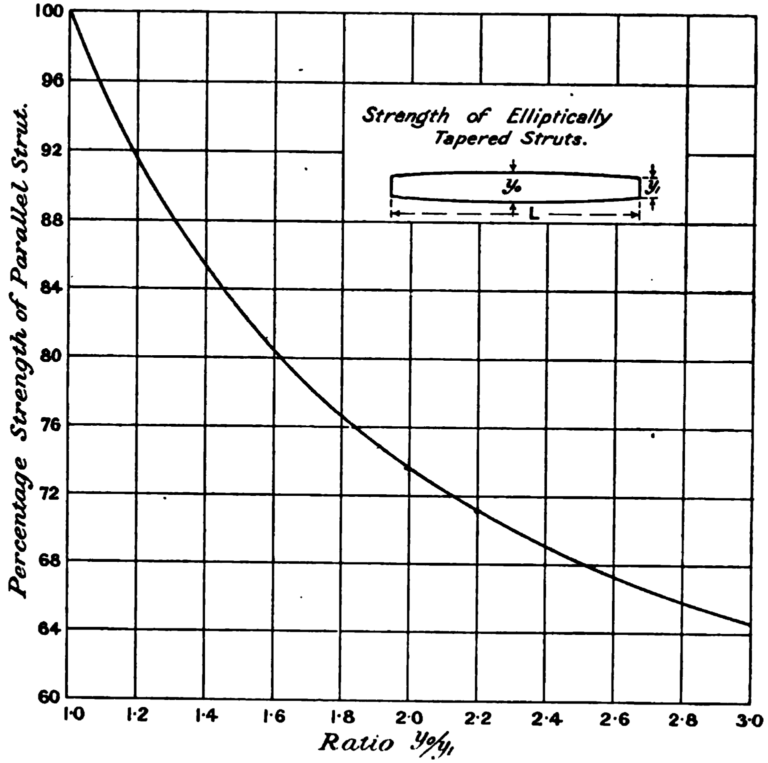

FIG. 195.

Let AB in Fig. 196 represent a beam which is geometrically similar at every cross section having a linear taper from the centre to the ends.

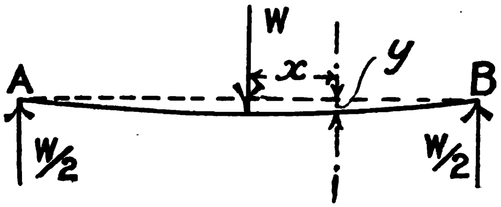

FIG. 196.

Let S_0 = a dimension at the centre section.
kS_0 = the corresponding dimension at end section.
S_x = corresponding dimension at x from centre of beam.

Then $$S_x = S_0\left\{1 - \frac{2x}{l}(1-k)\right\}$$

Put $$\frac{2}{l}(1-k) = a$$

so that $$S_x = S_0(1-ax)$$

Let $$I = nS_x^4 = nS_0^4(1-ax)^4$$

Adopting the notation of Fig. 196.

$$\mathrm{EI}\,.\,\frac{d^2y}{dx^2}=\mathrm{M}=-\frac{\mathrm{W}}{2}\left(\frac{l}{2}-x\right)$$

therefore
$$\mathrm{E}\frac{d^2y}{dx^2}=\frac{-\frac{\mathrm{W}}{2}\left(\frac{l}{2}-x\right)}{n\mathrm{S}_o{}^4(1-ax)^4}=\frac{\mathrm{W}}{2n\mathrm{S}_o{}^4}\left\{\frac{\frac{l}{2}-x}{(1-ax)^4}\right\}$$

$$\mathrm{E}\frac{dy}{dx}=-\frac{\mathrm{W}}{2n\mathrm{S}_o{}^4}\left\{\frac{l}{6a(1-ax)^3}-\frac{1}{3a^2(1-ax)^3}+\frac{1}{2a^2(1-ax)^2}\right\}+\mathrm{P}_1$$

when

$$x=0,\ \frac{dy}{dx}=0\quad\text{and}\quad \mathrm{P}_1=\frac{\mathrm{W}}{12\,na\,\mathrm{S}_o{}^4}\left(l+\frac{1}{a}\right)$$

$$\mathrm{E}y=-\frac{\mathrm{W}}{2n\mathrm{S}^{o4}}\left\{\frac{l}{12a^2(1-ax)^2}-\frac{1}{6a^3(1-ax)^2}+\frac{1}{2a^3(1-ax)}\right\}+\mathrm{P}_1x+\mathrm{P}_2$$

when

$$x=\frac{l}{2},\ y=0\quad\text{and}\quad \mathrm{P}_2=\frac{\mathrm{W}}{2n\mathrm{S}_o{}^4}\left\{\frac{1}{3a^3\left(1-\frac{al}{2}\right)}-\frac{l(al+1)}{12a^2}\right\}$$

$$y=\frac{\mathrm{W}}{2n\mathrm{ES}_o{}^4}\left\{\frac{1}{3a^3\left(1-\frac{al}{2}\right)}+\frac{al+1}{6a^2}\left(x-\frac{l}{2}\right)+\frac{6\,ax-al-4}{12a^3(1-ax)^2}\right\}$$

The maximum deflection occurs at the centre where $x=0$, and is given by

$$y_{\max}=\frac{\mathrm{W}}{2n\mathrm{ES}_o{}^4}\left(\frac{l^3}{12(2-al)}\right)\quad.\ .\ .\ .\ .\ .\ (1)$$

Let S be the side of a parallel beam of the same shape and length which gives the same deflection $y_{\max}$ under the same load W as the tapered beam just considered, W being applied centrally. The deflection of the parallel beam

$$y_{\max}=\frac{\mathrm{W}l^3}{48n\mathrm{ES}^4}\quad.\ .\ .\ .\ .\ .\ .\ .\ (2)$$

Equating (1) and (2)

$$\frac{\mathrm{W}l^3}{48n\mathrm{ES}^4}=\frac{\mathrm{W}l^3}{24n\mathrm{ES}_o{}^4(2-al)}$$

or
$$\mathrm{S}^4=\mathrm{S}_o{}^4\left(1-\frac{al}{2}\right)$$

but
$$a=\frac{2}{l}(1-k)$$

therefore
$$\mathrm{S}=\sqrt[4]{k}\,.\,\mathrm{S}_o$$

Extending this result to struts it may be taken that a straight parallel strut whose side is $\sqrt[4]{k}\,.\,\mathrm{S}_o$ is approximately of the same strength as a linearly tapered one, varying from S_o at the centre to $k\mathrm{S}_o$ at the ends.

In using this formula the factors generally known will be the length of the strut, the load to be carried, and the limit to the size of the end dimension (fixed by fitting considerations).

The first operation is to find the size of parallel strut required to carry the load. This will give the value of S.

Then
$$S^4 = kS_o^4 = S_o^3 S_e$$
where S_e is the end dimension;

therefore
$$S_o^3 = \frac{S^4}{S_e}$$
giving S_o, the required dimension.

172. Most Economical Straight Tapered Strut.—Following on the work just done it is possible to determine the best proportion for a linear tapered strut.

Let l = the length of the strut assumed sufficiently great to apply Euler's formula;

P = the buckling load.

Then, with the same notation as before,
$$P = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 n E S^4}{l^2}$$
or
$$S^4 = \frac{Pl^2}{\pi^2 n E}$$
but
$$S^4 = kS_o^4$$
therefore
$$kS_o^4 = \frac{Pl^2}{\pi^2 n E} \quad \ldots\ldots\ldots\ldots \quad (1)$$

The volume of the tapered strut is given by
$$V = 2b\int_o^{\frac{l}{2}} S_o^2\left\{1 - \frac{2x}{l}(1-k)\right\}^2 . dx$$
$$= b\frac{S_o^2 l}{3}(k^2 + k + 1)$$

where b is a correcting constant for the area.

Substituting from (1)
$$V = \frac{bl}{3}\sqrt{\frac{Pl^2}{\pi^2 n E}} \cdot \frac{1}{\sqrt{k}}(k^2 + k + 1)$$
$$= C(k + k^{\frac{1}{2}} + k^{-\frac{1}{2}})$$
$$\frac{dV}{dk} = C\left(\frac{3}{2}k^{\frac{1}{2}} + \frac{1}{2}k^{-\frac{1}{2}} - \frac{1}{2}k^{-\frac{3}{2}}\right) = 0 \text{ for minimum V}$$
or
$$k^{-\frac{3}{2}}(3k^2 + k - 1) = 0$$
therefore
$$3k^2 + k - 1 = 0$$
or
$$k = \cdot 434$$

The volume of this strut

$$= \frac{b \cdot S_o^2 l}{3}(k^2 + k + 1)$$

$$= \cdot 541\, b \cdot S_o^2 l$$

The volume of the equivalent parallel strut is $V_p = blS^2$.

Substituting, $V_p = \cdot 659\, b.\, S_o^2 l$.

Therefore the saving in weight is about 18 per cent.

173. Solid and Laminated Streamline Interplane Struts.—The commonest form of interplane strut for the smaller types of aeroplanes is the streamline section made in one piece of wood. The moment of inertia of a streamline section may be taken approximately as $BD^3/25$ as shown in Chapter IX. It is sometimes convenient, however, to construct these members in laminations, and Fig. 197 shows a number of types which are satisfactory. In building laminated struts the following points should be borne in mind, as the success, or otherwise, of the construction depends largely on them.

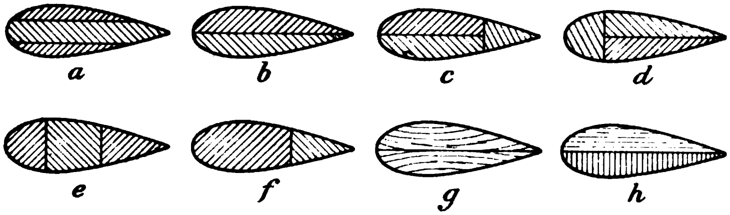

FIG. 197.

(1) The individual laminæ should have the same moisture content.

(2) The timber should be of the same general quality and texture to prevent unequal expansion under varying moisture conditions.

(3) The material must be straight-grained and, if possible, riven material should be used.

(4) The timber should be in radial planks if possible, *i.e.* it should have edge grain. Tangential planks (flower-grained) should especially be avoided for the outer layers.

(5) The end grain rings should be disposed so as to make angles of from 30° to 90° with the joints to avoid warping of the cross section. The arrangement of grain shown at *g* and *h* is bad, and must not be allowed.

The gluing of laminated struts requires great care, and only the best quality glue should be used. After the struts are finished, bands of glued fabric should be applied at a number of points along the member. These bands should be composed of fabric from 1 to 1½ inches wide, and the turns should be lapped to a half width of fabric so that on completion a double thickness of fabric is obtained.

174. Hollow Streamline Struts.—These are constructed in two parts, as shown in Fig. 198, which are either spindled to a streamline section as at A, or have one or more semicircular hollows as at B and C. The two halves are joined along the symmetrical axis by glue and are usually wrapped with glued tape to prevent any opening.

This type of strut may be either parallel or tapered as in the solid form. If the former design is used the strength calculations present no other difficulty than the estimation of the moment of inertia, which entails some amount of work of a routine nature.

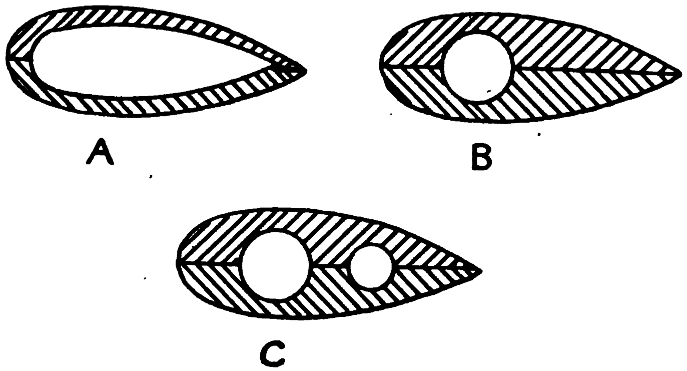

FIG. 198.

If all normal sections of the strut are geometrically similar the design of a tapered member is carried out in exactly the same way as for the solid, as already explained. If the spindling is as shown at B and C, however, the problem is rendered extremely complicated, and test results are the safest criteria as to the strength of the design.

175. Built-up Struts.—The two types previously dealt with are in common use in many aeroplanes, but in larger craft especially, the weight becomes excessive for the strength required, and struts of a built-up character should be adopted. One such form is shown in Fig. 199, which is a design used by Messrs. Handley Page in the 0·400 type aeroplane. AA represents the strut proper, consisting of two channel members of spruce glued together along the edge and keyed to help this joint. These pieces are rebated at the joints E to admit fairing strips C and C, which are glued and bradded to the strut. At intervals these fairing nose and tail pieces are supported by very thin lightened diaphragms D, D.

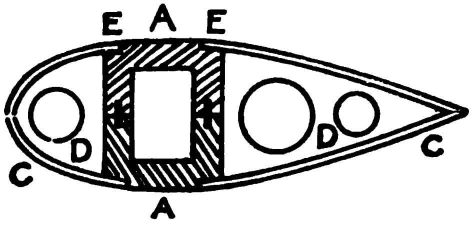

FIG. 199.

The whole structure is then wrapped with glued tape, and the strut is complete. This is a particularly neat construction, and has proved very sound in practice. The strut portion is left solid at the ends, tapered down to fit the sockets, and the fairing brought up as close as possible to the incidence wires (Fig. 200).

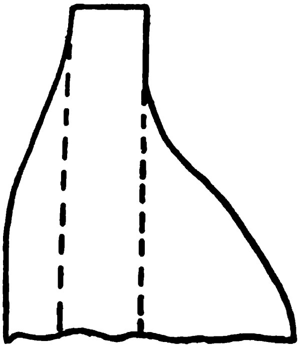

FIG. 200.

176. McGruer Bent-wood Struts.—Another good form of construction is that known as the McGruer bent-wood strut.

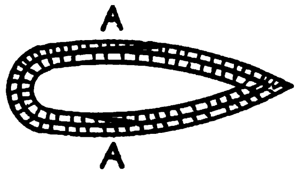

FIG. 201.

This type of construction is more suitable for longerons and fuselage struts, perhaps, than for interplane struts, and it will be treated when dealing with these parts of the structure as regards strength.

The system consists of moulding layers of specially selected spruce to the form required, the layers being spliced as at A (Fig. 201). Two or more layers are used to obtain the necessary strength. The strips are bent under heat and do not rely on the splices to retain their shape. The thickness of the layers varies in different sizes, and tests show the strength of these struts to be very good.

177. Composite Struts.—A form of strut introduced by Messrs. Sopwith consists of a steel or duralumin **I** section, or two channels back to back forming an **I** section, into which wooden nose and tail pieces are fastened, either by small bolts at intervals, or better still by being forced into position. The whole strut is then wrapped with glued tape, and the result is a very good strong strut (Fig. 202).

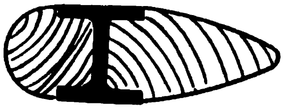

FIG. 202.

178. Metal Struts.—Steel or duralumin tubes faired off to a streamline shape to reduce resistance make excellent struts. From the standpoint of end fittings they are probably the most satisfactory, since these can be made so much smaller and simpler than for ordinary wooden struts. As regards weight for strength, also, they compare very favourably with the latter type for all except the smaller aeroplanes. A great point in their favour also is the reduction of head resistance gained by their use.

In designing a strut of this type the whole load should be assumed to be carried on the tube, as it is not wise or economical to make the fairing of such cross sectional dimensions that it can bear any appreciable amount.

179. Drag Struts.—These members are usually too short to be treated as Euler struts, and so either the Rankine formula or one of its modifications must be used in design.

The simplest form of drag strut is the parallel square section spruce member, the strength of which can be estimated directly by use of the formula; but there are considerable advantages to be gained by tapering these members, first in the weight of the strut itself, and secondly in the reduction of the size and weight of the end fittings. For these tapered struts, just as for interplane struts, there is a form of most economical taper, and they may be designed along similar lines. This usually entails too much care, however, both in design and in manufacture, and the taper adopted is either a curve, which is put in to the designer's taste, or a linear taper from the centre to the ends.

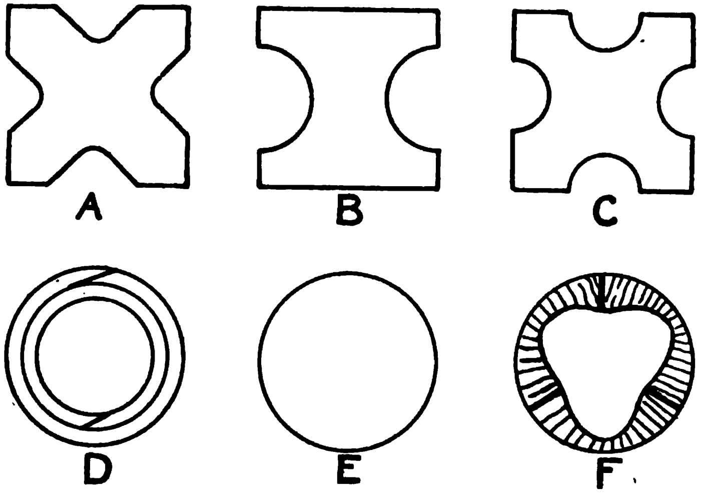

Fig. 203.

In the first case the curve on page 227 may be used to give a good approximation to the strength of the member. The second case, *i.e.* a strut with a linear taper, has been treated in para. 171.

Other forms of drag struts are in common use in addition to the square strut, and a few sections are shown in Fig. 203.

A is a section known as the cruciform. B is the ordinary section spindled on two faces only, while C is spindled on all sides. E is a solid round wooden section; D is the McGruer bent-wood strut. This form of construction has been mentioned previously under interplane struts. The process of manufacture has the effect of raising the value of E above the usual figure, and in some tests it was as high as 2×10^6 lbs. per sq. in. With this figure the usual formula may be used for calculation purposes. The following tables give the results of tests made on this type of strut, and shows the high test values obtained.

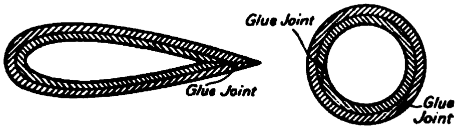

Fig. 204.

Fig. 204 shows the types of struts tested, and the tables give the dimensions and crippling loads when the struts were pivoted at their ends.

TABLE 1.—Streamline Struts (Spruce).

Length	76·0"	76·25"	64·0"	64·0"
Major axis	4·92"	5·04"	6·5"	6·5"
Minor axis	1·735"	1·784"	2·225"	2·185"
Weight (lbs.)	3·375	3·625	4·563	4·75
Crippling load (tons) . . .	0·732	0·88	2·525	2·2

TABLE 2.—Circular Struts (Spruce).

Length . . .	11½"	$22\frac{11}{16}$"	19⅝"	14⅛"	31⅛"	13¾"	25¼"	40⅛"	39"	$49\frac{3}{16}$"
External Diam. .	0·84"	0·98"	1·08"	1·13"	1·16"	1·23"	1·27"	1·47"	1·58"	1·72"
Thickness . .	0·095"	0·108"	0·22"	0·24"	0·26"	0·24"	0·26"	0·27"	0·26"	0·32"
Crippling load (tons)	0·4	0·8	1·16	1·14	0·475	1·72	1·395	1·18	1·44	1·33

To ascertain how closely the crippling loads, as calculated, compared with those found by actual tests, the modulus of elasticity and ultimate stress of specimens of wood, taken from two struts, were determined by experiment. The two struts had the dimensions given in the following table, which also gives the calculated and actual crippling loads.

In making such calculations it should be noted that discrepancies arise in practice because the moisture content of the wood seriously affects its modulus of elasticity.

TABLE 3.—Circular Struts (Spruce).

	No. 1.	No. 2.
Outside diameter	1·665"	1·670"
Inside diameter	1·235"	1·241"
Area, sq. in.	·984	·98

Areas were determined by weighing and were checked by direct measurements.

	No. 1.	No. 2.
Collapsing load	1323 lbs.	1308 lbs.
Load per sq. in.	1345 lbs.	1331 lbs.
Free length (pin centres)	60·82"	60·82"
Radius of gyration	·52"	·521"
Eulerian value	1418 lbs.	1512 lbs.
(Calculated from "E" determined from a piece of the tube)		
Per cent. difference from Eulerian value . .	.5·2	11·5

F shows a form of construction introduced by Messrs. Vickers which is known as the Ryan acetated strut. It consists of three or four sectors glued together at the edges and wrapped with glued tape. These struts behave exactly as a similar section member made from the same quality timber, and the taping appears to have very little effect in increasing the strength. It is, however, very valuable in protecting such a strut against damp and unfavourable weather conditions.

180. Fairey Strut.—A special form of drag strut has been used with considerable success by the Fairey Aviation Co. It is usually oval in section and is hollow, as shown in Fig. 205. The strut is formed in two halves, and instead of the halves being cut to fit together along the junction line, they are made with a bevel as shown at (*a*). The halves are laid together with the centres touching, and with the tongues well glued into position. They are then bound with glued tape, working from the centre towards each end simultaneously, thus forcing the edges together. The effect of this is to put an initial tension in the outer fibres of the strut, and test figures show very good results obtained by this method.

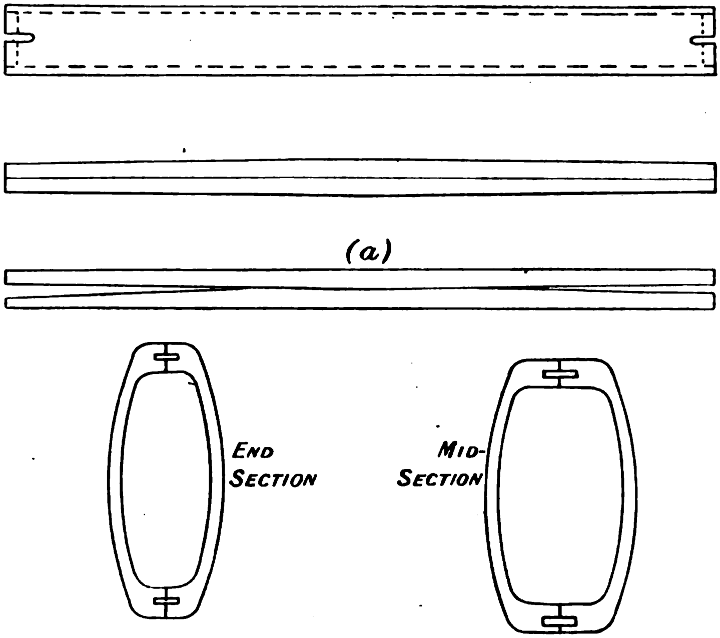

Fig. 205.

Steel tubes are also commonly used as drag struts. The strength of these can be obtained as in the case of steel tubular interplane struts.

181. Compression Ribs.—All the foregoing types of drag strut have the objection that, being small in their end dimensions, they do not offer any support to the spar against a torsional movement. Although with well-designed main plane fittings this should not be a serious point, it is one which sometimes requires consideration. The point is illustrated in Fig. 206, where a steel tubular strut is attached by a socket fitting to a spar. It will be seen that very little support is given to a twisting action of the spar.

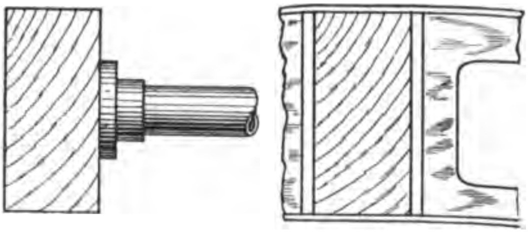

Fig. 206.

Chiefly for this reason a form of strut known as the compression rib is very commonly adopted. This is a rib made to the same profile as the ordinary ribs, but specially strengthened to resist compression, and usually having no nose or tail pieces.

The types in common use are shown in Fig. 207. A represents a single web type in which the web is made of thick material and may be either solid or have lightening holes drilled in it as at E. C shows a type having two webs, which may be either lightened or solid as in the previous one. B is a box form, and the webs may be as in the other two mentioned, or be latticed as at F.

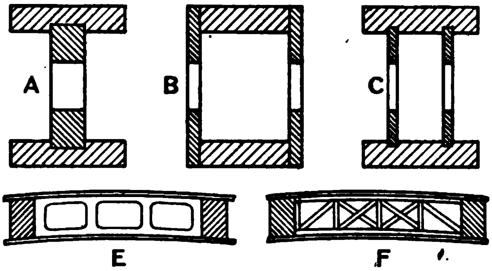

Fig. 207.

The tendency for these members is to fail sideways under a compressive load, but since they are attached to the fabric firmly they are stabilized very considerably against such lateral collapse, and so calculations are of very little use. If there is any doubt as to the strength of a design it should be tested to destruction, and the result taken as a guide. In addition to the stabilizing offered by the fabric it is very usual, especially in the case of ribs made as shown at A, to run light stringers of spruce along the whole wing midway between the spars. These stringers are attached to the top and bottom flanges of the ribs, and of course add considerably to the strength. Instead of wooden stringers, crossed tape after the style of the herring-bone strutting used in wooden floor construction is often employed, as shown in Fig. 208, and this has the same stabilizing effect on the former and compression ribs.

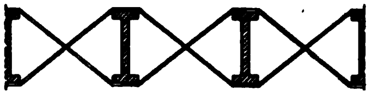

Fig. 208.

182. Fuselage Longerons.—It has been shown that both the top and bottom longerons are liable to take compressive loads. They must therefore be designed as struts. In the complete frame the longerons are continuous members having a different compression in each bay. If the longerons are treated as pin-jointed at the vertical struts attachment points, the loads are easily found, and these have been shown to be very nearly identical with the correct ones, taking account of the continuity over the supports.[1]

183. The Effect of Fabric.—If no fabric is on the fuselage and a load is applied the tendency of the top longeron will be to buckle as shown in Fig. 209 (*a*), *i.e.* with an opposite curvature in each bay, and points of inflection at each point of support. That this must be so is seen at once by considering the energy stored in the system under an end load. It is clear that the longerons will bend in such a way as to store the minimum energy, and this is done when the curvature is as shown.

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[1] A.C.A. Reports and Memoranda, No. 320, by H. L. Cowley.

Turning to the case, however, when the fabric is properly attached to the structure the position is altered. The bays cannot now deflect outward owing to the restraining influence of the fabric, and hence the form which the longeron takes up is somewhat as shown in Fig. 209 (*b*). Here there are two points of inflection between adjacent strut positions. The effect of fabric is therefore to reduce the equivalent length of the struts, and so the simple Rankine formula requires to have its constant modified.

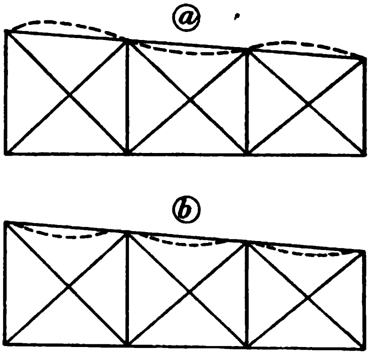

FIG. 209.

The actual section of fuselage longerons varies with different firms and different types of aeroplane.

The commonest type is a solid square section increasing in size from the tail end to the centre sections, and then decreasing again to the nose of the aeroplane. The variation in section is done either by a steady taper or by a series of parallel sections and tapering portions alternating with each other. This is simply a matter of design, as once the frame diagram of the fuselage is fixed the loads in the members are fixed, and the size of these members must be sufficient to take them.

Another form of cross section is the ordinary spindled type, although, since a longeron is equally likely to fail in any direction, it is better to employ a section which has equal moments of inertia about all axes, such as a square or circular one. The spindled form is, however, very common and makes a good job (Fig. 210).

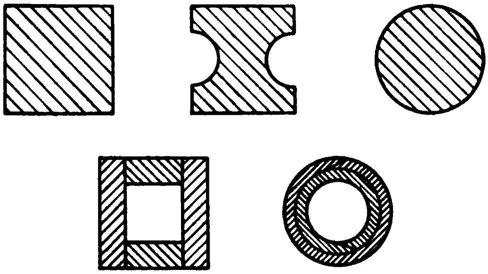

FIG. 210.

The solid circular section may also be used, but the difficulty of attaching fittings to this shape rather weighs against it, and also it is not as economical in material as the square form.

For larger aeroplanes the hollow square section makes a very good, sound job, and if left solid at points where fittings occur, the strength is very satisfactory. This is constructed in the same manner as a box spar previously noted.

The McGruer bent-wood tubes and the Ryan acetated tubes, both previously mentioned, have also been used successfully as longerons. These, however, present the same difficulty as regards the design of fittings which is shared by all circular members.

184. Splices in Longerons.—It is often necessary to splice a longeron either because it is desired to use shorter lengths of timber, or because ash is frequently used for one part of the structure where a bend occurs, and spruce for the remaining comparatively straight portion.

The splicing should be done along the same broad principles laid down for main spars. A very satisfactory splice is the straight scarf joint glued and wood-screwed, the whole being then wrapped with well-glued tape in two layers. This scarf should have a slope not greater than 1 in 9. The longer the splice the more efficient it appears to become, and one construction as shown (Fig. 211) with a slope of about 1 in 13 gave an efficiency, compared with the unspliced member when tested in pure compression, of about 113 per cent. The position for a splice should be about one-third of the distance from one vertical strut position to the next, which is approximately the position of the point of inflection.

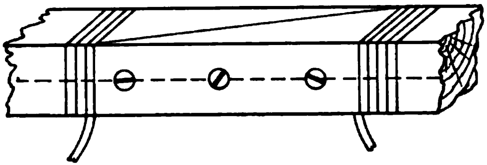
FIG. 211.

185. Fuselage Struts.—These members follow the general rule for short struts and are designed as such. In section they can be any of the shapes shown above for longerons. In addition to these they are sometimes made of the cruciform section shown in Fig. 203.

They may be parallel throughout their length, but are frequently tapered. If the former, the Rankine formula can be used directly in their design. Fabric cannot support these struts as it does the longerons, since it has no stabilizing effect against collapse inwards. If the struts are tapered the formulæ and curves previously quoted may be used to determine the equivalent parallel strut to which the Rankine formula must be applied.

In connection with these members it should be particularly noted that, if for any reason the bracing wires in the fuselage are stronger than necessary to carry the loads upon them, the adjacent struts should be made as strong as the wires. This is important, since in trueing up a fuselage the bracing wires are often pulled up very tightly, and the struts will buckle under their load if they are not designed of sufficient strength to withstand it.

CHAPTER XIX

DETAIL FITTING DESIGN

186. Importance of Correct Design.—With an increase in the size of aeroplanes the question of correct fitting design becomes one of supreme importance. In small aeroplanes it is difficult to make the fittings sufficiently weak without using material of a ridiculously low gauge, but as the size increases there is a corresponding freedom in design, and considerable weight may be saved coupled with a greater security of the aeroplane by a proper observance of scientific principles.

Fittings are so varied in their design that it is impossible to go in detail into all of them. They all depend, however, on certain principles, and once these principles are thoroughly understood there should be no serious difficulty in designing a fitting to meet any conditions.

The primary object of a fitting is to take loads from one point of the main structure to another, and in designing the fitting for this purpose, there are three points which must be borne in mind—

(1) It must be amply strong in all directions to meet the loads coming upon it.

(2) It must be as light as possible.

(3) It must reduce head resistance to a minimum.

The loads which have to be catered for are tensions from the cables or wires and compressions from strut members. These loads have to be carried safely to spars, longerons, or other main members, and so to the body of the aeroplane.

187. Tension Lugs.—The tension from wires and so on is generally taken in the first place on a lug made of sheet metal, usually steel, formed as an integral part of the main fitting. This lug may, however, be considered as a unit by itself, and its design will be discussed in detail.

The researches of Lieut. Scoble, R.N.V.R., in this direction are of great value, and it was shown in a paper read by him to the Institution of Naval Architects,[1] that if a load is applied to a plate by means of a pin passing through a hole in the plate, the stress distribution was very irregular, and was always a maximum in the immediate neighbourhood of the hole. This means that if a lug is designed so that the maximum stress is kept below the elastic limit, the greater part of the metal will never yield its full value. If a slight settlement or giving of the metal takes place, however, it was shown that the stresses became equalized.

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[1] "The Design of Pin Joints based on Ultimate Stress," March 28, 1917.

It was therefore concluded that the correct criterion for the design of a tension lug was the ultimate strength, and not the elastic limit. This theory was tested, and it was found possible to design a fitting which developed its full tensile strength at fracture.

188. Lug of Uniform Strength.—The following method will enable a lug of uniform strength to be designed, but it will be seen later that this is not quite suitable for general aeroplane work.

In Fig. 212 let d = the diameter of the pin.
b = the breadth of the lug across the hole.
t = the thickness of the plate.
B = the breadth of the plate in the arm of lug.
f_t = the ultimate tensile stress of the plate.
f_s = the ultimate shear stress of the pin.
f_c = the ultimate crushing stress of the pin on the plate.

The tensile strength of the plate across the pin-hole is then

$$f_t(b-d)t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The strength of the pin in double shear is

$$\frac{f_s\pi d^2}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The crushing strength of the pin on the plate is

$$f_c . d . t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For uniform strength of the lug the values of (1), (2) and (3) above should be equal. Hence it is possible to determine the dimensions of the lug exactly, having decided on the thickness of plate to be used.

Thus from (2) and (3)

$$\frac{f_s\pi d^2}{2} = f_c dt$$

so that

$$d = \frac{2f_c t}{\pi f_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and from (1) and (3)

$$f_t(b-d)t = f_c dt$$

or

$$b - d = \frac{f_c d}{f_t}$$

and

$$b = d\left(1 + \frac{f_c}{f_t}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The ratio of $\frac{f_s}{f_t}$ varies from ·78 for black bar to ·67 for bright rolled bar. The ratio is higher for low than for high carbon steels.

The value of f_c may be taken as about 50 per cent. more than that of f_t.

A lug designed as above would be found to fail by the pin bursting forward, and in order to overcome this fault the radius of the plate should be set forward a distance as in Fig. 212, varying from $\frac{1}{16}''$ to $\frac{1}{8}''$ according to the dimensions of the lug. The dimension B is the same value as $(b - d)$.

It will be noticed that so far in the design no account has been taken of the load to be carried.

The equations quoted are sufficient in themselves to determine the proportions of a lug, and if one dimension is given the others are obtainable.

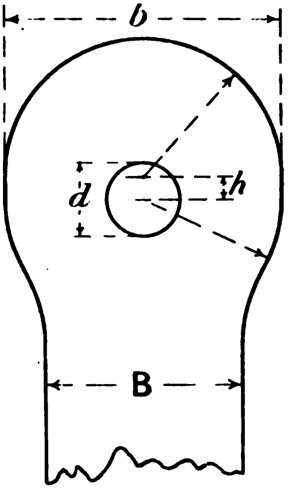


FIG. 212.

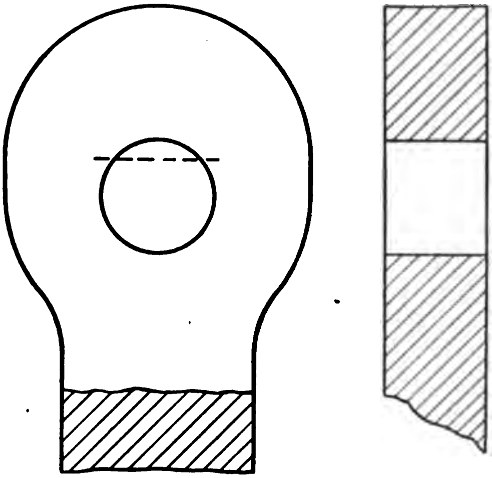


FIG. 213.

For example, if t the thickness of the plate is fixed, there is one lug, and one lug only in this size plate which is of uniform strength.

The load which this lug will take is

$$P = fBt$$

The lug of uniform strength is shown in Fig. 213, and it will be seen to be not of a design suitable for standard aeroplane work.

189. The Aeroplane Wiring Lug.—Having examined the case of the lug of uniform strength, we must turn our attention to the design of a lug suitable for aeroplane wires. In this problem the load P at which failure should occur is known, and the size of cable or wire to carry this load has already been determined.

The size of the cable will determine the fork-end to be used, and this being standard the pin diameter will be known.

Let the diameter of the pin $= d'$.

For clearance the diameter of the hole in the lug must be $\frac{1}{64}''$ greater,

i.e.
$$d = (d' + \tfrac{1}{64}'')$$

If f_t is the minimum ultimate tensile stress of the material of which the plate is made, then the area required in tension will be P/f_t square inches. The thickness of the plate t will now be chosen, and in selecting a suitable thickness it should be borne in mind that tolerances are allowed in manufacture, and the minimum thickness should be used in making calculations. The size of the shank can now be fixed.

For
$$Bt = \frac{P}{f_t}$$
or
$$B = \frac{P}{tf_t}$$

B should be calculated and the dimension fixed to the nearest $\frac{1}{32}''$ on the full size for practical manufacturing considerations.

The dimension b may now be determined since

$$(b - d) = B$$
or
$$b = B + d$$

Here again b is fixed to the nearest $\frac{1}{32}''$. The dimension h may now be fixed and can be made $b/12$, but in no case should it be less than $\frac{1}{32}''$.

The dimensions of the lug are now determined, but there are two points which should always be checked, viz. the bearing stress of the pin on the plate to see that this does not exceed a safe maximum, and the shearing strength of the pin. The formulæ for these two calculations are quoted in the previous paragraph.

Example.—As an example in the design of a wiring lug the calculation will be made for the following case :—

Ultimate load to be carried $P = 5500$ lbs.

Thickness of plate 8 gauge $= \cdot 160$ inch.

This load will require a $\frac{5}{16}''$ streamline wire, and the standard Admiralty fork end for this size gives a pin $\frac{7}{16}''$ diameter.

i.e. $\qquad d' = \cdot 437$

therefore $\qquad d = \cdot 437 + \cdot 016 = \cdot 453''$

Assuming the lug to be of mild steel, take $f_t = 28$ tons per square inch.

Then $\qquad (b - d)tf_t = 5500$

$$(b - d) = \frac{5500}{\cdot 16 \times 28 \times 2240} = \cdot 548''$$

or
$$b = \cdot 548 + \cdot 453$$
$$= 1\cdot 001 \text{ say } 1\tfrac{1}{32}'' \text{ to the next } \tfrac{1}{32}''$$

B will be $\cdot 548''$ or to the next $\frac{1}{32}''$, $B = \frac{9}{16}''$. The offset h should be $\frac{b}{12} = \frac{33}{32 \times 12} = \cdot 086''$, or to the nearest $\frac{1}{32}''$, $h = \frac{3}{32}''$.

The main dimensions are now settled and the lug may be drawn.

It still remains, however, to check the bearing stress of the pin on the plate, and the shearing stress on the pin.

(1) Bearing stress on the plate

$$f_B . t . d' = 5500$$

$$f_B = \frac{5500}{\cdot 16 \times \cdot 437} = 78{,}700 \text{ lbs. per square inch}$$

$$= 35 \text{ tons per square inch.}$$

The ultimate allowable is $1\frac{1}{2} f_t = 42$ tons per square inch, so this is satisfactory.

(2) Shear stress on the pin.

$$\frac{\pi d'^2 f_s}{2} = 5500$$

or

$$f_s = \frac{5500 \times 2}{\pi \times (\cdot 437)^2} = 18{,}340 \text{ lbs. per square inch}$$

$$= 8\cdot 2 \text{ tons per square inch, which is satisfactory.}$$

A lug is often formed by a tongue from the sheet composing the main fitting, and in this case the gauge of the metal is already determined. It sometimes happens that the thickness of the plate is insufficient for the bearing of the pin, while quite satisfactory for the other conditions. In this case it is common practice to braze a washer on to the lug to increase the bearing area for the pin (Fig. 214). This is not the best practice, and should be avoided if possible, but it is a convenient method of overcoming the high bearing stress.

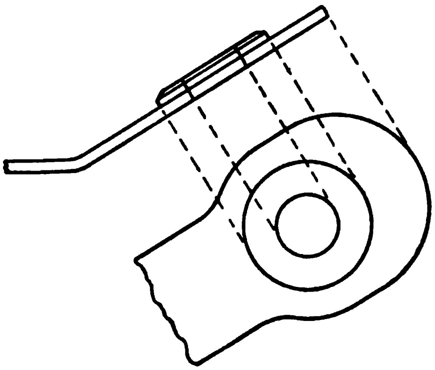


FIG. 214.

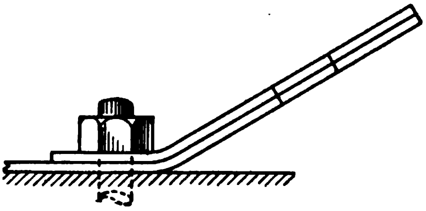


FIG. 215.

In some cases the main fitting is made of a thin gauge plate which is not suitable for making a wiring lug. In this case a reinforcing lug is used consisting of a separate piece of metal which, added to the thin main plate, gives the necessary strength.

In Fig. 215 the lowest part of the lug is turned up from the main fitting plate. The top part is a separate lug added to reinforce the lower one, and stopped beyond the main bolt attachment to the spar.

190. Bending of Lugs.—Wiring lugs should always be bent to the same angle as the wire which they serve, in order to prevent any bending on the plate. This is a most important point, and if not attended to the plates will straighten themselves in flight and cause trouble to the truth of the aeroplane.

The lugs are almost always at an angle to the main fitting, and there are one or two important points to observe in this connection.

The bend should start close up to the bolt, holding the fitting down, or otherwise the lug will straighten in flight and the fitting will be spoilt. The aeroplane will also be thrown out of truth. Fig. 216 shows the correct and incorrect ways of attaching a lug. The dotted line on the incorrect diagram shows what happens under a load on the lug. If for any reason the bolt must be a short distance from the bend in the plate, a stout washer should be used shaped to the bend to prevent opening out (Fig. 217).

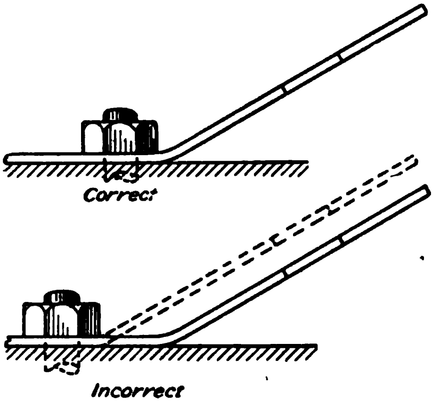


FIG. 216.

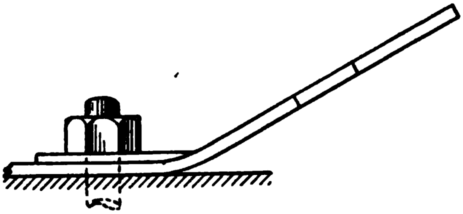


FIG. 217.

The line of the bending on a lug should always be at right angles to the pull, as otherwise a tendency to tear is introduced which may have serious consequences.

191. Transference of Loads to Timber.—The wiring lug having been made sufficiently strong to take the load from the cable, the next point to be looked into is that of getting the load from the wiring lug to the spar.

There are various forms of fittings in use, but finally all of the forces coming through them have to be transmitted to the timber of the spar. This is the difficult part of the problem, since timber is such a weak material compared with steel.

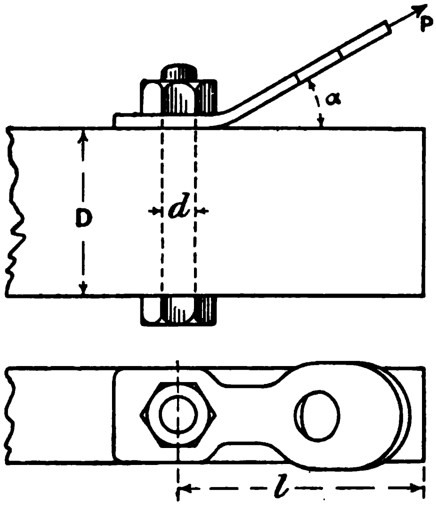


FIG. 218.

The standard method of connecting the fitting to the spar is by means of bolts, and the load must therefore be transmitted through these bolts. Fig. 218 shows a wiring lug attached directly to a spar by means of a single bolt. We have already seen how to make the lug of correct size to take the load P in the cable, and we must examine in detail the problem of transmitting this load to the spar.

The arrangement shown may fail in four ways.

(1) By the bolt failing in shear from the horizontal component of P.
(2) By the bolt failing in tension from the vertical component of P.
(3) By the bolt crushing into the timber.
(4) By the bolt shearing through the timber.

(1) In the first place the bolt must be able to take the horizontal component of P in single shear.

I.e. the inequality $\frac{\pi}{4} \cdot d_1^2 . f_s \nless P \cos a$ must be satisfied, where f_s is the ultimate shear stress of the metal of the bolt, d_1 is the diameter of the bolt at the bottom of the thread, and a is the angle of the pull to the horizontal.

(2) Let f_t be the ultimate tensile stress of the bolt metal.

Then the stress on the metal $f = \dfrac{P \sin a}{\frac{\pi}{4} d_1^2}$

This must be less than f_t or the bolt will fail.

(3) In investigating the bearing of the bolt upon the timber, it is necessary to make an assumption as to the distribution of pressure. In Fig. 219 is shown an approximate distribution of stress for various forms of loading.

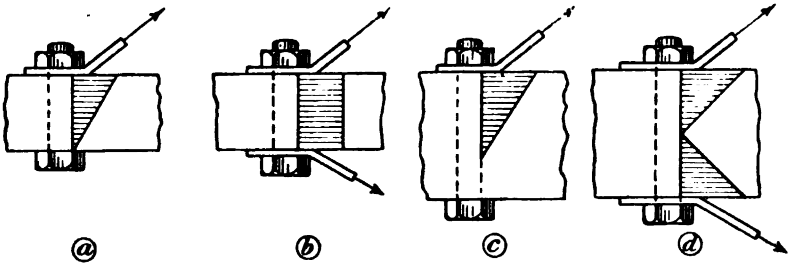


FIG. 219

(*a*) shows a bolt short in relation to its diameter with a pull on one end only, and the distribution is practically triangular.

(*b*) shows a similar bolt but having a pull at each end, this giving a fairly uniform bearing pressure.

(*c*) shows a bolt long in proportion to its diameter with a load at one end. The pressure distribution is again triangular, but the portion of wood farthest from the pull is unstressed.

(*d*) is a similar bolt to the one shown in (*c*), but loaded at both ends. The pressure distribution is now a double triangle.

These diagrams are of course only approximate, and considerable judgment is necessary in any given case to determine what is a good pressure distribution to adopt in calculating the strength required. Further, there are many intermediate distributions depending on the ratio of the length to the diameter of the bolt.

One fact should here be noticed. If a bolt commences to bend, the head is forced against the lug on one side. This increases the frictional force between the plate and the wood, and gives an additional resistance to failure. The greater the bending of the bolt the greater is this frictional force. This is the reason why bolts are often found bent without a failure of the fitting as a whole. This cannot, of course, be taken into account in design, but it is a fact worthy of note.

Returning now to a consideration of the case shown in Fig. 219 (*a*), the pressure distribution will be assumed triangular.

Let f_c = the ultimate crushing stress of wood.
f = the maximum stress developed by the load P.
The average stress developed is $f/2$.
The bearing area of wood = Dd.
The load producing crushing = P cos a.

Therefore $$\frac{f}{2} . \mathrm{D} . d . = \mathrm{P} \cos a$$

and $$f = \frac{2\mathrm{P} \cos a}{\mathrm{D} . d}$$

This value must not exceed f_c or crushing will ensue.

(4) The attachment shown may fail by the bolt shearing through the timber.

Let L be the distance from the centre line of the bolt to the end of the spar, then a strip of wood L in length and D in depth must be sheared along two faces for failure to occur, *i.e.* area resisting shear = 2DL.

The load producing shear = P cos a.

$$\text{The shear stress} = \frac{\mathrm{P} . \cos a}{2\mathrm{DL}}$$

and this must not exceed the ultimate shearing stress of the timber or failure will follow.

Of the four methods of failure enumerated, the first two are easily met. For small fittings there is, further, no difficulty in ensuring that the other two are dealt with. In larger fittings, however, the bearing pressure on the wood often becomes a very serious item, and must be met, either by increasing the number of bolts or the diameter of those already required, from other considerations.

A very good method of obtaining the necessary area is to use a hollow bolt, since this gives the diameter, but no increase in weight. The type of fitting will determine what is the best course to pursue in a given case, and no general rules are possible.

To increase the resistance to shear, either the bolts must be placed further from the end of the spar, or else extra bolts must be used. Care should be paid to the arrangement of bolts for this purpose, as otherwise the full value is not obtained.

In Fig. 220 (*a*) two bolts of equal diameter are arranged so that one is immediately behind the other. The shearing area is $2l_1D$ only, where D

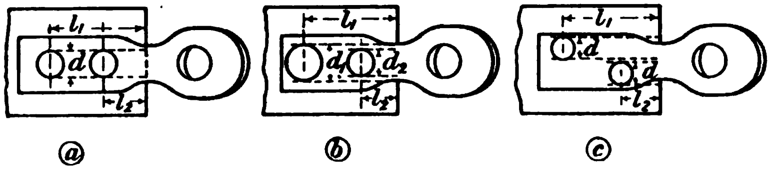


FIG. 220.

is the depth of the spar, whereas had they been staggered as in (*c*) the area would be $2l_1D + 2l_2D$.

Another method by which the latter result might be obtained is shown at (*b*), where the last bolt is made of larger diameter than the front one, and so the shear area is still $2l_1D + 2l_2D$.

In some cases an increase in bearing resistance is obtained by hard wood blocks glued to the spar as shown in Fig. 221.

Hard woods have a much greater strength in bearing than spruce, and in consequence a comparatively thin block will add very considerably to the strength of the fitting.

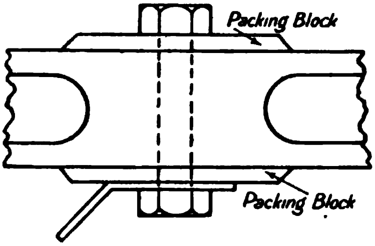


FIG. 221.

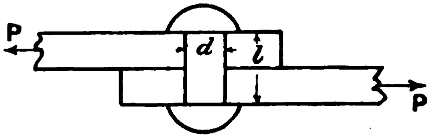


FIG. 222.

192. Design of Rivets and Pins.—The design of rivets and pins is a simple matter, and needs very little discussion. They should be used only in shear, a bolt being substituted if any tensile strength is required.

The distinction between a rivet and a pin is that the latter can be removed easily, in fact is intended to be so removed, while the former is fixed and can only be taken out by filing off the head.

Let d = the diameter of the rivet or the pin.
f_s = the ultimate shear stress of the pin.
f_b = the ultimate bearing stress of the pin.
l = length of pin in contact with the plates (Fig. 222).

Then the ultimate load which can be carried by the rivet in single shear will be $f_s \frac{\pi}{4} d^2$, and the ultimate load which can be carried by the rivet in bearing will be $f_b \cdot \frac{l}{2} \cdot d$, assuming the plates are of equal thickness.

193. Design of Bolts.—A bolt may be subjected to tension or shear, or both simultaneously, and in some positions has also to carry a bending moment. The design of a suitable bolt for the case where there is only tension or shear is simple.

If d is the diameter of the shank of the bolt, the diameter of the bottom of the thread may be either the same as this or less. In the case where it is less the thread is known as a minus thread, and this is the usual thing. Where it is the same as the shank the thread is called a plus thread. Let d_1 be the diameter of the bolt at the bottom of the thread. Then the tensile strength of the bolt is $P_t = \frac{\pi}{4} d_1^2 f_t$ where f_t is the ultimate tensile stress of the material.

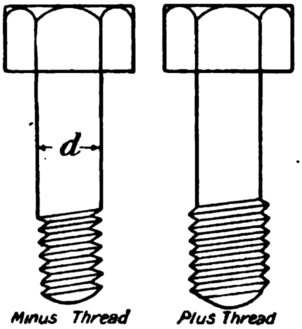


FIG. 223.

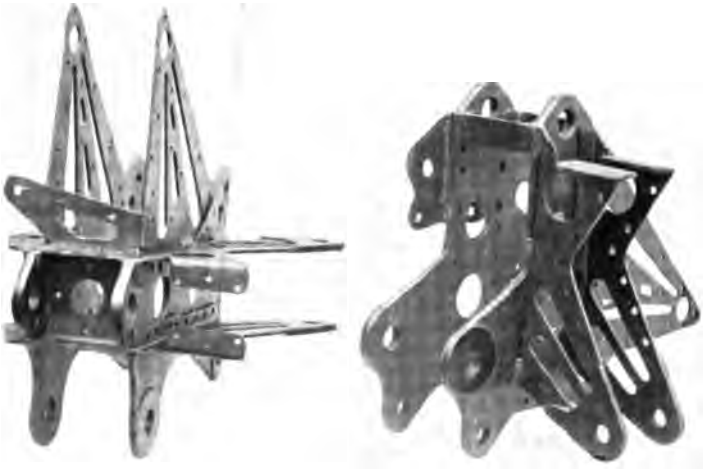
The table shows the diameter and areas at the bottom of the thread, etc., for standard bolts.

BRITISH STANDARD FINE THREADS.

Full diameter.	Number of threads per inch.	Cross-sectional area at bottom of thread.	Minimum ultimate strength of bolt.
		sq. mm.	(lbs.)
4 B.A. 3·6 mm.	—	6·20	753
2 B.A. 4·7 mm.	—	10·93	1,325
		sq. inch	
$\frac{1}{4}''$	26	·0316	2,480
$\frac{5}{16}''$	22	·0508	3,980
$\frac{3}{8}''$	20	·0760	5,960
$\frac{7}{16}''$	18	·1054	8,260
$\frac{1}{2}''$	16	·1385	10,860
$\frac{9}{16}''$	16	·1828	14,300
$\frac{5}{8}''$	14	·2235	17,500
$\frac{11}{16}''$	14	·2790	21,900
$\frac{3}{4}''$	12	·3250	25,500
$\frac{13}{16}''$	12	·3913	30,700
$\frac{7}{8}''$	11	·4520	35,400
$\frac{15}{16}''$	11	·5295	41,500
$1''$	10	·5971	46,800

The shearing strength of the bolt should also be estimated on the threaded section dimensions, and is $P_s = \frac{\pi}{4} d_1^2 f_s$, where f_s is the ultimate shear stress of the material.

Owing to the sudden change of the sectional area of material where the thread commences there is a concentration of stress, and this section is the weakest in the bolt in consequence. For this reason the values of f taken in design should be well within the limits of the material used, and no chances should be taken with the physical constants of the metal.



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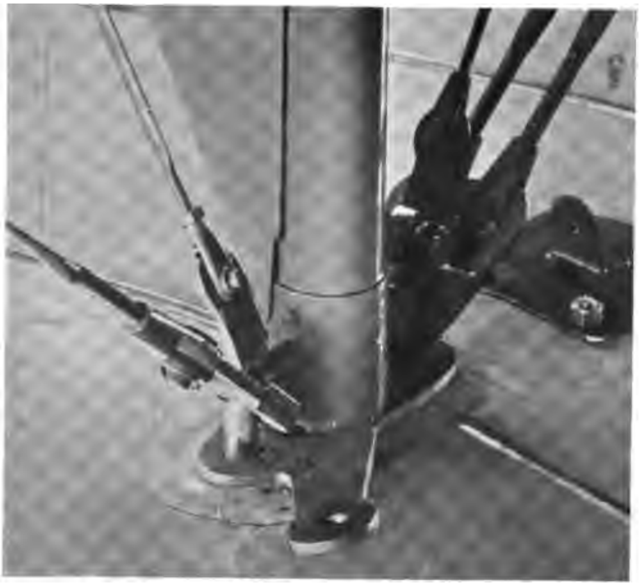
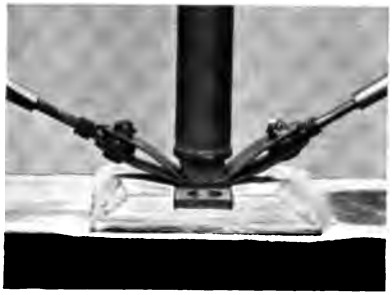


2

Plate XVII.—Main Plane Fittings.

1. Handley Page. **2. Short.**



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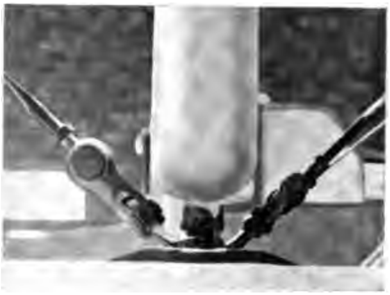

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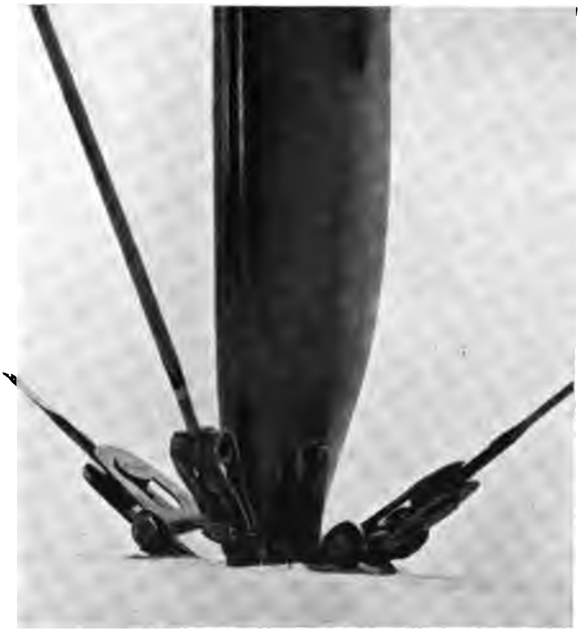

3

Plate XVIII.—Main Plane Fittings.

1. Short (drag bracing). 2. Phœnix. 3. Beardmore.

A hollow bolt is designed in exactly the same way except of course that

$$P_t = \frac{\pi}{4}(d_1^2 - d_0^2)f_c$$

$$P_s = \frac{\pi}{4}(d_1^2 - d_0^2)f_s$$

where d_0 is the diameter of the hole through the bolt.

194. Strut Sockets.—If a member in the structure of an aeroplane is always in compression the design of the end fittings presents no difficulty. It has low loads coming upon it, and its main function is to register the strut end in position on the main member to which it is attached.

Wooden interplane struts may be fitted to the spar by means of a thin gauge steel socket of streamline shape, which is welded to the main fitting plate at one or two points. A pin is usually passed through the socket and the strut end to prevent any tendency to separate when the wires are slackened off.

Steel interplane struts are usually socketed with a short length of steel tube, and the standard aeroplane structural tubes are so arranged that this telescoping or socketing can be done with all sizes, using the standard sections both for the main member and the socket.

In Plates XVII. and XVIII. varieties of main plane fittings are shown.

Fuselage struts can be registered by turning up small lugs from the main clip plate, thus forming a kind of socket into which the end of the strut fits tightly.

Another method is to take the vertical plane bracing wires from two small bolts between wiring plates and to use these bolts to form a registration for the strut.

Still another form of fixing is by means of a socket made out of tinned steel plate of very thin gauge, either spot-welded or soldered to the clip.

If steel tubular struts are used the sockets are also of steel tube, but as a general rule if this type of member is used in fuselage construction the longerons are of steel tube as well, and the fittings then become junction pieces made of tube. In Plate XIX. are shown a variety of fuselage fittings.

195. Tension Tubes.—Some members in the aeroplane structure, whilst often in compression, have under certain conditions of loading to act as tension members.

In this case the design of the end fittings is of considerable importance. Such cases occur in the centre section struts, which in normal flight are in tension, but which in a steep nosedive are subjected to compression, and in stay tubes from the hull of a boat seaplane to the wings, which in landing carry heavy compressive loads but in flight are in tension.

The design of these tube socket ends has been studied by Lieut. W. D. Scoble, R.N.V.R., and he found that the design carried out on ordinary lines agreed well with the results of experiment. The usual type of joint is one in which the tube is pinned to the socket by a number of small pins.

In a design of this sort the difficulty arises from the fact that the bearing area of the pins on the tube is small and the stress is very high, even when other conditions are satisfactorily met. This may be overcome by sweating a liner inside the tube and so increasing the bearing area at the fitting end. This case will be considered and Lieut. Scoble's method will be followed closely. Fig. 224 shows a tube with a short liner at one end which is fixed into a socket tube.

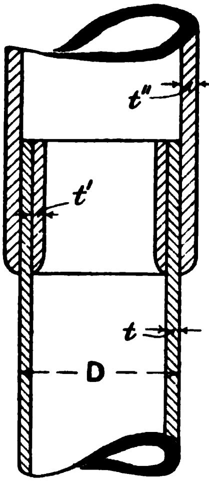

FIG. 224.

Let D = the diameter of the tube.
t = the thickness of the tube.
f_t = the ultimate tensile stress of the tube.
f_s = the ultimate shear stress of the tube.
f_c = the ultimate crushing stress of the tube and liner.
t' = the thickness of the liner.
$f_{t'}$ = the ultimate tensile stress of the liner.
$f_{s'}$ = the ultimate shear stress of the liner.
t'' = the thickness of the socket.
$f_{t''}$ = the ultimate tensile stress of the socket.
$f_{s''}$ = the ultimate shear stress of the socket.
$f_{c''}$ = the ultimate crushing stress of the socket.

The tube is to be pinned to the socket by n pins, of diameter d, in single shear.

$f_{s'''}$ = the ultimate shear stress of the pins.

The strength of the tube in direct tension

$$P = \pi D t f_t \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

This is the load which the fitting should stand.

The shearing strength of the pins is $\frac{n\pi}{4} d^2 f_{s'''}$ . . . . (2)

The crushing strength of the tube and liner is $nd(t + t')f_c$ . . . . (3)

The crushing strength of the socket is $ndt''f_{c''}$ . . . . . . (4)

The tensile strength of the socket is $(\pi D - \frac{n}{2} d)t''f_{t''}$ . . . . . . (5)

assuming that there are two rows of pins.

The above equations enable the design to be made. It is clear that (2), (3), (4) and (5) should each be at least equal to (1) to ensure that the socket enables the tube to develop its full strength.

Example.—As an example design a socket for a tube 2" by 22*g*.

(1) The strength of the tube $= \pi D t f_t$

$$= \pi \times 2 \times \cdot 028 \times 30 = 5 \cdot 26 \text{ tons.}$$

(2) Assume 10 pins are used. Then from (2) above

$$\frac{n\pi}{4} d^2 f_{s'''} = 5 \cdot 26$$

$$10 \frac{\pi}{4} d^2 . 24 = 5 \cdot 26$$

$$d^2 = \frac{5 \cdot 26 \times 4}{10 \times \pi \times 24} = \cdot 028$$

Therefore $d = \cdot 167$, say $\frac{3}{16}$"

(3) To find the thickness of liner use equation (3) above.

$$nd(t+t')f_c = 5{\cdot}26$$

$$10 \times \tfrac{3}{16}({\cdot}028 + t')60 = 5{\cdot}26$$

$$t' = \frac{5{\cdot}26 \times 16}{60 \times 10 \times 3} - {\cdot}028 = {\cdot}019\text{; say } 24g.$$

The thickness of the socket is given by (4) and (5) above, and the maximum value should be taken.

From (4) $\qquad 10 \times \tfrac{3}{16} \times t'' \times 60 = 5{\cdot}26$

Therefore $\qquad t'' = {\cdot}047$

From (5) $\qquad \left(2\pi - \frac{5 \times 3}{16}\right)t'' \times 30 = 5{\cdot}26$

$$t'' = {\cdot}0328$$

The socket should be 18*g*.

In this case the strength of the tube has been taken on the minimum stress allowed by the specification for steel tubes. As a general rule, the strength developed will be more than this, and it is not a bad plan, therefore, to calculate the strength of the tube as a main structural member on the minimum value, but to design the socket on the assumption that it will develop the maximum strength.

In the case above, this would mean that, although the tube should be assumed to be capable of taking 5·26 tons only as a structural member, the socket would be designed on the assumption that the steel of the tube was the best quality, say 40 tons per square inch ultimate tensile stress. This, of course, only modifies the calculations slightly, and does not affect the method.

Lieut. Scoble recommends that the pins should be arranged as at (*a*), and not as at (*b*) (Fig. 225). If the holes are staggered as at (*b*) the joint fails by a fracture through the ten pin holes, whereas if they are unstaggered the failure is only through five holes.

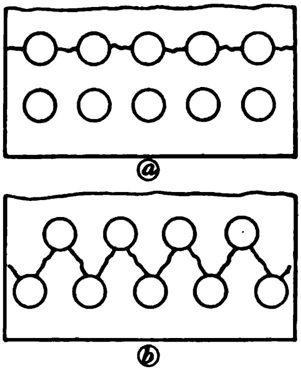

FIG. 225.

196. Soldered Joints.—In connection with his experimental work on the above, Lieut. Scoble made a series of tests on soldered joints, and these are quoted since they are of great interest The first set was made to determine the permissible clearance between the tube and socket in a soldered joint. A steel plug was soldered into a steel tube and a tension test applied. The results are given in the table at the top of p. 252.

It will be seen that the first five tests gave even results with the exception of No. 3, which was probably a bad specimen. The last two, where the clearance had been increased to ·03″, gave inferior results. It would appear that up to ·02″ clearance the strength to be expected is about 4000 lbs. per square inch, but beyond that there is a serious drop.

Test.	Tube, inside diameter.	Length of plug.	Size of plug.	Ultimate load (lbs.).	Ultimate stress (lbs.).	Remarks.
1	1·28″	One diam	·005″small	22,270	4320	Solder failed
2	"	"	·010 "	22,600	4390	Solder holding
3	"	"	·015 "	14,400	2800	Solder failed
4	"	"	·020 "	22,600	4390	"
5	"	"	·020 "	21,500	4170	"
6	"	"	·030 "	15,340	2980	"
7	"	"	·030 "	14,550	2820	"

The next series of experiments was a comparison of the strength when steel was welded to steel, and steel to aluminium manganese bronze. A steel tube of ·75″ inside diameter was used for the experiments; plugs were soldered in, and the specimens tested under tension as previously. Table 1 shows the results for steel–steel, and Table 2 for steel–manganese bronze.

TABLE 1.—Steel-Steel.

Test.	Length of plug.	Load (lbs.).	Shear stress on solder, lbs. per sq. in.
1	¼	3,350	5680
2	¼	3,620	6140
3	½	6,590	5590
4	½	6,430	5450
5	½	7,120	6040
6	¾	7,700	4350
7	¾	8,560	4840
8	1	11,490	4860
9	1	11,340	4800

TABLE 2.—Steel-Bronze.

Test.	Length of plug.	Load (lbs.).	Shear stress on solder, lbs. per sq. in.
10	¼	1100	1860
11	¼	1050	1780
12	¼	880	1490
13	½	1660	1410
14	½	1750	1480
15	½	2000	1695
16	½	2530	2140
17	½	2100	1780
18	¾	2350	1330
19	1	2570	1090
20	1	1920	814
21	1	3370	1430
22	1½	3020	852
23	1½	3510	991

These results are fairly steady in value and can be used in estimating the probable strength of a soldered joint, but it should be fully appreciated that no aeroplane fitting should rely on soldering for its strength. The soldering should simply be an increased factor, and until it is possible to

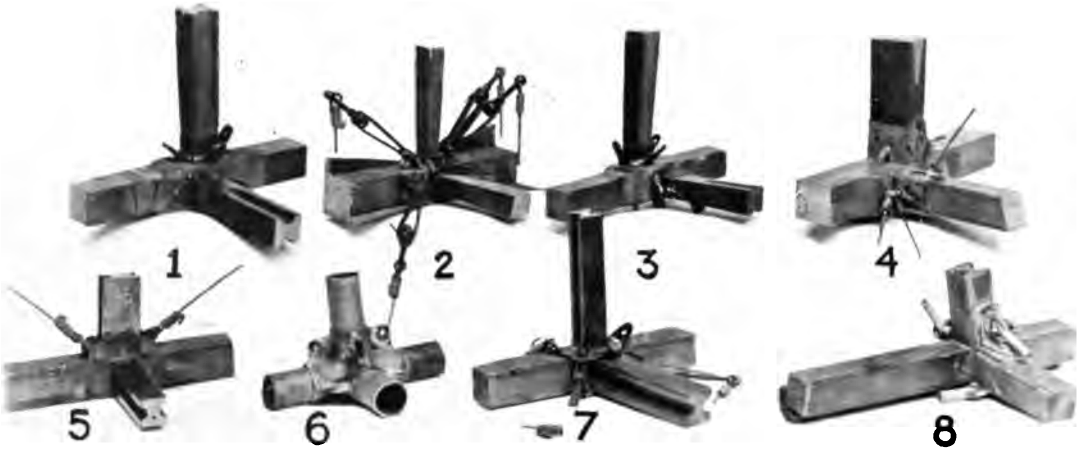

Plate XIX.—Fuselage Fittings.

1. Airco. 2. Avro. 3. Airco. 4. Short. 5. Sopwith, 6. Vickers.
7. Sopwith. 8. British and Colonial.

guarantee the perfection of every joint, welded or soldered, this condition must remain. In unimportant positions, *i.e.* where failure would not mean an accident to the aeroplane, it is permissible to employ soldering without any other method of attachment.

Steel tubes can be attached to other members without the use of a fitting, simply by means of a bolt, as shown in Fig. 226. A liner is soldered into the end of the tube, and the end is then hammered flat and drilled to take a bolt. This method is often adopted for raking struts, but is not so sound a type of construction as a proper socket, as there is a tendency to lever the nut off the bolt.

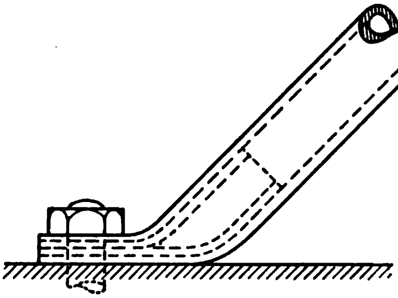

FIG. 226.

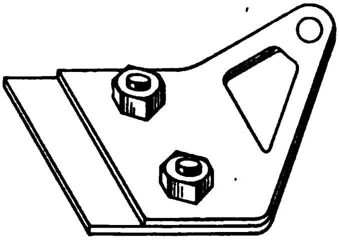

FIG. 227.

197. Local Strengthening and Lightening.—Owing to the number of members which are attached to the same fitting, each of them having a different load to transmit, it is clearly impossible to make the fitting of absolutely uniform strength throughout.

The fitting is often made up of a single sheet of metal, which forms the base of the design. This is then bent to shape so that the various lugs are at the correct angle for taking the wires they serve.

Some parts of such a fitting will be too strong, and some will be too weak, and modifications are introduced to meet these cases.

Where metal is unstressed, or where too large a section is left to take the load being transmitted, lightening holes may be drilled to save weight.

There is one important point which must be borne in mind when settling the position of these lightening holes: they must on no account be drilled in the direct line which a load would take from its point of application on the fitting to the point where it is transferred to the spar or longeron.

Local strengthening of a fitting may be done in one or two ways. The brazing on of washers to a tension lug is one of these which has already been noted. A tension lug may be reinforced as shown in Fig. 227, where a lug of exactly similar shape is cut out of another piece of steel plate and fastened on top of the original lug. This second or reinforcing lug is only continued far enough to be properly gripped by the main bolts, through which the pull is transmitted to the spar. The two lugs may be spot-welded together at the edges, although such attachment is not really necessary.

198. Bending Allowance in Plate.—The great majority of fittings are designed so that they will develop on to a plane. By this means stampings can be made and the fittings bent to shape.

In setting out a development, however, for manufacturing purposes, allowances must be made for bending. These bending allowances are of great importance if the fitting is required to fit the spar or longeron, and Fig. 228 gives the amount of such allowances for various gauge plates.

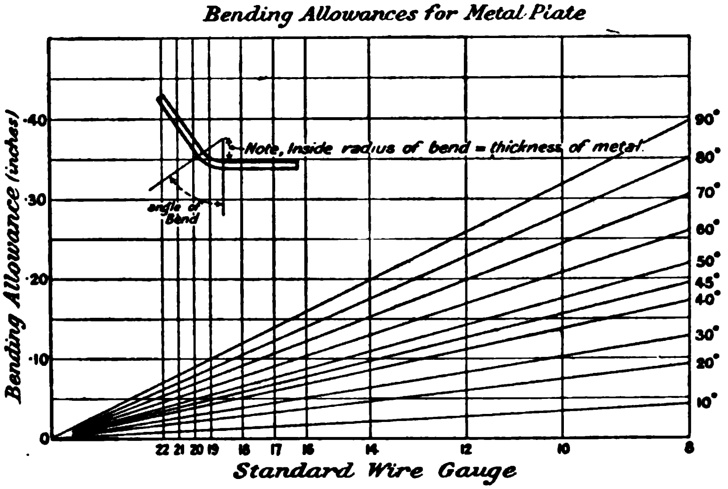

FIG. 228.

199. Countersunk Wood Screws.—Countersunk wood screws often play an important part in the attachment of fittings to spars and longerons. They are made in various sizes, each size distinguished by a number. The dimensions A, B, C, D (Fig. 229) are fixed for every standard size, but a variety of overall length may be obtained in any standard. In specifying wood screws, it is only necessary to state the size number followed by the overall length, *e.g.* No. 8, 4½″ C.S.K. wood screw. The table gives the principal standard sizes and dimensions.

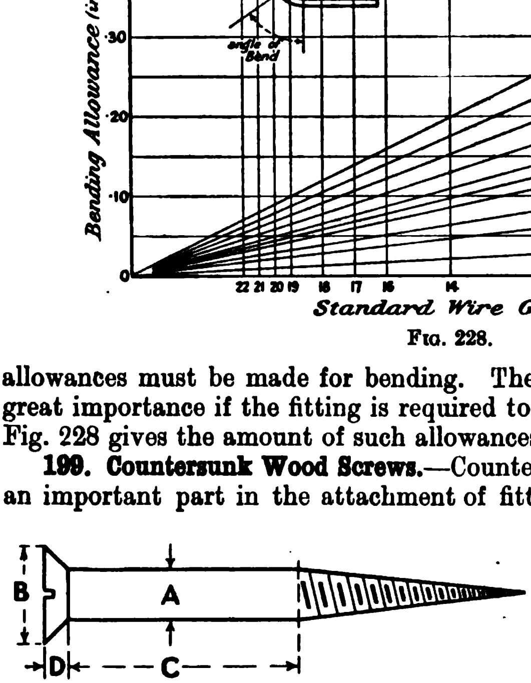

FIG. 229.

Size number.	A	B	C	D	Size number.	A	B	C	D
1	·063	·121	·16	·03	18	·32	·60	·50	·155
2	·08	·14	·16	·04	20	·34	·64	·56	·16
4	·11	·21	·16	·042	22	·36	·72	·66	·18
6	·13	·27	·16	·06	24	·38	·73	·85	·20
8	·18	·32	·26	·065	26	·425	·74	·85	·21
10	·20	·37	·28	·08	28	·44	·85	1·75	·22
12	·225	·43	·30	·10	32	·5	1·0	2·4	·26
14	·245	·48	·50	·12	40	·6	1·35	2·5	·4
16	·27	·56	·50	·15					

Few tests are available on the strength of wood screws, but the following table was obtained by Lieut. Halse, R.N.V.R., for the Air Department of the Admiralty, and gives some ideas as to the shear strength obtainable.

Size of screw.	Number of screws.	Breaking pull.	Wood.	Load per screw.
				lbs.
3/8″ No. 2	5	615	Mahogany	123
”	5	620	”	124
3/8″ No. 1	6	595	”	99
”	6	630	Walnut	105
3/8″ No. 0	6	605	Mahogany	101
”	6	545	Walnut	91
1/4″ No. 2	6	535	Mahogany	89
”	6	495	Walnut	82
1/4″ No. 00 . . .	8	565	Mahogany	71
”	8	545	Walnut	68

Tension tests carried out by Mr. Harris Booth are quoted in the following table.

Size	Length.	Safe load in lbs.
No. 4	5/8″	120
No. 4	3/4″	142
No. 6	3/4″	160
No. 6	1″	200

200. Friction Clip.—Occasionally a fitting is attached to a wooden member simply by the tightness with which it is fastened on. The hold is then only a friction grip. This is generally found in a fuselage clip, but it is extremely bad practice, as the wooden member may shrink and the fitting become loose, or it may even work loose in ordinary service. It should not, therefore, be used.

201. Terminal Connections of Wires, Cables, etc.—Upon the form which the connection between the tension member and the fitting takes, rests the efficiency of the tension unit. When proper care is taken over this important item, there is no reason why the full strength of the wire should not be developed, but if the connection is badly made the efficiency may drop to a very low figure.

A method commonly adopted in connecting a streamline wire or swaged rod to its fitting is by the use of a machined universal end, as shown in Fig. 230. The forked part of this fits over the lug to which it is pinned, and the screwed end of the streamline wire is taken by the screwed trunnion. A specially designed lock-nut completes the fitting.

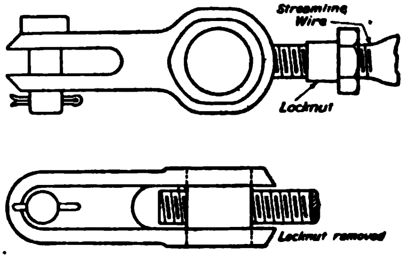

Fig 230.

This fitting allows the wire to be trued up, and the fact that it is a universal joint permits vibration of the wire without undue danger. The screwed end of the wire is stronger than the main portion, and so, by a proper design of universal joint, the full strength of the wire may be developed. Cables are connected to the lug direct at one end and through a turnbuckle for adjustment purposes at the other. When attached to a plate, a thimble or bush is needed in the hole in the lug to give a reasonable radius round which the cable is bent. Whether attached to a lug or a turnbuckle the cable is threaded through and then spliced, forming an eye. The splice, in order to make a good job, should be wrapped or served with thin steel wire, and the whole then carefully soldered up solid.

Solid wires may be fastened in a variety of ways, as shown in Fig. 231.

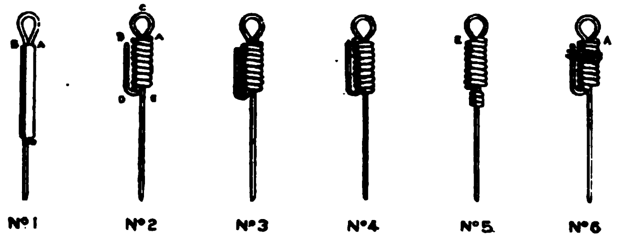

FIG. 231.

An exhaustive series of tests on these connections was carried out by Messrs. Roebling Sons Co., and the results reported to the National Advisory Committee for Aeronautics of the U.S.A. The following table is compiled from the Report.

Type.	Description.	Average efficiency.	Range of efficiency.	Points of fracture.	Remarks.
1	Plain ferrule of flat strip. Free end bent back. Whole terminal dipped in solder.	80 %	60 %—90 %	A or B	Very uncertain fastening
2	Oval spring wire ferrule of nine convolutions, A, B, and C, same radius. No solder.	65 %	60 %—75 %	A or Slipped	
3	Radius of A and B different from C.	62 %	60 %—65 %	Slipped	
4	Same as 2 but with right-hand ferrules.	60 %	59 %—61 %	Slipped	
5	Free end wrapped round main stay wire.	72 %	65 %—75 %	B	
6	Loose end tied down on ferrule by fine annealed wire.	70 %	68 %—78 %	A	

The tests showed that the highest efficiency was obtained from type No. 1, which was soldered, and another point brought out was that failure occurred in the other types at A and B through insufficient friction between the ferrule and the wire. This defect is overcome, in British practice, by soldering the whole terminal, and an efficiency of 100 per cent. is then readily obtainable.

CHAPTER XX

TESTING

202. Classification of Tests.—Tests may be conveniently divided into two distinct classes. First, those made to determine the suitability of a material for any particular purpose; and secondly, those made to determine the strength of a finished article or unit in a structure. In the first class would be included the standard tension tests on strips of metal cut from a batch, made to enable a judgment to be formed as to the reliability of the whole batch. The second group would include such tests as those on a lug carrying a bracing wire, a rib for the main planes and a destruction test on the completed aeroplane.

203. Tests on Materials.—Material testing is of very great importance in all engineering work, and especially so in aeroplane construction, where the minimum possible sizes must be worked to in design, and where a faulty piece of material may cause a serious and often a fatal accident. The materials used at the present time in aircraft construction are principally wood, steel and aluminium alloys. The main structure is usually built of wood, and, although this material will gradually be displaced by the metals referred to above, it is as yet by far the most important material with which the designer of aeroplanes has to deal. In this chapter, then, somewhat special attention will be paid to the tests which are necessary to ensure that timber is sufficiently sound for aeroplane building, especially as this is usually not dealt with in standard works.

It has appeared undesirable to the authors to give an account of the routine tests, such as tension and compression tests on metals, since these are dealt with so fully in all works on materials and structures. For information on these tests, therefore, the reader is referred to any of the standard books.

204. Timber Tests.—In order to determine whether a piece of timber is suitable for aeroplane structural members it is necessary to carry out tests as follows :—

(1) A test to determine the moisture content.
(2) A density test.
(3) A test for straightness of grain.
(4) A test to determine the modulus of elasticity.
(5) A test for ultimate compressive strength.
(6) A brittleness test.

205. Moisture Content of Timber.—Before timber can be used in any constructional members it is essential that it shall be dried. This is done either under natural conditions or by kiln drying. If the latter is necessary, owing to pressure of time, it must be done under very stringent conditions, as otherwise the timber may be ruined for the purpose required. As timber dries its strength increases, so that it is necessary to specify what degree of moisture is contained when considering the results of strength tests. As wet timber is weaker than dry, it is also essential that no material which contains more than a certain percentage of moisture should be allowed in construction. The importance of the test for moisture content thus becomes evident. The method of conducting this test is as follows: A piece of material should be selected from the batch of timber to be tested and a sample cut from it. This sample should be taken at least 2 inches from the end of the plank, and at least $\frac{1}{4}$ inch from the surface or edge of the plank. The sample should be then cut into small strips about the size of a matchstick, and these strips then weighed. They are then placed in an oven at a constant temperature of 100° C. and dried until their weight remains constant.

If the original weight of the sample $= W_W$
and the final weight $= W_D$

Then the moisture percentage is the ratio

$$\frac{W_W - W_D}{W_D} \times 100$$

A reference to the specifications for various woods in Appendix II. at the end of the book will show the limits laid down for the value of the moisture content under different conditions.

206. Test for Density.—This test may be carried out by any of the standard laboratory methods for determining specific gravity or density, and since these methods are fully described in all text-books on physics, it is unnecessary to quote them here. It is essential, however, that the results of tests should be expressed in relation to the moisture content of the sample, since this has considerable bearing on the density. If the results are needed for comparison work on timbers, or for other exact scientific purposes, it is necessary to give the density of the "oven-dried" timber, but for practical purposes it is better to know this value for the timber in the condition in which it will be used.

It will be seen on reference to the specification in Appendix II. at the end of the book, that the densities of ash and spruce are specified for a moisture content of 15 per cent.

207. Straightness of Grain.—Timber may be faulty either in the plank or in the tree, due to the grain not being straight. A tree may grow in such a way as to give a wavy or spiral grain, or a plank may be cut from a straight-grown tree in such a way that its length is not parallel to the grain. Both these defects give much the same result in a test. The elastic modulus, elastic limit, and the ultimate strength of a beam cut from such material are considerably lowered, and the specimen may break by a shear failure on the tension side at a very much lower figure than would be given by a straight-grained piece of timber. It is often very difficult to detect the presence of crooked grain, or to see that a plank has not been cut parallel to the grain, and, in consequence, some of this timber gets passed as satisfactory. It is safe to say that a majority of the accidents which have occurred in aeroplanes in the past due to faulty material have been caused by one of these defects. It is essential, therefore, to be able to detect such material and, fortunately, there is a very simple test which should be applied to as many samples of timber as possible. This is the splitting test, and it consists of cutting off a short length of the timber from one end of the plank and splitting it in two directions at right angles, by a blunt chisel and mallet. The splits reveal the true direction of the grain, and the deviation from parallelism with the length of plank must not exceed 1 in 20 for spruce or 1 in 10 for ash. The simplicity of this test, and the rapidity with which it can be carried out, renders it advisable to insist that it is done on all main structural members of an aeroplane. For this purpose it is only necessary that such members should be cut about 2 inches over length in the first place, this extra piece being cut off when the shaping has been roughly done and tested for straightness of grain. If this practice were followed as a matter of course, it is certain that accidents due to faulty wood would be very considerably reduced.

208. Modulus of Elasticity.—The factor governing the strength of a long strut is the modulus of elasticity of the material of which it is made, and since, in an aeroplane, many such members are employed, it is essential to determine the value of this elastic modulus accurately. This is best done by a bending test, but in the conduct of the test it is important to note the following points. In the first place, if a load be applied to the centre of a beam or evenly distributed along it, a portion of the deflection at any point will be due to shear. It is important, therefore, to eliminate this part of the deflection, since the simple formula used for bending deflections takes no account of it. Unless corrections are applied, a serious error in the estimation of the value of the modulus of elasticity may result. The test should be conducted, therefore, by applying the load at two points, *i.e.* the beam is supported at two points and two equal loads symmetrically placed with regard to the centre line of the beam are applied (Fig. 232). This eliminates all shearing force between the two applied loads.

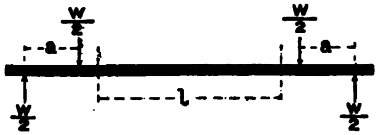

FIG. 232.

The deflection of a length between the loaded points is measured, and this deflection is then independent of shear. The loads should be applied through saddles to prevent local failure of the beam by crushing, and should be in the neutral plane of the beam.

In order to avoid end ways stresses the points of application of the load should be of the nature of ball or roller bearings. A suitable apparatus for carrying out this test is shown in Fig. 233. The load is applied in the usual manner in a standard testing machine.

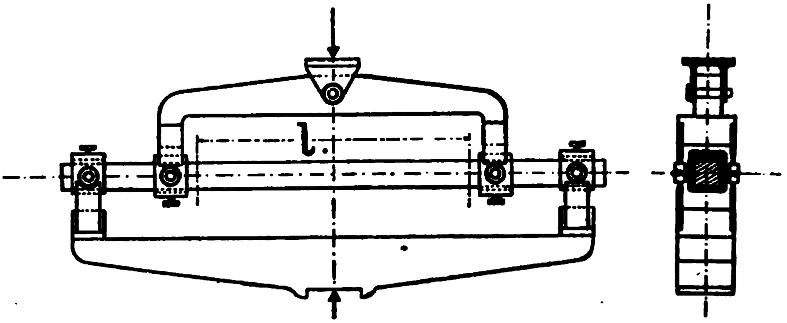

Fig. 233.

The deflection is measured by a deflection meter over any length l of the beam included between the points of application of the load. A suitable form of deflection meter for this purpose is shown in Fig. 234.

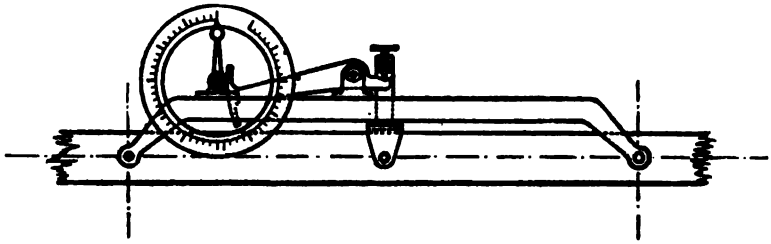

Fig. 234.

A series of deflections for varying loads should be measured and plotted. This deflection-load curve should be appreciably straight. Then, taking any point on this curve,

let W = the applied load.
a = the distance from the support to the point of application of half the total load (Fig. 232).
l = the distance on which the deflection is measured.
d = the deflection.
I = the moment of inertia of the beam.
$= \frac{bd^3}{12}$ for a rectangular section where b = breadth and d = depth of section.

The value of E is given by the formula

$$E = \frac{Wal^2}{16Id}$$

In order to judge the quality of a batch of timber it is necessary to make several tests of this sort on samples taken haphazard from the consignment. The average of such results may be considered a fair value for the batch.

The standard size of the test piece, as laid down by the Air Board specification for silver spruce, is 40" × 1" × 2", and the diagrams of apparatus, etc., given above are taken from the same source.

It is, of course, not essential to use the exact form of apparatus shown in the figures. If the essential points are grasped and carried out it is immaterial what are the exact types of instruments used.

209. Compression Test.—Timber being much weaker in compression than in tension, it practically always fails first in bending on the compressive fibres. Some cases arise where, due to a combined bending and tensile load, the tension fibres may fail, but such cases are rare. The compression test is, for this reason, of primary importance, and is one of the principal methods of determining the quality of a piece of timber. The standard compression test piece laid down by the Air Board specification for silver spruce is shown in Fig. 235.

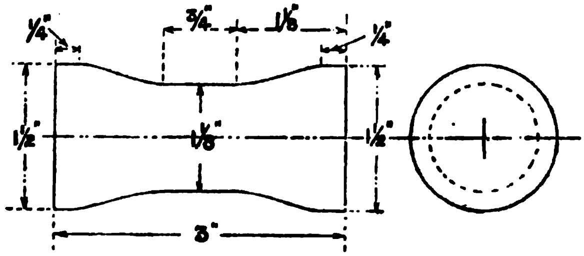

FIG. 235.

In making these test pieces it is essential that the ends shall be quite flat, and if the faces are parallel the test can be carried out between the plates of an ordinary testing machine. If there is any error in the parallelism of the faces, however, or if the testing machine does not apply an even load, an eccentric load is obtained resulting in too low a figure for the crushing stress. To eliminate errors from such causes the specimen may be fitted with end collars and ball bearings, as shown in Fig. 236. This will ensure a truly axial load. The rate of loading has a very considerable effect upon the results obtained. If the loading be too rapidly done the results are too high; and conversely too slow an application of the load gives too low results. The limits laid down by the Air Board specification are that the loading shall take place at a speed between 3000 and 6000 lbs. per minute. In these, as in all compressive tests, it is important to watch the beam of the machine as the failing load is reached, as otherwise too heavy a load may be applied, with a risk of a false estimate of the quality of the material being made. Reliance should not be placed upon the result of a

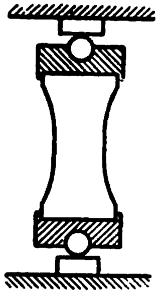

FIG. 236.

single test, but several specimens should be prepared from the sample, and an average of the results taken.

In making compression tests on ash, the specimens are specified to be 1″ × 1″ × 2″, and this shape test piece may also be used for spruce, but the results given in the latter case are always lower than those obtained on the turned specimen. Failure always occurs in the rectangular blocks by crushing at the ends.

210. Test for Brittleness.—Brittleness may be the cause of serious danger in aeroplane structures, and the elimination of any timber which may be in the brittle state is therefore important. An examination of a piece of good timber which has been fractured by bending will show a compression failure on one side and a long, fibrous tension break on the other. In a sample of brittle material this long fracture will be replaced by a very short break, and the compression failure also will not be so well marked as in the good material.

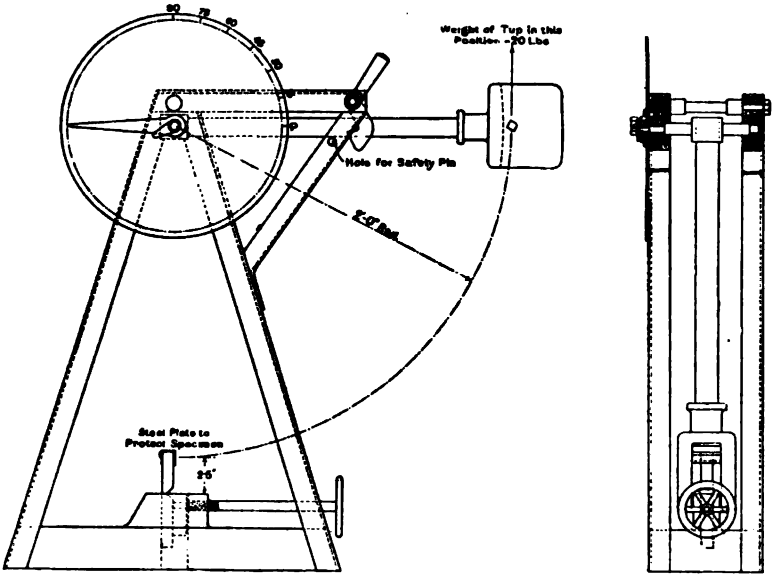
90
75
60
45
30
Weight of Tup in this Position -20 Lbs
Hole for Safety Pin
2′-0″ Rad.
Steel Plate to Protect Specimen
2·5″

FIG. 237.

The test which determines the quality of the timber in this respect is known as the impact or the notched bar test. A piece of the material of specified size has a V-shaped or other form of notch cut in it, and the specimen is broken by an impact blow. The blow is delivered by a weight moving through a known distance, and the energy required to fracture the specimen is thus determined. The amount of this energy is used as a criterion of the soundness of the timber as regards brittleness.

A form of testing machine for this purpose is shown in Fig. 237, in which the blow is delivered by means of a weight swinging on an arm from the horizontal position. On striking the specimen a certain portion of the energy stored in the weight is absorbed, and the amount of this absorbed energy is determined by the distance the weight moves through after impact.

In the type of machine shown in Fig. 237,

Let W = the weight of the tup in lbs.
Let R = the radius of the arm in feet.

Then if the weight starts from the horizontal position, the kinetic energy stored in it immediately before impact is WR foot-lbs.

Suppose after impact the weight swings forward through θ degrees.

The vertical height travelled by the weight = $R(1 - \cos\theta)$.

The energy remaining after impact is

$$WR(1 - \cos\theta)$$

Therefore the energy absorbed by impact = $WR\cos\theta$.

The scale on the machine shown is graduated to read the angle a which the arm makes with the horizontal when it comes to rest, and not θ, the angle to the vertical.

The energy absorbed by impact is then $WR\sin a$.

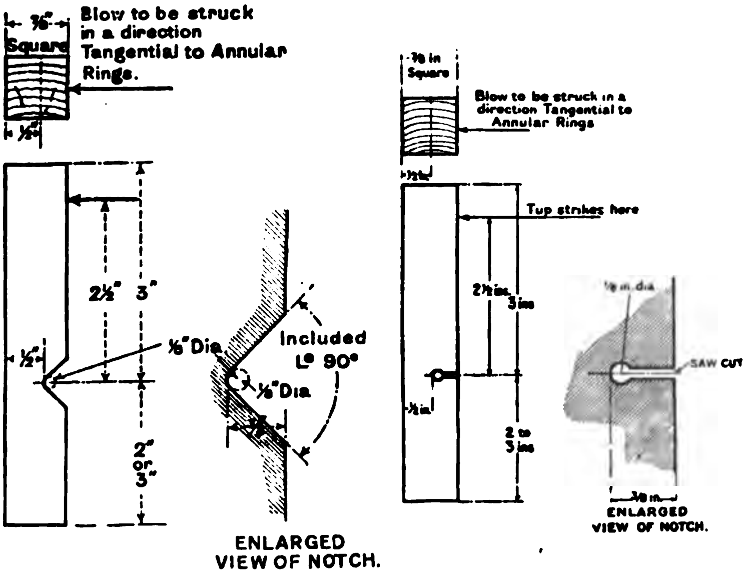

Fig. 238.

Forms of test specimens for either spruce or ash are shown in Fig. 238. These specimens must be cut so that their sides are radial and tangential to the annular rings, and the blow must be struck in a tangential direction, as shown in the figure. For ash it is specified that a specimen as shown above shall absorb not less than 10 foot-lbs., and spruce not less than 8 foot-lbs. Although a test as described is necessary to obtain quantitative results, a good idea as to the quality of a specimen may be obtained by the examination of a fracture. A test piece as shown in Fig. 238 should be broken with a smart blow from a hammer, and if the material is good a fibrous fracture will result. If it is brittle the break will be short.

211. Tests on Three-Ply.—The importance of this material in aeroplane construction and the great differences in quality which can be obtained under varying conditions of manufacture, render systematic testing necessary. The following tests are those prescribed by the Air Board specification for three-ply.

(*a*) *Moisture Content.*—A small specimen of the three-ply, about two inches square, is heated in an oven at 105° C. for 10 hours, and the loss of weight in this period must not exceed 15 per cent. of the original weight. This test ensures that the clause requiring a moisture content of not more than 15 per cent. of the total weight is complied with.

(*b*) *Shearing Test.*—The cement joints between the veneers of the plywood must be stronger than the wood itself. The function of the cement is to transmit shear from one veneer to the next, and this test is designed to ensure that the quality of the material used and the workmanship during cementing is the best possible.

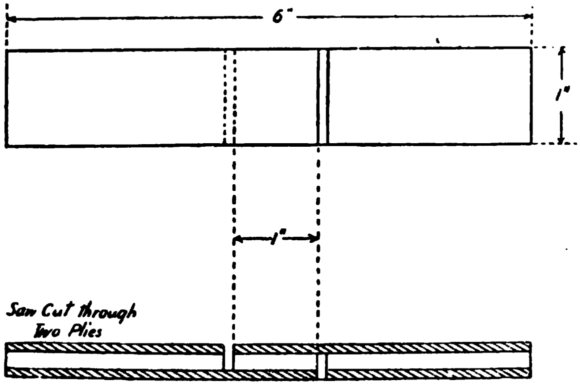

FIG. 239.

At least three test pieces of the dimensions shown in Fig. 239 should be cut from every batch of material under examination. Saw cuts through two plys are made in two places, $\frac{1}{2}''$ each side of the centre of the specimen. A tension load applied to the test piece can then only be transmitted by shear across the cement joints.

The load must be applied evenly and without jerks, and the failing stress of two of the three specimens must be not less than 150 lbs. per square inch.

(*c*) *Absorption Test.*—This is a test designed to ensure that the material will not absorb an undue amount of moisture, and by so doing cause an opening up of the veneers.

A test piece 6" × 6" should be immersed in water at 45° C. For veneers up to $\frac{1}{30}''$ thick the time of immersion should be three hours, and for veneers above this thickness, six hours. At the end of the period there must be no visible signs of any opening at the edges of the specimen.

(*d*) *Bending Test.*—For plywoods of ⅛″ thick and under, the following test should be carried out. A specimen 12″ × 2″ is cut so that the grain of the outer plies is running across the length of the strip. This specimen is bent on the rim of a former 18″ in diameter, first one way and then the other. There must be no sign of a split, crack or fracture of any kind, and the plies must not part under the test.

212. Time Effect in Timber Testing.—The effect which the time taken in carrying out a test on timber has upon the results obtained has been pointed out by Messrs. A. A. Griffiths and C. Wigley in a paper presented to the Advisory Committee for Aeronautics.[1] This paper gives the results of experiments made to determine the accuracy of certain assumptions made as to the behaviour of wood under test, and the discovery of the importance of the time factor was made as the result of the experiments.

In commencing bending tests to determine the value of the modulus of elasticity (E), the plan adopted was to apply a central load to the beam. A large time effect was observed on the deflection, however, *i.e.* the deflection continued to increase with no addition of load. This was put down as due to the shear stress, and the method was abandoned. Two point application of the load was next tried, so that between the loaded points the shear stress was eliminated. This was found to reduce the time effect, but there was still an increase in the deflection after a period during which the load had been kept on the specimen. This was due to the shear stress in the portions of the beams between the applied loads and the reactions. The curvature of the beam on the portion between the loaded points, *i.e.* the region of constant bending moment, was constant for loads within the elastic limit. This seemed to show conclusively that timber in shear is very much influenced by the time factor, and this was proved in further experiments on the torsional strength of wood. In the torsion experiments a time effect was noticed for the smallest loads, and comparisons were made to determine the error introduced by this effect. In order to do this a vibration method was used which enabled the "instantaneous" value of the modulus of torsional rigidity (N) to be obtained. On comparing this with the values of N found by the static method, it was discovered that the time effect was such that it reduced the apparent value of N by 20 per cent. in most cases, and in some by 30 or 40 per cent.

213. Temperature Effect in Testing.—It was thought by the experimenters referred to in the last paragraph, that some of the irregularities they had discovered in the torsion—time curves might be due to temperature changes, and to verify this, specimens were tested surrounded by a water jacket which could be heated. The results obtained showed a great increase in the rate of time effect, and the apparent value of N in one specimen at 90° C. was only about one-eighth of the value at normal temperature. On the reduction of the temperature to normal again, the rate of increase of torsion with time was very slow. From these results the experimenters formed the general opinion that there is a definite limit to the amount of torsion of a bar under a given torque, but that the temperature governs the time it will take in reaching this limit. In the case of normal temperatures it takes a long time, whereas at high temperatures it reaches it comparatively quickly.

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[1] "A Preliminary Investigation of Certain Elastic Properties of Wood," A.C.A. Report, No. T. 1077, by A. A. Griffiths and C. Wigley.

214. Tests on Parts of Aeroplanes.—Passing now to the second class of tests mentioned in para. 202, it is necessary to consider some of the methods used in dealing with complete parts of an aeroplane. It is obvious in a case of this sort, however, that many ways are possible of conducting such tests, and no definite rules can be laid down as to the best method. Any methods given in this chapter must therefore be considered only as indication of a suitable means. Special cases will always arise, and the designer must elaborate his own tests to meet these. It should be remembered, however, that in all cases it should be the aim to reproduce as accurately as possible in a test the condition under which the member is loaded when in use, and the test should be as simple as possible.

215. Tests on Spars.—Tests on main plane spars may be subdivided into tests on the complete spar, and tests on a specimen only taken from the spar. In both cases, however, it is necessary to get a combined lateral load and a tension or compression on the spar. As a general rule, it is the compression which is a source of anxiety, and not the tension, so the latter case is not worth considering. In the first place, take the case of a specimen of the spar only, *i.e.* a section cut from the whole member.

From the bending moment diagram of the aeroplane for which the spar is designed, the distance between the points of inflection and the maximum bending moment are obtained, and also the loading per foot run and the end compression.

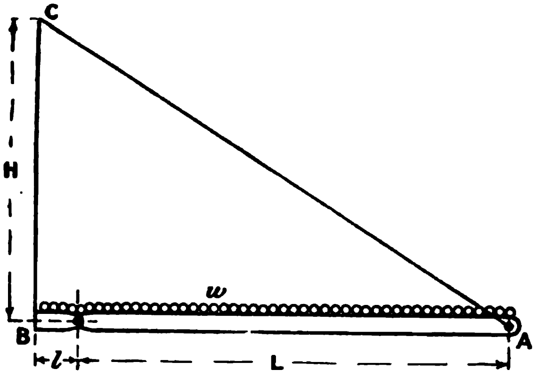

Fig. 240.

The specimen to be tested should be at least equal in length to the distance between the points of inflection.

Two methods may be used in testing such a specimen.

First Method.—The spar should be cut and fitted with end attachments which give as nearly as possible pin-joint conditions. The distance L between the pin centres should be equal to the distance between the points of inflection in the spar.

One end is pinned to a wall bracket B, rigidly attached to a vertical wall BC (Fig. 240).

The other end is supported by a cable AC, C being the point of attachment of the cable to the wall.

Let the distance from the pin to the wall $= l$ feet.
Let the height of C above B $= H$ feet.
Let the distributed load on the spar under normal flying conditions $= w$ lbs. per ft.
Let the end compression in the spar under normal flying conditions $= P$ lbs.

If a distributed load w lbs. per ft. be placed along the beam AB the maximum bending moment due to it is $\frac{wL^2}{8}$, and this is the maximum bending moment due to the distributed load in flight. In addition, the end thrust has to be P at the same time, and this can be arranged by a correct proportion of the height H to the length $(L + l)$.

The reaction at A under the loading w is $\frac{wL}{2}$

This is taken by the wire AC.

The horizontal component of the wire pull

$$= \left(\frac{L + l}{H}\right) \times \frac{wL}{2}$$

This must be equal to P. So that

$$\frac{wL(L + l)}{2H} = P$$

or

$$H = \frac{wL(L + l)}{2P}$$

from which H can be calculated.

Suppose the spar stands up to a distributed load of W lbs. per foot.

Then the maximum bending moment due to this lateral load

$$= \frac{W}{w} \times \text{normal bending moment.}$$

and the end thrust $= \frac{W}{w} \times$ normal end thrust.

Therefore $\frac{W}{w}$ is the load factor of the spar. It should be noted in connection with this test that the end A requires some stabilizing against lateral deflection. This may be done by two blocks of wood shaped to a knife edge placed one on each side of the spar at the end A. If the length AB is such that in the actual aeroplane it is supported sideways by a drag strut and bracing, other stabilizing blocks such as just described should be placed at suitable positions, as otherwise a lateral failure may occur. This test is easily fitted up and does not require the use of a testing machine. It is therefore convenient in works where such apparatus is not available.

Second Method.—This method is due to Professor Lea, of Birmingham University, who devised it to carry out tests on sample steel spars. AB in Fig. 241 is the specimen of a length equal to the distance between the points of inflection. Links EA, EB are pin-jointed to the specimen and to each other, and the end E is gripped in one jaw of the testing machine.

To the centre of the specimen another link CD is pin-jointed, and the end D is taken in the other jaw of the machine. On the application of a load, a bending moment is induced in the specimen by the pull of the link CD, and the horizontal components of the loads in EA and EB are taken as a compression in AB. By a proper regard to the angles of the inclined links the proportion of the end thrust to bending moment may be adjusted. Instead of having a single link CD, two may be used evenly spaced about the centre line of the specimen and connected by a bridge piece attached to another link which is held in the testing machine. By this means a constant bending moment is obtained in the specimen at the centre portion. The arrangement is shown in Fig. 242.

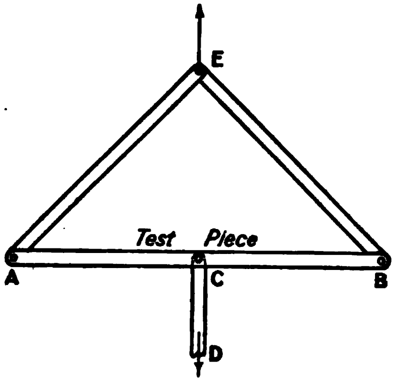

FIG. 241.

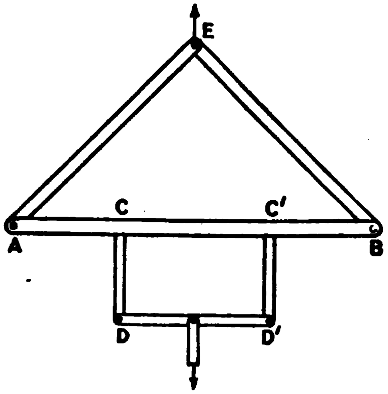

FIG. 242.

216. Tests on Complete Spars.—Sometimes it is desired to test a complete spar and a modification of the first method of testing parts of such members may then be used. At the same time it is far preferable to test a complete member in relation to the whole structure, as will be described when dealing with destruction tests on the complete aeroplane. If it is necessary, however, to test one spar only the following will give an approximately correct idea of the strength. In Fig. 243, ABCD is the spar, B and C being strut positions, and D the point of attachment to the centre section.

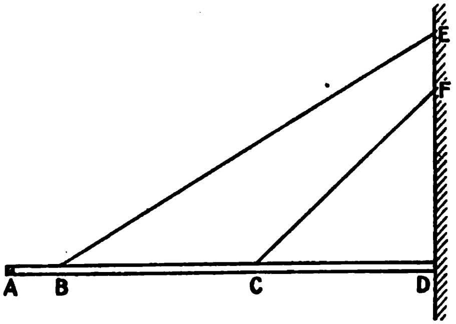

FIG. 243.

D should be attached to a wall by the same fittings as would be used in the actual aeroplane. B and C are cable attachments, and the cables

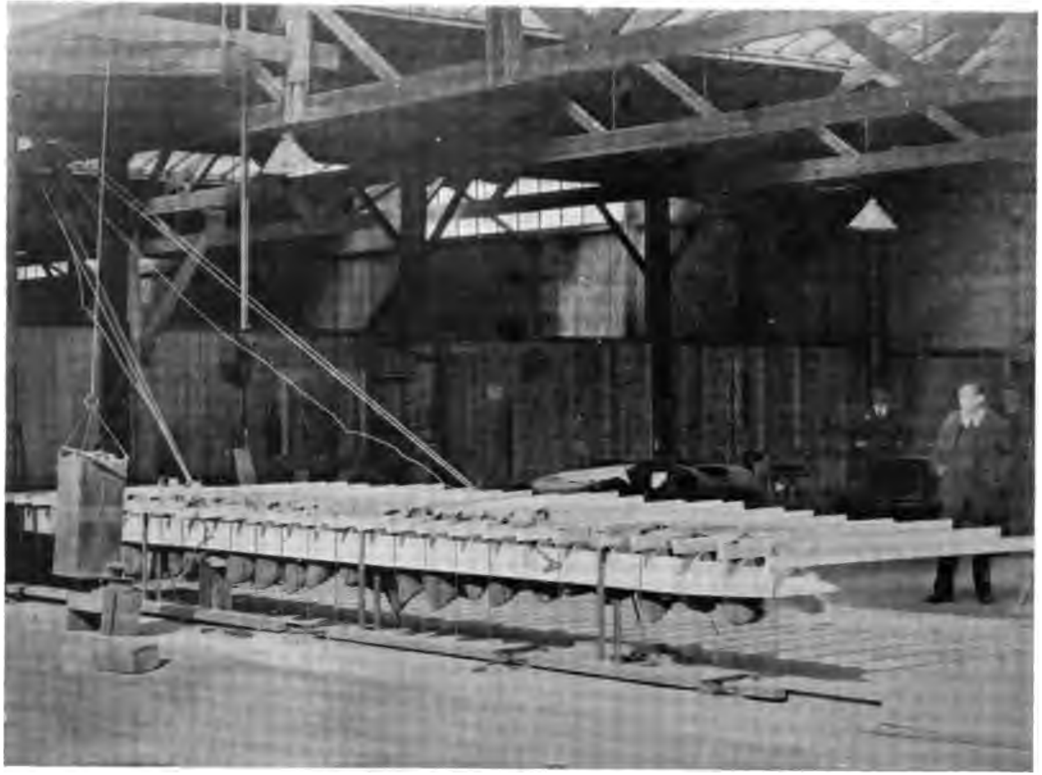

PLATE XX.—Fairey Spar Test.

BE and CF are arranged at such angles as will give the correct end compressions in BC and CD when a loading corresponding to that in the actual aeroplane is applied along the spar. The angles of these cables are calculated in the same way as previously described for a small specimen. The loading along the spar from A inwards should be graded according to the wing tip loading curves shown in Chapter XI. At interplane strut positions B and C, and any other points where drag struts occur, the spar should be supported sideways by knife-edged or rounded blocks of wood fastened rigidly to the floor or platform on which the test is being carried out.

217. Fairey Method of Spar Testing.—By the courtesy of Mr. C. R. Fairey, of the Fairey Aviation Co., a description of a method of spar testing originated by him is included in this chapter. A complete extension plane spar of a large boat seaplane was built and the correct fittings and bracing wires attached. This was inverted and erected on the upright of a large compression testing machine, as shown in Fig. 244 (A). The spar was braced to the top pin of the testing machine, and counterbracing wires were taken back to the ground on the opposite side. The upright VC could be adjusted to the vertical by means of strainers in the counterbracing wires.

The loads were applied at points corresponding to rib positions in the actual aeroplane, levers being mounted at each point as shown at B in Fig. 244. To a girder M, half embedded in concrete, rods MN were attached to each lever NP, the end N being slotted for the rods. Above the slot was a washer and knife edge, as shown at C in Fig. 244. The load consisted of bags of sand Q, hung on steel rods bridged between each pair of levers, wooden distance pieces also being used to space the levers. By moving these bags along the levers different loads could be applied to the spar. Uprights on the spar enabled the deflections to be measured. A photograph showing the actual test is given in Plate XX.

The great objection to a test on a single spar is the fact that the deflections at the strut points do not correspond with those in the actual aeroplane, as the geometrical proportions of the frames are so widely different.

218. Testing of Main Plane Ribs.—Tests of main plane ribs have been carried out with a very high degree of accuracy by Mr. W. D. Douglas of the R.A.E. by methods developed by him. The method is outlined briefly, but for a full description the reader is referred to a report by Mr. Douglas.[1]

The factors to be considered in a complete test are—

(*a*) The distribution of air pressure over the upper and lower surfaces of the plane and the internal pressure.

(*b*) The initial stresses in the fabric.

(*c*) The method of attachment of the fabric to the ribs.

(*d*) The distribution of stress in the top and bottom fabric.

(*e*) The method of attaching the ribs to the spars.

With regard to (*a*), a pressure plotting for R.A.E. 15 aerofoil has been

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[1] "Strength Test of Main Plane Ribs," by Wm. D. Douglas, A.C.A. Report, T. 1138.

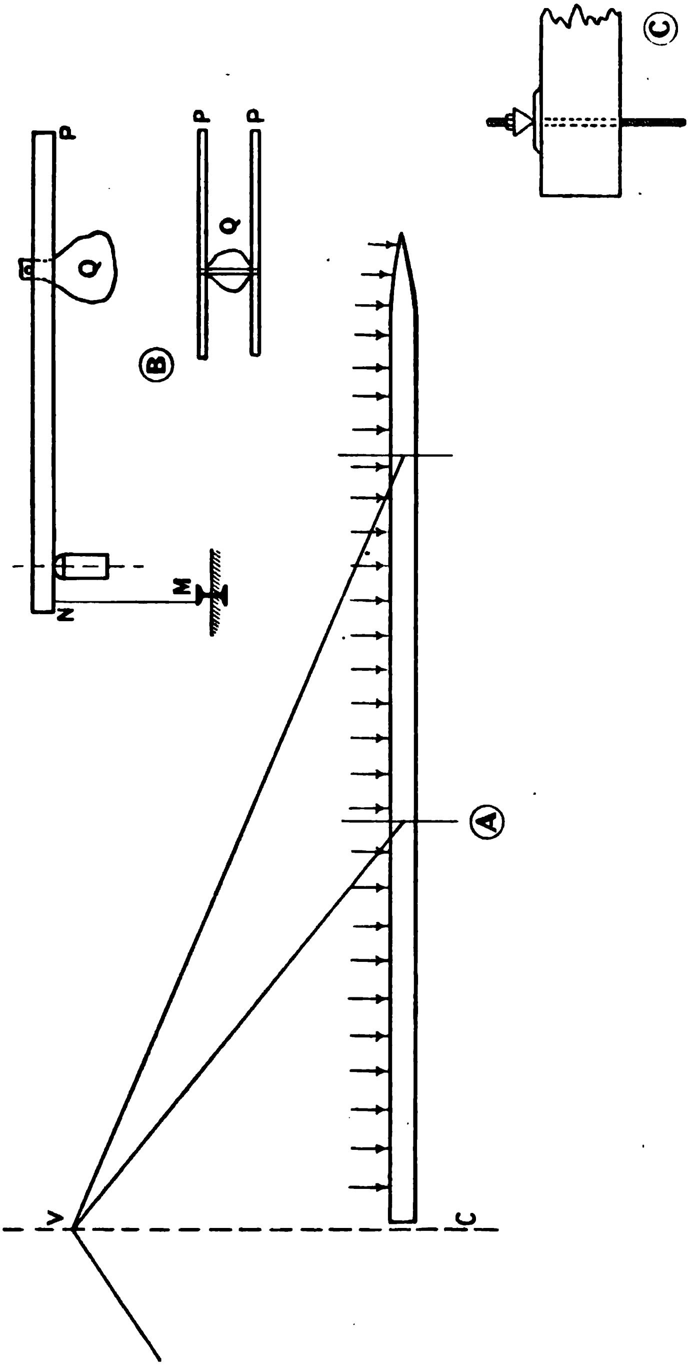


Fig. 244.

adopted as a standard by the R.A.E. This plotting was made at an angle of incidence of 16°, and is shown in Fig. 245. The initial stresses in the fabric are actually reproduced by covering the test section of the plane (which should be at least equal to the chord in length) with fabric.

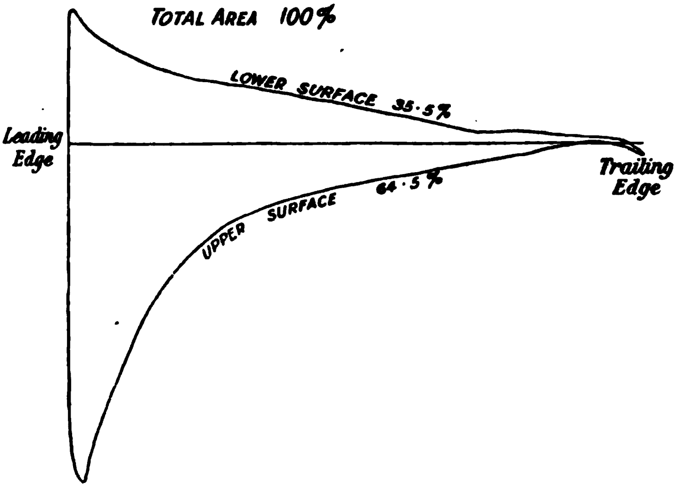


FIG. 245.

After an examination of the conditions of flight the conclusion is arrived at that the method of attachment of the ribs to the fabric does not affect the problem seriously. If b is the distance between two adjacent ribs it is assumed that the distribution of stress in the fabric may be obtained as follows (Fig. 246 (A)).

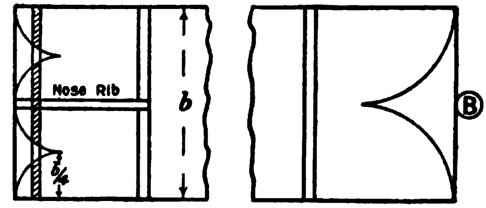


FIG. 246.

Arcs of radius $\frac{b}{2}$ are drawn as shown, cutting an elementary strip AB in D and C. Then the load on AB transferred to the ribs LT and ER is given by $\frac{AD+CB}{AB}$, the remainder being carried direct to the leading edge. The direct vertical forces and their distribution along the rib can now be calculated, and also the reactions at L, E, T and R due to the distributed load along LE and TR, so that the whole vertical loading on the rib is known. If intermediate nose ribs are provided, the arcs are drawn as in Fig. 246 (B) instead of as at (A), but the procedure is otherwise identical. There is also a

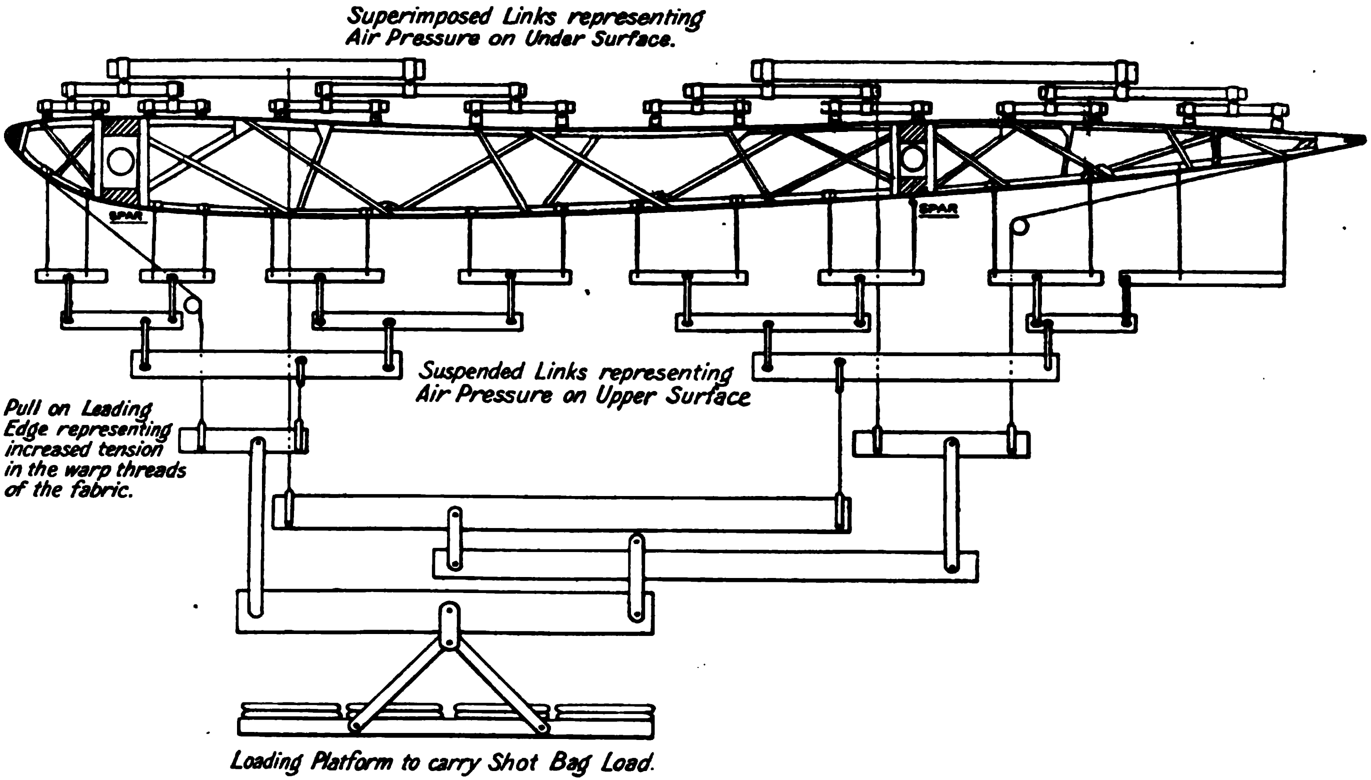

Superimposed Links representing Air Pressure on Under Surface.
SPAR
SPAR
Suspended Links representing Air Pressure on Upper Surface
Pull on Leading Edge representing increased tension in the warp threads of the fabric.
Loading Platform to carry Shot Bag Load.

Fig. 247.

difference in tension of the top and bottom fabric which throws a horizontal component of load on these members. This is approximately met by applying the loads (vertical and horizontal) to the leading and trailing edges through fabric tension members, which are tangential to the leading edge upper surface. These strips are doped to the bottom fabric. The distributed vertical loads are applied to both flanges of the ribs by means of a link mechanism. This consists of a series of super-imposed levers fitted with grooves and knife edges. These levers are built up over the inverted ribs, the distances between knife edges being so arranged that by the application of one or more concentrated loads a system of small forces is produced on the rib corresponding to the pressure distribution required. The links rest on small blocks of walnut spaced along the ribs as necessary. Two ribs are usually tested, these being in the centre of the length of plane referred to above. The plane is mounted on trestles, being supported under the front and rear spars. The test load applied gives at the same time the vertical loads and the inclined loads at the leading and trailing edges, as will be clear from the diagrammatic view of the arrangement in Fig. 247.

219. Tests on Fittings.—In testing a complete fitting the method of procedure must depend largely on the design of the fitting, and such a test often requires great ingenuity on the part of the designer in order to obtain satisfactory results. A simple tension lug is, of course, dealt with by attaching one end to a jaw of the testing machine, and applying the load through the appropriate cable properly attached as in the actual aeroplane. When, however, it is required to obtain the effect both of a wire pull and a strut compression, special apparatus must be rigged up. It is always advisable to reproduce as closely as possible the actual conditions obtaining in the aeroplane for which the fitting is designed, and for this purpose it is generally wise actually to fit up a portion containing the fitting, and so arrange the test loads that they are proportionate to the loads coming on during flight.

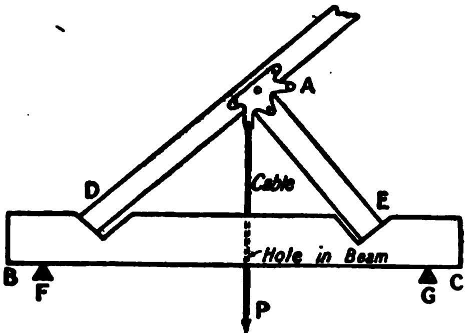


FIG. 248.

Fig. 248 shows one arrangement for testing a fuselage clip in an ordinary beam testing machine. The beam BC is notched at D and E, and supported in the testing machine at F and G. AE represents the vertical strut of the fuselage and DA the longeron, the fitting to be tested being represented by A. The correct size wire or cable is attached to the fitting and connected to the shackle of the testing machine. When a load is applied to this wire the strut and the longeron are thrown into compression as in the actual aeroplane, the ratio of these loads being governed by the angles of AE and AD to the vertical pull.

It is not possible within the limits of this chapter to give further examples, and, in any case, each fitting requires special treatment and the design of a special test, so that generalizations are of very little use.

220. Tests on Axles.—The specifications for axle tubes issued by the Air Ministry call for a test upon each axle. This test consists of a proof bending moment applied to one end of every axle and a test at both ends for one axle in every ten. Fig. 249 shows a diagrammatic view of a suitable machine for this test.

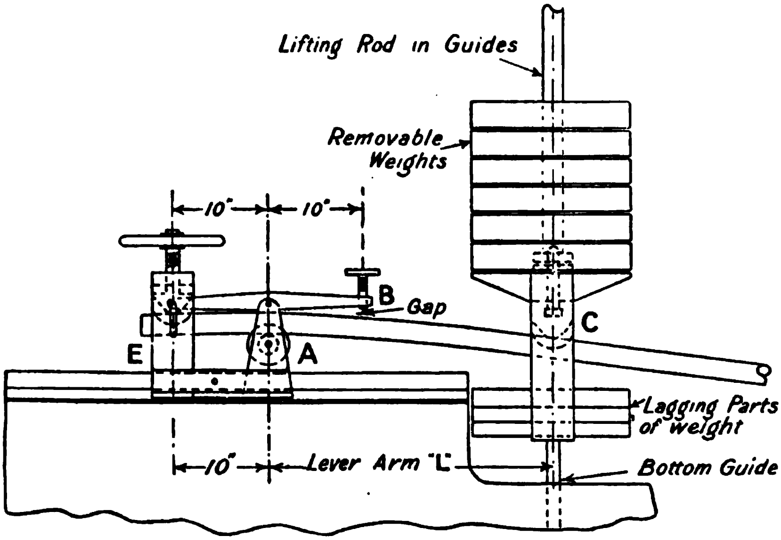


FIG. 249.

Contact between the tube and the support at A is specified not to extend more than $\frac{3}{4}$-inch along the tube. The load should be applied in two parts, the first part being from one-quarter to one-half the proof load, as convenient. The gauge at B should then be adjusted, and the remainder of the load put on the specimen. After an interval of at least five seconds it should be removed and the permanent set measured by the gauge at B. This set must not exceed $\frac{1}{16}$ inch. The schedules of axle tubes contained in Appendix II. at the end of the book give particulars as to the proof loads required for the different size tubes.

CHAPTER XXI

DESTRUCTION TESTS

221. Destruction and Proof Loading Tests.—An aeroplane, perhaps more than any other engineering structure, must be designed with the one object of obtaining the maximum strength for the minimum weight of material. Since there is usually no factor of safety on many parts of the aeroplane in the sense that this term is generally understood by the structural engineer, every endeavour must be made to conduct tests upon completed units and complete aeroplanes whenever possible. This chapter will be devoted to a brief description of the methods which may be used in conducting these tests.

The object of a full scale test may be either to determine the actual load factor which the aeroplane has, or to proof load the structure to a given factor to ensure that it will stand such a load without serious strain. In either case the procedure will be the same except that in the first class of test the loading will be continued until failure occurs, whereas in the second case the test will be stopped when the proof load has been reached.

222. Tests upon the Main Plane Structure.—It is usual when conducting tests upon the main plane structure to strip all fabric off the wings and test the bare skeleton. The object of this is twofold. In the first place it is impossible to observe what is happening to the structure when the fabric is in position, and in the second place the value of fabric is a variable factor. When it is new and well doped it undoubtedly exercises a considerable effect towards strengthening the whole structure, but after it has been in service for any period it is apt to become "soggy," *i.e.* the stiffening effect of the dope wears off with a consequent slackening of the fabric. If an aeroplane is tested, therefore, with the fabric on, a totally wrong result may be obtained, and so in all good testing the fabric is first stripped off the wings.

Suppose it is desired to test the strength of the aeroplane when it is flying at the coarsest angle of attack possible, *i.e.* when the centre of pressure is at the most forward position. First estimate the direction of the resultant load on the aerofoil by compounding the lift and drag for the angle being considered; *e.g.* let AB (Fig. 250) be the aerofoil inclined at the angle a for which the test is required.

Then at the centre of pressure C, corresponding to this angle, there are acting the vertical lift L and the horizontal drag D. The resultant of these makes an angle β to the vertical such that $\beta = \cot^{-1}\frac{L}{D}$. Since $\frac{L}{D}$ is known from the aerofoil characteristics β can be found.

The resultant force CE acts at angle $(\alpha - \beta)$ to the normal to the plane and has a component along it acting towards the leading edge. In estimating D, allowance should be made for the drag of the interplane struts and wires. These are the conditions which must be reproduced in a test, and so the wings are mounted to make an angle $(\alpha - \beta)$ to the horizontal, as shown in Fig. 251. The wings are inverted so that load can be applied to the under side of the aerofoil to represent the air loads in flight.

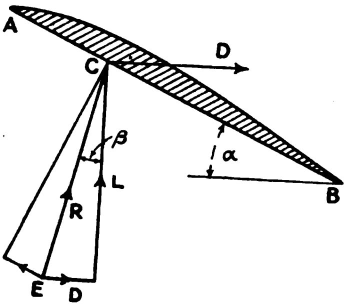


Fig. 250.

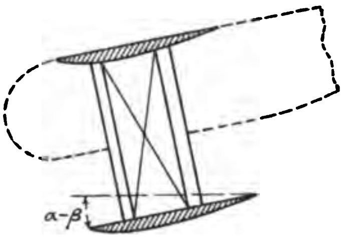


Fig. 251.

In mounting the wings it is best to retain the fuselage in position, as this gives the proper centre section fittings. The wings are supported at the centre section, which should be stiffened up to carry the reaction of the weight upon the wings. This is legitimate as the conditions of loading at the centre are not the same as in normal flight but very much heavier.

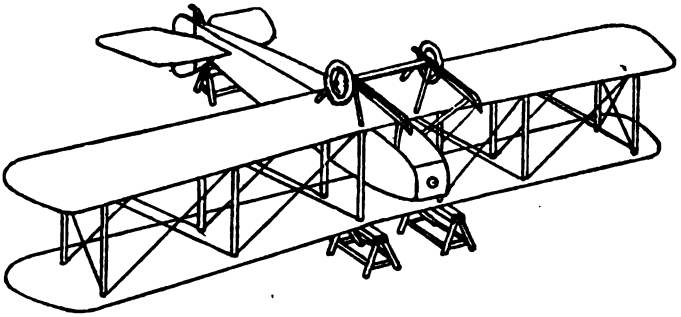


Fig. 252.

Fig. 252 shows an aeroplane erected for a test of this sort. It will be noticed that the supports are under the centre section and the tail. It is a wise precaution to arrange trestles under the wings at such a height that there is no danger of the wings touching them during the test, but as close as possible without any fear of this, in order to catch the structure when failure occurs and so prevent any more damage than necessary.

Blocks may be placed on the trestles, these being removed as the test proceeds. The aeroplane having been inverted and properly supported at the correct angle, it should be trued up. This should be fully done before inversion, and then it need only be checked when in position. The position of the loads should now be marked on the planes so that no delay is caused in carrying out the test. The load is usually applied by means of shot bags, and these should be laid along a line following the centre of pressure on the wings, the intensity of loading being graduated at the wing tips to agree with an experimental loading curve such as given in Chapter XI.

The loads for successive factors are determined as follows :—

Let $W =$ the total weight of the aeroplane.
$w =$ the total weight of the wing structure.

Then unit load, *i.e.* the load stressing the wings in normal horizontal flight, is $(W - w)$.

For a load factor N the load stressing the wings is

$$N(W - w)$$

When the aeroplane is inverted the wings are already being stressed by their own weight, so that for unit factor the extra load required to be added is

$$(W - w) - w = (W - 2w)$$

For a load factor 2 the total load on the wings in shot bags will be

$$2(W - w) - w$$

and for a load factor N the shot bags to be added must weigh

$$N(W - w) - w$$

These loads should be calculated and tabulated so that it is known exactly what load is required for any given factor.

Unit load should be placed on the wings, dividing it between the top and bottom wings in the ratio of the flight loads on them. If possible, this load should be left on the structure for an hour or two and then removed. After removal the aeroplane should be trued up again if necessary and the test recommenced. After each loading it is customary to observe the deflection at various points of the structure, and this is best done by means of a dumpy level. When the aeroplane is properly trued up the datum is taken for the various positions where deflection measurements are required and readings are taken at intervals as needed. The best method to adopt for this part of the experiment is to paste graduated slips of paper on to the various points of the aeroplane where observations are required and read these slips through the level. The most useful points for taking deflections on the main plane structure are the interplane strut positions and half-way along each bay on the spars. It should be one man's job to look after this part of the test, and he should not have any more to do than read the deflections and enter them on the test sheet. Loading is continued to destruction or until proof load is reached. If the former, careful notes and, if possible, photographs should be made of the point of fracture, and the aeroplane thoroughly inspected for any indication of failure at other points. The advantage of the trestles under the wings will now be apparent, since the structure can probably be repaired sufficiently for further tests if these are required, especially if premature failure has been caused by the collapse of a fitting or small member in the structure. Plate XXI. shows a photograph taken during the progress of a main plane test at the Royal Aircraft Establishment.

223. Test of Nose-Diving Conditions.—This test is an extremely difficult one to arrange, since it implies a down load on the front truss, an up load on the rear truss, a large drag on the wings and a down load on the tail plane simultaneously.

An ingenious method of arranging such a test was devised by Mr. W. D. Douglas, who is in charge of the testing of this sort at the Royal Aircraft Establishment, Farnborough. The aeroplane was suspended through its centre of gravity, a spring balance being arranged in the suspension which was always kept reading the weight of the aeroplane. Up loads on the rear truss were applied through a series of levers, and down loads on the front truss by shot bags in the usual way. The drag was represented by making the suspension non-vertical, so that there was a component along the chord. This was balanced by a cable through the centre of propeller thrust, and thus all conditions were met. The tail bore against a framing so that when the loads were applied and the weight of the aeroplane taken up on the spring balance mentioned above, the tail balance load was automatically given. The test conducted by Mr. Douglas by means of this apparatus was entirely successful, and although it is an expensive method there is, up to the present, no alternative for obtaining destruction test figures for this condition of flight.

224. Down Load on Front Truss.—This is a test which is of value as it gives the strength of the wings to resist the down loads which occur on the front truss of an aeroplane in high-speed flights. The load on the front truss in a nose dive should be calculated by the methods of para. 41, and these figures taken as a basis for the test loads. The aeroplane is erected in its normal flying attitude and the loads placed along the front truss, being of course distributed between the top and bottom planes in the usual ratio. This test is identical with the ordinary up load test except for the reversed attitude of the aeroplane and therefore requires no further description.

225. Test of Fuselage for Down Load.—The fuselage should be stripped of its fabric and mounted the correct way up. It should be supported strongly at the centre section by means of suitable timbering and anchored down at the front, preferably through the engine plates or mounting. The tail forms a convenient platform upon which the load can be placed, but if there is any doubt as to its strength for this purpose it should be removed and a solid platform made to replace it. Shot bags may now be loaded on to the platform until failure occurs, deflections being measured at the required intervals of loading. This is a very simple test, and may be

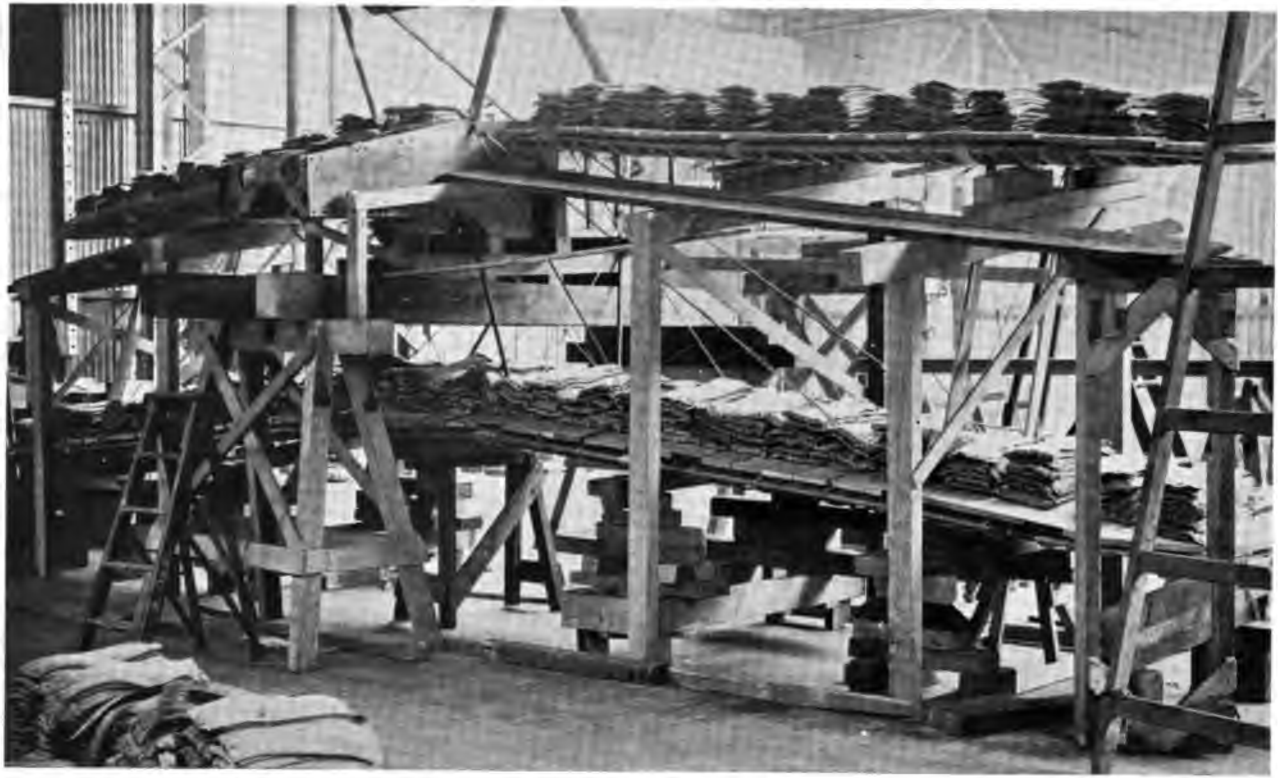


Plate XXI.—Destruction Test on Main Planes.

modified as desired to suit special conditions, *e.g.* the load may be applied through a series of pulleys, being measured by a spring balance. This method of testing is also applicable to the upward load on the fuselage caused by the tail skid, except that in this case the fuselage will be reversed.

226. Test of Fuselage in Torsion.—The fuselage should be erected and firmly anchored and supported, as in the previous test described. In this case the tail should, if possible, be left upon the structure together with all its bracing wires, struts, etc. In order to apply a torque to the fuselage, one side of the tail is loaded with shot bags, the other side being loaded in the opposite direction by means of a pulley attached to the roof. A spring balance in the pulley suspension cables records the load on the tail, and this should be adjusted so that the moment applied by the pulley is equal to the moment applied by the load on the other side. The most convenient way of doing this is to apply the pulley load and the shot loads at equal distances from the centre line of the fuselage, so that the loads will then only need to be kept equal. The shot bag load may be replaced by another pulley connected to the floor; but the first method is much simpler and very efficient. In this test it is important to measure the angular deflection of the fuselage, and this may be done by attaching a light pointer at the end and this, moving along a graduated arc, gives the twist directly. It is advisable to have another reading made at the point of support in order to correct any error from the movement of the structure at this place, since what will be required is the amount of angular distortion of the fuselage between the centre section and the tail plane.

227. Tests on Tail Plane.—Tests on the tail plane are best carried out by mounting it on the fuselage, which may be supported at any points necessary as long as the tail structure is not affected. The elevators should be in position and fixed in their neutral position by locking the control stick in the pilot's cockpit. Load is then applied to the surfaces according to a distribution curve, which may be either a true one found from wind channel experiments or a fictitious one, *e.g.* triangular from leading edge to trailing edge or evenly distributed. Load is applied until failure occurs, the amount giving a measure of the factor. Failure may occur in the tail or elevator structure or possibly in the control system, so this test properly carried out affords a check on the strength of the control cables, pulleys and control column.

228. Fin and Rudder.—Tests on these surfaces are carried out in an exactly similar manner to those on the tail plane and elevator just described, and therefore need no further explanation.

229. Chassis.—The chassis should be inverted and supported in such a position as corresponds to the attitude it would have in the condition which the test is meant to reproduce. A platform should be suspended from the axle at the points where the wheels are normally attached. This platform can be loaded with shot bags or other weights until failure of the chassis occurs and the load factor is thus determined.

CHAPTER XXII

PRINCIPLE OF LEAST WORK

230. Derivation of Principle.—The principle of Least Work is one which is shunned by the great majority of engineers as being impracticable, and in any case difficult to understand. As a matter of fact, the principle is one which is extremely easy to put into practice, and offers a simple method of solution to a large number of strength problems which arise in aeroplane structures. Unfortunately the method has never been simply treated in English books on engineering structures, and those engineers who have been sufficiently interested in the principle to attempt to obtain it from mathematical books, have been discouraged by the extremely impractical way the theory has been presented.

The principle of Least Work is, however, one which will possibly be applied extensively in future to the problem of the aeroplane, and therefore should be quite familiar in theory and practice to the aeronautical engineer. Put briefly, it is a theory which enables the engineer to find the stresses in structures which are statically indeterminate, *i.e.* in structures with too many members.

In an aeroplane, for example, the incidence wires are redundant in most conditions of flight, but these wires, though neglected by the ordinary methods of strength calculation, undoubtedly carry loads. The principles of statics fail to find these loads, which will in large aeroplanes be important and have a considerable effect upon the strength, and therefore the weight, of the structure.

It will be assumed in the beginning that all members of the structures with which we are dealing are capable of taking compressive or tensile loads. If a member, as for example a wire of an aeroplane, can only take tension, then the particular problem is simplified. Each member, too, is assumed to be pin-jointed. The final general statement of the principle of Least Work will be deduced from particular examples, as this method offers considerable advantages over that in which the general statement is made and then proved.

Consider Fig. 253. It consists of a rigid body R connected to another fixed rigid body K by four members, AD, AC, BD, BC, all of which can take tension or compression. The points of attachment are A, B, C, D as shown.

Let forces, gradually increasing from zero to P_1 and P_2 be applied at E and F, and let the final positions of the points of application of these forces be *e* and *f*. Let the points A and B move from these, their original positions, to *a* and *b* respectively. Let the final loads, tensions in this particular case, in the members be T_1, T_2, T_3, T_4. The structure is now in equilibrium under the six forces T_1, T_2, T_3, T_4, P_1 and P_2. Throughout, the actual displacement of the points A, B, E and F are assumed to be small.

Draw *ag* perpendicular to AD, *ah* perpendicular to AC, *bj* perpendicular to BD, *bl* perpendicular to BC, *em* and *fn* perpendicular to the lines of action of the forces P_1 and P_2.

Then it is clear that the total work done against the external forces by stretching the members AD, AC, BD, BC is equal to the total work done by the external forces P_1 and P_2.

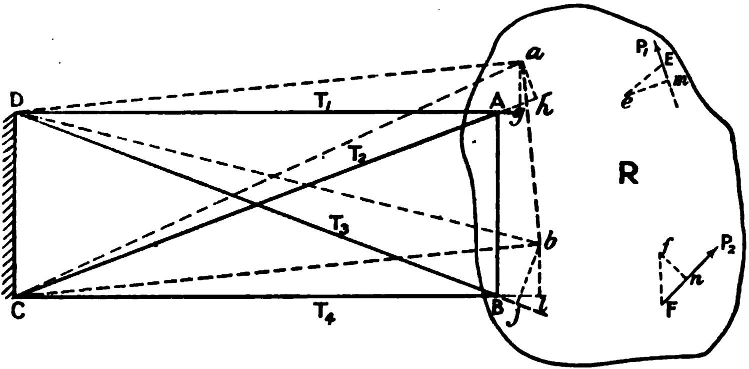


Fig 253.

i.e. $\frac{1}{2}(T_1 . Ag + T_2 . Ah + T_3 . Bj + T_4 . Bl) = \frac{1}{2}(P_1 . Em + P_2 . Fn)$

or $$T_1e_1 + T_2e_2 + T_3e_3 + T_4e_4 = P_1y_1 + P_2y_2 \quad . \quad . \quad . \quad (1)$$

where e_1, e_2, e_3, e_4 are the extensions of the members AD, AC, BD, BC, and y_1, y_2 the displacement of the points of application of the forces P_1, P_2 in their lines of actions.

It has been assumed in finding (1) that there were no initial loads in the members, but it is clear that the equation holds even if there had been, so long as $\frac{1}{2}T$ is replaced by the mean load during the displacement, and *e* is taken to mean the change in length occurring in that member due to the gradual application of the forces P_1 and P_2. It must be quite clearly understood in this case that *e* is not the total difference between the final length of the member and its length under no stress whatever. It is important to remember this, for it is sometimes necessary to consider the stresses in an aeroplane due to initial tensions in the wires caused by bad trueing up.

It is always assumed in the examination of aeroplane structures that any deformation that takes place does so within the elastic limits of the materials employed. Within these limits, by Hooke's Law, the extension or shortening of a member is proportional to the load producing it, *i.e.*

$$\frac{\text{Stress}}{\text{Strain}} = E = \frac{\frac{T}{a}}{\frac{e}{l}} = \frac{Tl}{ae}$$

where a is the area of the cross section of the member in suitable units, l its original length, T the load producing the extension e, and E is Young's Modulus.

I.e.

$$e = \frac{Tl}{aE}$$

From (1) we get

$$\frac{T_1^2 l_1}{a_1 E_1} + \frac{T_2^2 l_2}{a_2 E_2} + \frac{T_3^2 l_3}{a_3 E_3} + \frac{T_4^2 l_4}{a_4 E_4} = P_1 y_1 + P_2 y_2 \quad . \quad . \quad (2)$$

So, if there were any number of external forces and members in the structure, it could be shown that

$$\Sigma \frac{T^2 l}{aE} = \Sigma P y = W$$

$\frac{1}{2}\underline{P}y$?

It must here be emphasized that if the structure is redundant before being loaded in any way then all redundant members must be of just the right length to fit in their places, without inducing any stress in any other members. Otherwise any redundant member that has to be forced into position must induce stresses in the other members, and due account of these initial stresses must be taken when writing down the total internal work done by a slight increase of the external forces.

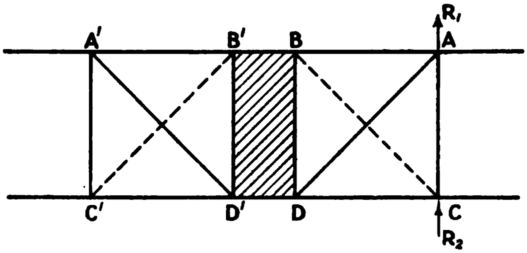


FIG. 254.

As a particular case consider the front truss of a one bay aeroplane shown in Fig. 254.

BB′D′D is the centre section and may be considered rigid for present purposes, AA′, CC′ are the top and bottom spars, AD, A′D′ the flying wires, BC, B′C′ the weight wires, R_1. R_2 reactions at A and C respectively.

Then it is clear from considerations of the loading that, in normal flight, the wires BC, B′C′ are slack, and that there is no end load in DC. Then neglecting the work done due to bending the spars (it is not important, but will be considered later) we have

$$\frac{T^2_{AC} \cdot AC}{a_{AC} \cdot E_{AC}} + \frac{T^2_{AD} \cdot AD}{a_{AD} \cdot E_{AD}} + \frac{T^2_{AB} \cdot AB}{a_{AB} \cdot E_{AB}} = R_1 y_1 + R_2 y_2 \quad . \quad . \quad (3)$$

where T_{AC} . . . , and so on, are the loads in the respective members, and y_1, y_2 the deflections at A and C. Also from the geometry of the figure we have

$$\begin{aligned} y_1 - y_2 &= \text{extension of AC} \\ &= \frac{T_{AC} \cdot AC}{a_{AC} \cdot E_{AC}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4) \end{aligned}$$

From these two equations the deflections at A and C may easily be found.

Now the statement that the external work, *i.e.* the work done by the external forces on a body, is equal to the internal work, *i.e.* the work done by elongating or shortening the internal members, is made on the assumption that the body is supported by fixed points, or rests on fixed, smooth surfaces. In the first case, as the points cannot move, no work is done by the reactions at the supporting surfaces. In the second, as the points move on smooth surfaces, the reactions are always at right angles to the direction of motion, and therefore, again, the work done by those reactions is zero.

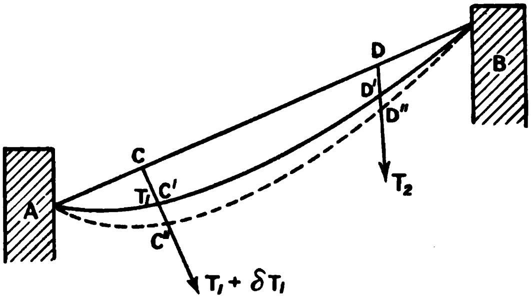


Fig. 255.

Let Fig. 255 represent any body fastened to fixed supports at A and B.

Let ACDB represent the structure under no forces.
AC′D′B „ .. „ forces T_1, T_2 applied at C′, D′
AC″D″B „ „ „ „ $T_1 + dT_1$, T_2 „

As T_1 increases from 0 to T_1 its mean value is $\frac{1}{2}T_1$, and the work done in the displacement CC′ is $\frac{1}{2}T_1 \cdot CC' = \frac{1}{2}T_1 y_1$ say.

As it increases from T_1 to $T_1 + dT_1$, its mean value is $T_1 + \frac{1}{2}dT_1$ and the work done is

$$(T_1 + \tfrac{1}{2}dT_1)\, C'C'' = (T_1 + \tfrac{1}{2}dT_1)\, dy_1, \text{ say}$$
$$= T_1 dy_1, \text{ neglecting quantities of the second order.}$$

The work done by T_2 in the displacement DD′ is

$$\tfrac{1}{2}T_2 \cdot DD' = \tfrac{1}{2}T_2 y_2 \text{ say.}$$

The work done by T_2 kept at the same value, due to the small increase in T_1, is

$$T_2\, D'D'' = T_2 dy_2 \text{ say}$$

Therefore the total work done due to the small increase in T_1 is

$$T_1 dy_1 + T_2 \cdot dy_2$$

This must equal the total increase in the internal work done on the members of the structure $= dW$.

$$dW = T_1 dy_1 + T_2 dy_2 \quad \dots\dots \quad (5)$$

This result might have been obtained in another way. Suppose that the structure initially had no loads whatever on it, and loads were then gradually applied at C and D, increasing (in the same fixed ratio to each other) from zero to $T_1 + dT_1$ and T_2 respectively, then the work done

$$= W_1 = \tfrac{1}{2}(T_1 + dT_1)(y_1 + dy_1) + \tfrac{1}{2}T_2(y_2 + dy_2)$$
$$= \tfrac{1}{2}T_1y_1 + \tfrac{1}{2}T_2y_2 + \tfrac{1}{2}y_1dT_1 + \tfrac{1}{2}T_1dy_1 + \tfrac{1}{2}T_2dy_2$$

also $W = \tfrac{1}{2}T_1y_1 + \tfrac{1}{2}T_2y_2$

But $dW = W_1 - W$

Therefore $dW = \tfrac{1}{2}y_1dT_1 + \tfrac{1}{2}T_1dy_1 + \tfrac{1}{2}T_2dy_2$

From (5), however,

$$dW = T_1dy_1 + T_2dy_2$$

Therefore $dW = y_1dT_1$

or $\dfrac{dW}{dT_1} = y_1$

This is a most important result. It is a statement of the fact that the displacement of the point of application of any one force T (measured in the line of action of that force) can be obtained by differentiating the total work done with respect to T.

Though the actual proof has been carried out with only two forces, T_1 and T_2, the same result could have been obtained for any number of forces, so long as each is independent of the others.

Now consider any structure with one redundant member.

Let AB (Fig. 256) represent the redundant member in position before loading, and A′B′ its position after loading. Let A move to A′ and B to B′, and let A′C, B′D be the perpendiculars from A and B on the original line of the redundant member. Let T be the final load in the bar.

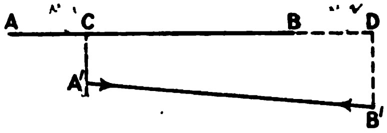


Fig. 256.

Then the component displacement of the point A, in the direction AB of the load in the member is $AC = y_1$ say, and the component displacement of B, in the same direction is $BD = y_2$ say.

Then the extension of the member AB as a result of the gradual application of the load T in it is

$$-y_1 + y_2 = e = \frac{Tl}{aE}$$

where l is the length of the member, a the area of the cross section, and E is Young's Modulus.

Now suppose the member AB is removed and equal and opposite external forces equal to T applied to the remaining part of the structure at A and B. Then it is clear that on this supposition the loads in the remaining members are unaltered, *i.e.* T may now be considered an external force, at either point of application. Suppose the force at one end, for the sake of clearness in the argument, be called T_1 and at the other end T_2, so that $T_1 = T_2 = T$. Then T_1 and T_2 are now external forces, and either may be supposed to vary independently of the other, without altering the equilibrium of the rest of the structure.

Suppose, to fix ideas, the member is in tension. The argument is precisely the same if it is in compression. As already shown, if W′ is the total internal work of all the members except AB

$$\frac{dW'}{dT_1} = +y_1$$

and

$$\frac{dW'}{dT_2} = -y_2$$

But

$$y_2 - y_1 = e = \frac{Tl}{aE}$$

$$-\frac{dW'}{dT_2} - \frac{dW'}{dT_1} = \frac{Tl}{aE}$$

or

$$\frac{dW'}{dT_1} + \frac{dW'}{dT_2} + \frac{Tl}{aE} = 0 \qquad \qquad (6)$$

But

$$T_1 = T_2 = T$$

Therefore

$$\frac{dW'}{dT_1} + \frac{dW'}{dT_2} = \frac{dW'}{dT}$$

Also

$$\frac{Tl}{aE} = \frac{d}{dT}\left(\frac{l}{2aE} \cdot T^2\right)$$

I.e. is the result of differentiating with respect to T the work done on the redundant member by T.

It follows, therefore, that the left-hand side of (6) is the result of differentiating the total internal work of all the members, including the redundant member.

So equation (6) may be written

$$\frac{dW}{dT} = 0$$

This is the Principle of Least Work.

It is a theoretical statement of the practical fact that if an elastic structure is in a state of stable equilibrium under any forces whatever, then the work stored is the smallest amount possible.

It is extremely simple to use in actual practice, though if there are many redundant members it may become laborious. For only one or two redundant members, however, the method of Least Work is preferable to guess work, however great the experience upon which that guess work is based.

Here is the simple rule for using the method of Least Work. Cut out all the redundant members and replace them by forces T_1, T_2, T_3 . . . at each end. The structure is now a static one and the loads in the members may be found by the ordinary methods of statics. Write down the total internal work done, including that on the redundant bars, by the forces T_1, T_2, T_3, . . . The result will be in terms of T_1, T_2, T_3, etc.

Differentiate this result W with respect to each of the forces T_1, T_2, T_3, . . . and equate to zero.

i.e. put

$$\frac{dW}{dT_1}=0$$

$$\frac{dW}{dT_2}=0$$

and so on. The number of equations will be the same as the number of unknowns, which can then be found.

Consider an actual example shown in Fig. 257.

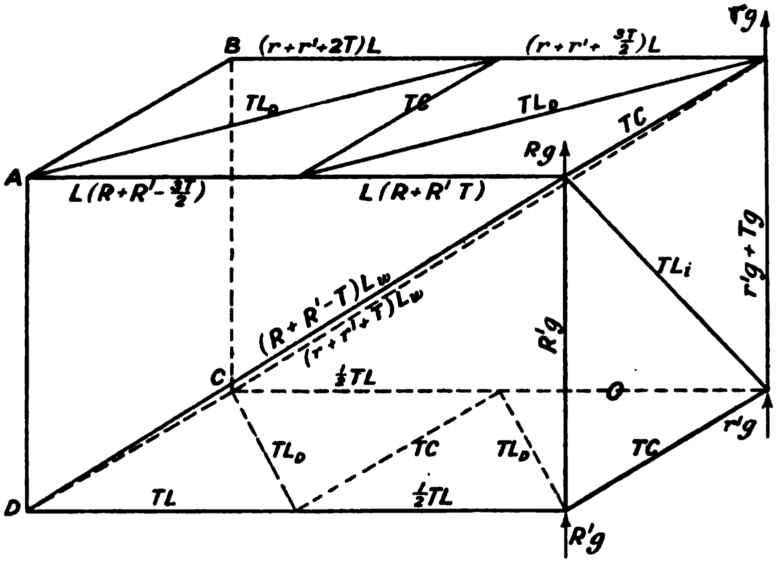


FIG. 257.

Let ABCD be the points of attachment of the planes of a one-bay aeroplane to the centre section. For the purposes of calculation these points may be considered fixed.

It will be found advantageous to express the load in any member in terms of the length of that member. Thus the load in the incidence wire is expressed as T × length of the wire, and so on. It also simplifies the general expressions obtained if all vertical reactions are expressed in terms of the gap, as shown. For illustration purposes it has been assumed that the top and bottom planes are equal in length, and divided into two equal drag bays, and that the aeroplane is not staggered. The following notation is used :—

TL_i = the tension in the incidence wire.
Rg, $R'g$, rg, $r'g$ = the vertical reactions.
L = the length of the bay.

C = the distance between spar centres.
g = the gap.
L_D = the length of the drag wire.
L_w = the length of the flying wire.
L_i = the length of the incidence wire.

Then by the ordinary methods of statics we can easily find the load in the various members of the structure in terms of the reactions and T.

Consider the case when the centre of pressure is forward. It is clear that the incidence wire, from the top front spar to the bottom rear spar, is in play. The corresponding loads in terms of T and the external reactions are shown in the figure, and from these it is easy to write down the total internal work done. Let this be W.

Then $dW/dT = 0$ gives a linear equation in T.

As a first approximation to the load in the incidence wire it may be assumed that the struts and spars do no work. This approximation gives a slightly less load in the incidence wires than is actually the case, and so far as testing the spars and interplane struts is concerned errs on the side of safety. In the case shown this assumption is made and

$$T = \frac{\dfrac{(R+R')L^3_w}{A_w} - \dfrac{(r+r')L^3_w}{A'_w}}{\dfrac{L^3_i}{A_i} + \dfrac{L^3_w}{A_w} + \dfrac{L^3_w}{A'_w} + \dfrac{2L^3_D}{A_D} + \dfrac{2L^3_D}{A'_D}}$$

where A_i, A_w, A'_w . . . are the areas of the wires.

As another example, consider the work done on the pin-jointed strut with a concentrated side load shown in Fig. 190, Chapter XVIII.

The work done due to W and P is for the span AW (as shown in the next paragraph).

$$\frac{1}{2EI}\int_0^a M^2 dx = \frac{1}{2EI} \cdot \frac{W^2 \sin^2 \mu b}{\mu^2 \sin^2 \mu(a+b)}\int_0^a \sin^2 \mu x \, dx$$

$$= \frac{W^2}{4P} \cdot \frac{\sin^2 \mu b}{\sin^2 \mu(a+b)}\int_0^a (1 - \cos 2\mu x)\, dx$$

$$= \frac{W^2}{4P} \cdot \frac{\sin^2 \mu b}{\sin^2 \mu(a+b)}\left(a - \frac{1}{2\mu}\sin 2\mu a\right)$$

Similarly for the span WB.

Therefore the total work done

$$= \frac{W^2}{4P \sin^2 \mu(a+b)}\left\{a \sin^2 \mu b + b \sin^2 \mu a - \frac{1}{\mu}\left(\begin{matrix}\sin^2 \mu b \sin \mu a \cos \mu a \\ + \sin^2 \mu a \sin \mu b \cos \mu b\end{matrix}\right)\right\}$$

$$= \frac{W^2}{4P \sin^2 \mu(a+b)}\left\{a \sin^2 \mu b + b \sin^2 \mu a - \frac{1}{\mu}\sin \mu a \sin \mu b \sin \mu(a+b)\right\}$$

231. Work due to Bending.—Consider a short length of a beam over which the bending moment is constant = M. Let the radii of curvature R at the extremities of this small length make an angle θ with one another, as shown in Fig. 258.

Then work due to bending = average couple × θ

$$= \frac{M}{2} \cdot \theta = \frac{M}{2} \cdot \frac{l}{R} \text{ since } l = R\theta$$

But
$$\frac{M}{I} = \frac{E}{R}$$

or
$$\frac{1}{R} = \frac{M}{EI}$$

So that work done $= \frac{M}{2} \cdot \frac{lM}{EI} = \frac{M^2 l}{2EI}$

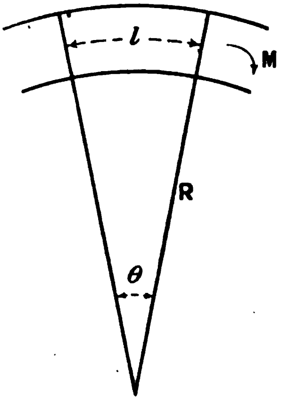


FIG. 258.

and hence if M varies over the beam the total work done $= \int \frac{M^2 dx}{2EI}$, the integration being carried over the whole length of the beam.

As an example we will consider the work done on a continuous beam without end loads.

Let ABC (Fig. 259) be a continuous beam, supported at A, B and C.

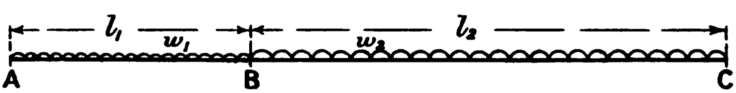


FIG. 259.

Let the spans be l_1, l_2, the loadings w_1, w_2, and the moments of inertia of the sections I_1 and I_2 respectively. Suffixes will be dropped where no ambiguity occurs.

Take origin at B for BC, and adopting the signs and notation of Chapter X., then at any point distant x from B we have—

$$M_x = EI\frac{d^2y}{dx^2} = \frac{-w_2 l_2 x}{2} + \frac{w_2 x^2}{2} + M_B + (M_C - M_B)\frac{x}{l_2}$$

Also the work done in bending $= \int \frac{M^2 dx}{2EI}$

Therefore the work done due to bending

$$=\frac{1}{2EI}\int_0^l\left\{-\frac{wlx}{2}+\frac{wx^2}{2}+M_B+(M_C-M_B)\frac{x}{l}\right\}^2dx$$

The definite integral is

$$\frac{1}{240EI}(-10wl^3M_B-10wl^3M_C+40M_BM_Cl+40M_C^2l+40M_B^2l+w^2l^5)$$

So that the total work done on the span AC is

$$\frac{1}{240E_1I_1}(-10w_1l_1^3M_A-10w_1l_1^3M_B+40M_AM_Bl_1+40M_A^2l_1+40M_B^2l_1+w_1^2l_1^5)$$

$$+\frac{1}{240E_2I_2}(-10w_2l_2^3M_B-10w_2l_2^3M_C+40M_BM_Cl_2+40M_B^2l_2+40M_C^2l_2+w_2^2l_2^5)$$

APPENDIX I

BERRY FUNCTIONS

TABLE I

$f(\theta) = 6\,(2\,\theta \operatorname{cosec} 2\,\theta - 1)\,/\,(2\,\theta)^2$, $\phi(\theta) = 3\,(1 - 2\,\theta \cot 2\,\theta)\,/\,(2\,\theta)^2$,
$\psi(\theta) = 3\,(\tan\theta - \theta)\,/\,\theta^3$ (θ in radians)

$\theta°$	$f(\theta)$	$\phi(\theta)$	$\psi(\theta)$
0	1·0000	1·0000	1·0000
1	1·0001	1·0001	1·0001
2	1·0006	1·0003	1·0005
3	1·0013	1·0007	1·0011
4	1·0023	1·0013	1·0019
5	1·0036	1·0020	1·0030
6	1·0051	1·0029	1·0044
7	1·0070	1·0040	1·0060
8	1·0092	1·0052	1·0078
9	1·0116	1·0066	1·0099
10	1·0144	1·0082	1·0123
11	1·0175	1·0100	1·0150
12	1·0209	1·0120	1·0179
13	1·0246	1·0140	1·0210
14	1·0286	1·0163	1·0245
15	1·0329	1·0188	1·0282
16	1·0376	1·0214	1·0322
17	1·0427	1·0243	1·0365
18	1·0481	1·0273	1·0411
19	1·0538	1·0306	1·0460
20	1·0599	1·0341	1·0512
21	1·0664	1·0377	1·0568
22	1·0734	1·0416	1·0628
23	1·0807	1·0458	1·0690
24	1·0884	1·0502	1·0756
25	1·0966	1·0548	1·0825
26	1·1052	1·0596	1·0898
27	1·1143	1·0647	1·0976
28	1·1239	1·0701	1·1058
29	1·1340	1·0758	1·1144
30	1·1446	1·0817	1·1234
31	1·1558	1·0879	1·1329
32	1·1675	1·0945	1·1428
33	1·1799	1·1013	1·1533
34	1·1928	1·1086	1·1643
35	1·2065	1·1161	1·1759
36	1·2208	1·1241	1·1880
37	1·2359	1·1324	1·2007
38	1·2517	1·1412	1·2140
39	1·2683	1·1503	1·2281
40	1·2858	1·1600	1·2429
41	1·3042	1·1701	1·2584
42	1·3236	1·1807	1·2747
43	1·3440	1·1918	1·2918
44	1·3654	1·2035	1·3099
45	1·3880	1·2158	1·3289
46	1·4118	1·2288	1·3489
47	1·4370	1·2425	1·3700
48	1·4635	1·2568	1·3922
49	1·4915	1·2720	1·4157
50	1·5211	1·2879	1·4405
51	1·5524	1·3048	1·4666
52	1·5856	1·3226	1·4944
53	1·6208	1·3415	1·5237
54	1·6582	1·3615	1·5549
55	1·6979	1·3827	1·5880
56	1·7403	1·4052	1·6232
57	1·7853	1·4291	1·6607
58	1·8335	1·4546	1·7007
59	1·8850	1·4818	1·7434
60	1·9401	1·5109	1·7891
61	1·9994	1·5421	1·8381
62	2·0631	1·5755	1·8908
63	2·1318	1·6115	1·9476
64	2·2060	1·6503	2·0089
65	2·2865	1·6922	2·0753
66	2·3741	1·7377	2·1474
67	2·4695	1·7872	2·2260
68	2·5739	1·8412	2·3119
69	2·6886	1·9005	2·4061
70·0	2·8152	1·9657	2·5100
70·1	2·8286	1·9725	2·5210
70·2	2·8421	1·9795	2·5321
70·3	2·8557	1·9865	2·5433
70·4	2·8695	1·9936	2·5546
70·5	2·8835	2·0008	2·5660
70·6	2·8976	2·0080	2·5776
70·7	2·9118	2·0153	2·5893
70·8	2·9262	2·0227	2·6011
70·9	2·9407	2·0302	2·6130

TABLE I—*continued*

θ°	$f(\theta)$	$\phi(\theta)$	$\psi(\theta)$
71·0	2·9554	2·0378	2·6250
71·1	2·9703	2·0454	2·6372
71·2	2·9853	2·0531	2·6495
71·3	3·0005	2·0609	2·6620
71·4	3·0159	2·0688	2·6746
71·5	3·0314	2·0767	2·6873
71·6	3·0471	2·0848	2·7001
71·7	3·0630	2·0929	2·7131
71·8	3·0790	2·1011	2·7263
71·9	3·0953	2·1095	2·7396
72·0	3·1117	2·1179	2·7530
72·1	3·1283	2·1264	2·7666
72·2	3·1451	2·1350	2·7804
72·3	3·1621	2·1437	2·7943
72·4	3·1792	2·1524	2·8084
72·5	3·1966	2·1614	2·8226
72·6	3·2142	2·1704	2·8370
72·7	3·2320	2·1795	2·8516
72·8	3·2500	2·1887	2·8663
72·9	3·2683	2·1980	2·8812
73·0	3·2867	2·2074	2·8963
73·1	3·3054	2·2170	2·9116
73·2	3·3243	2·2266	2·9270
73·3	3·3434	2·2364	2·9427
73·4	3·3627	2·2463	2·9585
73·5	3·3823	2·2563	2·9746
73·6	3·4022	2·2664	2·9908
73·7	3·4223	2·2767	3·0072
73·8	3·4426	2·2871	3·0238
73·9	3·4632	2·2976	3·0407
74·0	3·4841	2·3082	3·0577
74·1	3·5052	2·3190	3·0750
74·2	3·5266	2·3299	3·0925
74·3	3·5483	2·3410	3·1102
74·4	3·5702	2·3522	3·1282
74·5	3·5925	2·3635	3·1464
74·6	3·6150	2·3750	3·1648
74·7	3·6379	2·3866	3·1834
74·8	3·6610	2·3984	3·2023
74·9	3·6845	2·4104	3·2215
75·0	3·7083	2·4225	3·2409
75·1	3·7323	2·4348	3·2606
75·2	3·7568	2·4472	3·2806
75·3	3·7815	2·4598	3·3008
75·4	3·8066	2·4726	3·3213
75·5	3·8321	2·4855	3·3421
75·6	3·8579	2·4987	3·3631
75·7	3·8841	2·5120	3·3845
75·8	3·9107	2·5255	3·4062
75·9	3·9376	2·5392	3·4282
76·0	3·9650	2·5531	3·4505
76·1	3·9927	2·5672	3·4731
76·2	4·0208	2·5814	3·4961
76·3	4·0494	2·5959	3·5194
76·4	4·0784	2·6107	3·5431
76·5	4·1078	2·6256	3·5671
76·6	4·1376	2·6408	3·5914
76·7	4·1680	2·6561	3·6162
76·8	4·1987	2·6718	3·6412
76·9	4·2300	2·6876	3·6667
77·0	4·2617	2·7037	3·6926
77·1	4·2940	2·7201	3·7189
77·2	4·3267	2·7367	3·7456
77·3	3·3600	2·7535	3·7727
77·4	4·3938	2·7707	3·8003
77·5	4·4281	2·7881	3·8283
77·6	4·4631	2·8058	3·8568
77·7	4·4986	2·8237	3·8857
77·8	4·5346	2·8420	3·9151
77·9	4·5713	2·8606	3·9450
78·0	4·6086	2·8795	3·9754
78·1	4·6465	2·8987	4·0063
78·2	4·6851	2·9182	4·0377
78·3	4·7243	2·9380	4·0697
78·4	4·7643	2·9582	4·1022
78·5	4·8049	2·9788	4·1353
78·6	4·8462	2·9997	4·1690
78·7	4·8883	3·0210	4·2032
78·8	4·9311	3·0426	4·2381
78·9	4·9748	3·0647	4·2737
79·0	5·0192	3·0871	4·3098
79·1	5·0644	3·1100	4·3467
79·2	5·1105	3·1333	4·3842
79·3	5·1575	3·1570	4·4224
79·4	5·2053	3·1812	4·4613
79·5	5·2541	3·2058	4·5010
79·6	5·3038	3·2309	4·5415
79·7	5·3545	3·2565	4·5828
79·8	5·4062	3·2826	4·6248
79·9	5·4589	3·3092	4·6677
80·0	5·5127	3·3363	4·7115
80·1	5·5675	3·3640	4·7561
80·2	5·6235	3·3922	4·8017
80·3	5·6807	3·4211	4·8482
80·4	5·7390	3·4505	4·8957
80·5	5·7986	3·4805	4·9442
80·6	5·8595	3·5112	4·9937
80·7	5·9217	3·5426	5·0442
80·8	5·9853	3·5746	5·0959
80·9	6·0502	3·6073	5·1488
81·0	6·1166	3·6408	5·2028
81·1	6·1845	3·6750	5·2580
81·2	6·2540	3·7100	5·3145
81·3	6·3251	3·7458	5·3722
81·4	6·3978	3·7824	5·4314
81·5	6·4722	3·8199	5·4919
81·6	6·5485	3·8582	5·5538
81·7	6·6265	3·8975	5·6173

TABLE I—*continued*

θ°	$f(\theta)$	$\phi(\theta)$	$\psi(\theta)$
81·8	6·7065	3·9378	5·6823
81·9	6·7885	3·9790	5·7489
82·0	6·8725	4·0213	5·8172
82·1	6·9587	4·0646	5·8873
82·2	7·0471	4·1091	5·9591
82·3	7·1378	4·1547	6·0328
82·4	7·2309	4·2015	6·1084
82·5	7·3265	4·2496	6·1861
82·6	7·4246	4·2989	6·2658
82·7	7·5255	4·3496	6·3478
82·8	7·6292	4·4017	6·4320
82·9	7·7359	4·4553	6·5186
83·0	7·8456	4·5104	6·6077
83·1	7·9584	4·5671	6·6994
83·2	8·0746	4·6255	6·7938
83·3	8·1943	4·6856	6·8910
83·4	8·3176	4·7475	6·9911
83·5	8·4447	4·8114	7·0944
83·6	8·5759	4·8772	7·2008
83·7	8·7112	4·9451	7·3107
83·8	8·8508	5·0152	7·4241
83·9	8·9951	5·0876	7·5412
84·0	9·1442	5·1624	7·6622
84·1	9·2983	5·2398	7·7874
84·2	9·4578	5·3198	7·9168
84·3	9·6229	5·4026	8·0508
84·4	9·7939	5·4884	8·1896
84·5	9·9711	5·5773	8·3335
84·6	10·1549	5·6694	8·4827
84·7	10·3457	5·7651	8·6375
84·8	10·5438	5·8644	8·7982
84·9	10·7497	5·9677	8·9653
85·0	10·9638	6·0750	9·1391
85·1	11·1867	6·1867	9·3200
85·2	11·4189	6·3031	9·5084
85·3	11·6610	6·4245	9·7049
85·4	11·9137	6·5511	9·9098
85·5	12·1776	6·6833	10·1239
85·6	12·4535	6·8215	10·3478
85·7	12·7423	6·9662	10·5821
85·8	13·0448	7·1178	10·8275
85·9	13·3621	7·2767	11·0849
86·0	13·6954	7·4436	11·3552
86·1	14·0457	7·6190	11·6394
86·2	14·4144	7·8037	11·9384
86·3	14·8032	7·9984	12·2537
86·4	15·2135	8·2038	12·5865
86·5	15·6473	8·4210	12·9384
86·6	16·1067	8·6510	13·3109
86·7	16·5939	8·8949	13·7060
86·8	17·1116	9·1541	14·1259
86·9	17·6627	9·4299	14·5727
87·0	18·2506	9·7241	15·0495
87·1	18·8791	10·0387	15·5592
87·2	19·5525	10·3757	16·1052
87·3	20·2758	10·7375	16·6917
87·4	21·0548	11·1273	17·3233
87·5	21·8961	11·5483	18·0054
87·6	22·8076	12·0043	18·7444
87·7	23·7983	12·5000	19·5478
87·8	24·8792	13·0408	20·4240
87·9	26·0631	13·6330	21·3839
88·0	27·3653	14·2844	22·4396

Between 88° and 90° the functions can be calculated by the approximate formulæ—

$$f(\theta) = \frac{54 \cdot 71}{90 - \theta}, \quad \phi(\theta) = \frac{27 \cdot 36}{90 - \theta} + 0 \cdot 61, \quad \psi(\theta) = \frac{44 \cdot 35}{90 - \theta} + 0 \cdot 26,$$

θ being in degrees.

TABLE II

$F(\theta)=6\,(1-2\,\theta\ \mathrm{cosech}\ 2\,\theta)/(2\,\theta)^2$, $\Phi(\theta)=3\,(2\,\theta\ \coth 2\,\theta-1)/(2\,\theta)^2$,
$\Psi(\theta)=3\,(\theta-\tanh\theta)/\theta^3$

θ radians	$F(\theta)$	$\Phi(\theta)$	$\Psi(\theta)$	θ radians	$F(\theta)$	$\Phi(\theta)$	$\Psi(\theta)$
·00	1·0000	1·0000	1·0000	·50	·8945	·9391	·9092
·01	0·9999	1·0000	0·9999	·51	·8906	·9369	·9059
·02	·9998	0·9999	·9998	·52	·8868	·9346	·9026
·03	·9996	·9998	·9996	·53	·8828	·9323	·8992
·04	·9992	·9996	·9994	·54	·8789	·9300	·8957
·05	·9988	·9993	·9990	·55	·8748	·9276	·8922
·06	·9983	·9990	·9986	·56	·8708	·9253	·8887
·07	·9977	·9987	·9981	·57	·8667	·9229	·8851
·08	·9970	·9983	·9975	·58	·8626	·9204	·8815
·09	·9962	·9979	·9968	·59	·8584	·9180	·8779
·10	·9953	·9973	·9960	·60	·8542	·9155	·8743
·11	·9944	·9968	·9952	·61	·8500	·9130	·8706
·12	·9933	·9962	·9943	·62	·8457	·9105	·8669
·13	·9922	·9955	·9933	·63	·8414	·9080	·8632
·14	·9909	·9948	·9922	·64	·8371	·9054	·8595
·15	·9896	·9940	·9910	·65	·8328	·9028	·8557
·16	·9882	·9932	·9898	·66	·8284	·9003	·8519
·17	·9867	·9924	·9886	·67	·8240	·8977	·8481
·18	·9851	·9915	·9872	·68	·8196	·8950	·8442
·19	·9834	·9905	·9857	·69	·8151	·8924	·8403
·20	·9816	·9895	·9842	·70	·8107	·8897	·8364
·21	·9798	·9884	·9826	·71	·8062	·8871	·8325
·22	·9779	·9873	·9810	·72	·8017	·8844	·8286
·23	·9758	·9862	·9793	·73	·7972	·8817	·8247
·24	·9738	·9850	·9775	·74	·7927	·8790	·8207
·25	·9716	·9837	·9756	·75	·7881	·8762	·8167
·26	·9693	·9824	·9736	·76	·7835	·8735	·8127
·27	·9670	·9811	·9717	·77	·7790	·8708	·8087
·28	·9646	·9797	·9696	·78	·7744	·8680	·8047
·29	·9621	·9783	·9675	·79	·7698	·8653	·8007
·30	·9595	·9768	·9653	·80	·7652	·8625	·7967
·31	·9569	·9753	·9630	·81	·7606	·8597	·7927
·32	·9542	·9737	·9607	·82	·7560	·8569	·7887
·33	·9514	·9721	·9583	·83	·7513	·8541	·7847
·34	·9486	·9705	·9558	·84	·7467	·8513	·7807
·35	·9457	·9688	·9533	·85	·7421	·8485	·7766
·36	·9427	·9671	·9507	·86	·7374	·8457	·7725
·37	·9396	·9653	·9481	·87	·7328	·8429	·7684
·38	·9365	·9635	·9454	·88	·7282	·8400	·7643
·39	·9333	·9617	·9427	·89	·7235	·8372	·7601
·40	·9301	·9598	·9399	·90	·7189	·8344	·7560
·41	·9268	·9579	·9371	·91	·7143	·8315	·7519
·42	·9234	·9559	·9342	·92	·7096	·8287	·7478
·43	·9200	·9539	·9312	·93	·7050	·8259	·7437
·44	·9165	·9519	·9282	·94	·7004	·8230	·7396
·45	·9129	·9499	·9252	·95	·6958	·8202	·7355
·46	·9094	·9478	·9221	·96	·6912	·8173	·7314
·47	·9057	·9456	·9189	·97	·6866	·8145	·7273
·48	·9020	·9435	·9157	·98	·6820	·8117	·7232
·49	·8983	·9413	·9125	·99	·6774	·8088	·7192

TABLE II—*continued*

θ radians	F (θ)	Φ (θ)	Ψ (θ)
1·00	·6728	·8060	·7152
1·01	·6683	·8031	·7112
1·02	·6637	·8003	·7072
1·03	·6592	·7975	·7031
1·04	·6547	·7946	·6991
1·05	·6501	·7918	·6950
1·06	·6456	·7890	·6910
1·07	·6411	·7861	·6870
1·08	·6367	·7833	·6830
1·09	·6322	·7805	·6790
1·10	·6278	·7777	·6750
1·11	·6233	·7749	·6711
1·12	·6189	·7721	·6672
1·13	·6145	·7693	·6633
1·14	·6102	·7665	·6594
1·15	·6058	·7637	·6555
1·16	·6015	·7609	·6516
1·17	·5972	·7582	·6477
1·18	·5929	·7554	·6438
1·19	·5886	·7527	·6399
1·20	·5843	·7499	·6360
1·21	·5801	·7472	·6322
1·22	·5759	·7444	·6284
1·23	·5717	·7417	·6246
1·24	·5675	·7390	·6208
1·25	·5633	·7363	·6170
1·26	·5592	·7336	·6133
1·27	·5551	·7309	·6096
1·28	·5510	·7282	·6059
1·29	·5469	·7255	·6022
1·30	·5429	·7229	·5985
1·31	·5389	·7202	·5948
1·32	·5349	·7175	·5912
1·33	·5309	·7149	·5875
1·34	·5269	·7123	·5839
1·35	·5230	·7097	·5803
1·36	·5191	·7071	·5767
1·37	·5152	·7045	·5732
1·38	·5114	·7019	·5697
1·39	·5075	·6993	·5662
1·40	·5037	·6967	·5627
1·41	·4999	·6942	·5593
1·42	·4962	·6916	·5559
1·43	·4924	·6891	·5525
1·44	·4887	·6866	·5491
1·45	·4851	·6840	·5457
1·46	·4814	·6815	·5423
1·47	·4778	·6790	·5389
1·48	·4742	·6766	·5355
1·49	·4706	·6741	·5321
1·50	·4670	·6716	·5288
1·51	·4635	·6692	·5255
1·52	·4600	·6668	·5222
1·53	·4565	·6643	·5189
1·54	·4530	·6619	·5157
1·55	·4496	·6595	·5125
1·56	·4462	·6571	·5093
1·57	·4428	·6547	·5061
1·58	·4395	·6524	·5030
1·59	·4361	·6500	·4999
1·60	·4328	·6476	·4968

TABLE III

Tanh θ

θ radians	Tanh θ	θ radians	Tanh θ	θ radians	Tanh θ	θ radians	Tanh θ
0·00	·00000	0·50	·46212	1·00	·76159	1·50	·90515
·01	·01000	·51	·46995	·01	·76576	·51	·90694
·02	·02000	·52	·47770	·02	·76987	·52	·90870
·03	·02999	·53	·48538	·03	·77391	·53	·91043
·04	·03998	·54	·49299	·04	·77789	·54	·91212
·05	·04996	·55	·50052	·05	·78181	·55	·91379
·06	·05993	·56	·50798	·06	·78566	·56	·91542
·07	·06989	·57	·51536	·07	·78946	·57	·91703
·08	·07983	·58	·52267	·08	·79320	·58	·91860
·09	·08976	·59	·52990	·09	·79688	·59	·92015
0·10	·09967	0·60	·53705	1·10	·80050	1·60	·92167
·11	·10956	·61	·54413	·11	·80406	·61	·92316
·12	·11943	·62	·55113	·12	·80757	·62	·92462
·13	·12928	·63	·55805	·13	·81102	·63	·92606
·14	·13909	·64	·56490	·14	·81441	·64	·92747
·15	·14888	·65	·57167	·15	·81775	·65	·92886
·16	·15865	·66	·57836	·16	·82104	·66	·93022
·17	·16838	·67	·58498	·17	·82427	·67	·93155
·18	·17808	·68	·59152	·18	·82745	·68	·93286
·19	·18775	·69	·59798	·19	·83058	·69	·93415
0·20	·19738	0·70	·60437	1·20	·83365	1·70	·93541
·21	·20697	·71	·61068	·21	·83668	·71	·93665
·22	·21652	·72	·61691	·22	·83965	·72	·93786
·23	·22603	·73	·62307	·23	·84258	·73	·93906
·24	·23550	·74	·62915	·24	·84546	·74	·94023
·25	·24492	·75	·63515	·25	·84828	·75	·94138
·26	·25430	·76	·64108	·26	·85106	·76	·94250
·27	·26362	·77	·64693	·27	·85380	·77	·94361
·28	·27291	·78	·65271	·28	·85648	·78	·94470
·29	·28213	·79	·65841	·29	·85913	·79	·94576
0·30	·29131	0·80	·66404	1·30	·86172	1·80	·94681
·31	·30044	·81	·66959	·31	·86427	·81	·94783
·32	·30951	·82	·67507	·32	·86678	·82	·94884
·33	·31852	·83	·68048	·33	·86925	·83	·94983
·34	·32748	·84	·68581	·34	·87167	·84	·95080
·35	·33638	·85	·69107	·35	·87405	·85	·95175
·36	.34521	·86	·69626	·36	·87639	·86	·95268
·37	·35399	·87	·70137	·37	·87869	·87	·95359
·38	·36271	·88	·70642	·38	·88095	·88	·95449
·39	·37136	·89	·71139	·39	·88317	·89	·95537
0·40	·37995	0·90	·71630	1·40	·88535	1·90	·95624
·41	·38847	·91	·72113	·41	·88749	·91	·95709
·42	·39693	·92	·72590	·42	·88960	·92	·95792
·43	·40532	·93	·73059	·43	·89167	·93	·95873
·44	·41364	·94	·73522	·44	·89370	·94	·95953
·45	·42190	·95	·73978	·45	·89569	·95	·96032
·46	·43008	·96	·74428	·46	·89765	·96	·96109
·47	·43820	·97	·74870	·47	·89958	·97	·96185
·48	·44624	·98	·75307	·48	·90147	·98	·96259
·49	·45422	·99	·75736	·49	·90332	·99	·96331
						2·00	·96403

APPENDIX II

ABRIDGMENTS OF MATERIALS SPECIFICATIONS ISSUED BY THE AIR MINISTRY

AIR BOARD

T. 1. *April*, 1917. *Abridged.*

SPECIFICATION FOR 35-TON CARBON STEEL TUBES

Hard Drawn and Blued. Carbon about 0·3 per Cent.

(*For Softened Tubes, see Specification T.* 21.)

If blued tubes are annealed, brazed, or welded, their strength will be reduced at the parts where they are so treated to the values given in softening test.

Accuracy of Form, Size, and Straightness.—(*a*) Round tubes are to be accurately circular.

(*b*) The mean outside diameter (*i.e.* the mean between the maximum and minimum diameter), at any point, is not to differ from the size ordered by more than ± 0·004 inch (or for tubes over 2 inches diameter by more than diameter/500). The mean inside diameter is not to be less than the correct outside diameter *minus* twice the maximum permissible thickness, nor greater than same *minus* twice the minimum permissible thickness.

(*c*) Oval tubes are to be of the correct form and dimensions within the tolerances specified in Schedule T. 11.

(*d*) No tube is to have a mean thickness less than the specified gauge, or exceeding it by more than ·004 inch except tubes thicker than ·060 inch, for which the tolerance is to be 7½ per cent. of their thickness. (Tubes ordered to be 22-gauge thick are to be ·025 inch thick with a tolerance of +·004, to agree with the dimensions set out in Schedule T. 10.)

(*e*) At no point in a tube is the thickness to fall short of the nominal thickness by more than 10 per cent. or exceed it by more than 15 per cent.

(*f*) The tubes are to be as straight as possible, and in no part of the length is the departure from straightness to exceed one six-hundredth of the length of that part.

Tests.—(*a*) The tubes are to comply with the following mechanical tests.

(*b*) *Tension and Compression Tests.*—Test pieces consisting of short lengths cut off the tube must give the following results, without further heat treatment or other manipulation:—

Ultimate strength in tension not less than 35 tons per square inch.
Yield point „ „ 30 „ „
Yield point in compression „ 30 „ „

(c) *Softening Test.*—This test is intended to prove that the tubes will not be unduly soft after they have been annealed, welded, brazed, or otherwise heated.

A test piece consisting of a suitable length cut off the tube selected, is to be heated to a full red heat at one end for at least five minutes while the other end remains cold, and is then to be allowed to cool freely in the air. The sample so treated must give the following results when tested in tension:—

Ultimate strength in tension not less than 28 tons per square inch.
Yield point ,, ,, 18 ,, ,,

(d) *Flattening Test.*—The tube is to be flattened at the end, or at any point where defective material is suspected, by a few blows (not more than six) till the sides are not more than three times the thickness of the metal apart. The tubes must stand this treatment without cracking.

(e) *Crushing Test.*—Samples of the tube selected are to be crushed endwise until the outside diameter is increased in one zone by 25 per cent., or until one complete fold is formed. The samples must stand this treatment without cracking.

AIR BOARD

T. 2. *October*, 1917. *Abridged.*

SPECIFICATION FOR NICKEL-CHROME STEEL AXLE TUBES

NOTE.—*Axle tubes are seriously weakened where drilled.*

Accuracy of Form, Size, and Straightness.—(*a*) The tubes are to be accurately circular.

(*b*) The mean outside diameter (*i.e.* the mean between the maximum and minimum diameter) at any section is not to differ from the size shown in Col. 2 of the Schedule by more than ± 0·005 inch.

(*c*) The ends of the tubes for a distance of 14 inches from each end are to be rounded by pressure, so that no diameter exceeds the nominal diameter (Col. 1).

(*d*) No axle is to exceed the maximum specified weight (calculated by multiplying its length by the weight per foot given in column 5 of the Schedule).

(*e*) At no point in a tube is the thickness to fall short of the nominal thickness by more than 10 per cent. or exceed it by more than 20 per cent.

(*f*) The tubes are to be as straight as possible, and in no part of the length is the departure from straightness to exceed one three-hundredth of the length of that part.

Mechanical Tests.—(*a*) The tubes are to comply with the following tests.

(*b*) *Proof Load.*—Every axle is to be tested by having a proof bending moment applied to it near one end, and at least one axle in ten is to be tested in this way at both ends. The proof load is given in column 7 and the leverage L is given in column 8 of the Schedule at the end of this specification.[1]

(*c*) *Tensile Test.*—One test piece to represent every 100 axles is to be cut from one of the tubes from which the axles have been cut; it is to be heat treated with the axles it represents and is then to be tested in tension; it must give the following results:—

Ultimate tensile strength . . .	Not less than 85 tons per square inch.
Elongation on 2 inches . . .	,, ,, 5 per cent.
,, 4 ,, ,, . .	,, ,, 3 ,,

---

[1] See Chapter XX., para. 220.

SCHEDULE

1.	2.	3.		4.	5.	6.	7.	8.
Nominal diam.	True outside diam. (Lim. ±·005 in.)	Thickness.		Min. area of section.	Max. weight per foot.	Modulus of section Z.	Proof load.	Proof load leverage L.
inches.	inches.	gauge.	inches.	inches.^{2}	lbs.	inches.^{3}	lbs.	inches.
2·375	2·368	10	$\cdot128{}^{-\cdot000}_{+\cdot010}$	·903	3·30	·482	2,410	35·0
2·165	2·158	10	$\cdot128{}^{-\cdot000}_{+\cdot010}$	·819	3·00	·394	2,290	30·0
2·165	2·158	12	$\cdot104{}^{-\cdot000}_{+\cdot008}$	·673	2·46	·331	1,930	30·0
2·165	2·158	14	$\cdot080{}^{-\cdot000}_{+\cdot006}$	·524	1·92	·264	1,540	30·0
1·75	1·743	10	$\cdot128{}^{-\cdot000}_{+\cdot010}$	·652	2·38	·247	1,730	25·0
1·75	1·743	12	$\cdot104{}^{-\cdot000}_{+\cdot008}$	·538	1·97	·209	1,460	25·0
1·75	1·743	14	$\cdot080{}^{-\cdot000}_{+\cdot006}$	·420	1·54	·168	1,170	25·0
1·75	1·743	16	$\cdot064{}^{-\cdot000}_{+\cdot005}$	·339	1·24	·138	964	25·0
1·50	1·493	14	$\cdot080{}^{-\cdot000}_{+\cdot006}$	·357	1·30	·120	966	21·7
1·50	1·493	16	$\cdot064{}^{-\cdot000}_{+\cdot005}$	·289	1·05	·099	797	21·7
1·50	1·493	18	$\cdot048{}^{-\cdot000}_{+\cdot004}$	·219	·80	·077	620	21·7
1·10	1·095	14	$\cdot080{}^{-\cdot000}_{+\cdot006}$	·256	·93	·061	807	13·2

ADMIRALTY, AIR DEPARTMENT

T. 5. *January,* 1916. *Abridged.*

PROVISIONAL SPECIFICATION FOR 50-TON CARBON STEEL TUBES

The tubes are to be delivered in the normalized condition. (Where brazed or welded their strength will be reduced to the value given in the softening test.)

General Condition.—The tubes are to be straight, smooth, true to section, of uniform sectional thickness, and of equal diameter throughout, free from scale, dirt specks, longitudinal seaming, lamination, grooving or blistering, both internally and externally.

Accuracy of Dimensions.—The mean diameter of any tube is not to differ from the size specified by more than ± ·002 inch.

The mean thickness of any tube is not to be less than the specified gauge, and is not to exceed it by more than ·004 inch in tubes thinner than ·08 inch, or by more than 5 per cent. in thicker tubes. Any variation of thickness due to eccentricity of the bore is not to exceed 20 per cent. of the specified thickness.

Tests.—The tubes are to comply with the following mechanical tests.

Tension and Compression Tests.—A test piece consisting of a short length cut off the tube must give the following results, without further heat treatment or other manipulation.

Normalized Tubes :—

Ultimate stress in tension not less than 50 tons per square inch.
Yield point ,, ,, ,, 45 ,, ,,
Yield point in compression ,, 45 ,, ,,

Softening Test.—Additional tensile tests are to be made when required, to prove that the tubes will not soften unduly when brazed or otherwise heated. For this purpose the test piece, before it is tested, is to be heated to a full red heat at one end, while the other end remains cold, and is then to be allowed to cool. When tested the ultimate strength is not to be less than 30 tons per square inch, and the yield point is not to be less than 18 tons per square inch.

Flattening Test.—A sample of the tube of length equal to its diameter is to be flattened till the sides are not more than eight times the thickness of the metal apart. The samples must stand this treatment without cracking.

AIR BOARD

T. 6. *April*, 1917. *Abridged.*

SPECIFICATION FOR MILD STEEL TUBES

HARD DRAWN AND BLUED. CARBON ABOUT 0·15 PER CENT.

(*For Softened Tubes, see Specification T.* 26.)

If blued tubes are annealed, brazed, or welded, their strength will be reduced at the parts where they are so treated to 20 *tons per square inch ultimate and* 11 *tons per square inch yield point.*

Tubes under ¾ inch outside diameter are difficult to make to this Specification, and should, unless strength is essential, be ordered to Specification T. 26.

General Condition.—The tubes are to be smooth, true to section, of uniform sectional thickness, and of equal diameter throughout, free from scale, dirt, specks, longitudinal seaming, lamination, grooving or blistering, both internally and externally.

Accuracy of Form, Size and Straightness.—(*a*) Round tubes are to be accurately circular.

(**b**) The mean outside diameter (*i.e.* the mean between the maximum and minimum diameter) at any point is not to differ from the size ordered by more than ± ·004 inch (or for tubes over 2 inches diameter, by more than diameter/500). The mean inside diameter is not to be less than the correct outside diameter *minus* twice the maximum permissible thickness, nor greater than same *minus* twice the minimum permissible thickness.

(*c*) Oval tubes are to be of the correct form and dimensions within the tolerances specified in Schedule T. 11.

(*d*) No tube is to have a mean thickness less than the specified gauge, or exceeding it by more than ·004 inch, except tubes thicker than ·060 inch, for which the tolerance is to be 7½ per cent. of their thickness. (Tubes ordered to be 22-gauge thick are to be ·025 inch with a tolerance of +·004 inch to agree with the dimensions set out in Schedule T. 10.)

(*e*) At no point in a tube is the thickness to fall short of the nominal thickness by more than 10 per cent. or exceed it by more than 15 per cent.

(*f*) The tubes are to be as straight as possible, and in no part of the length is the departure from straightness to exceed one six-hundredth of the length of that part.

Tests.—(*a*) The tubes are to comply with the following mechanical tests.

(*b*) *Tension and Compression Tests.*—Test pieces consisting of short lengths cut off the tube, must give the following results, without further heat treatment or other manipulation :—

Ultimate strength in tension not less than 30 tons per square inch.
Yield point ,, ,, ,, 28 ,, ,,
Yield point in compression ,, ,, 28 ,, ,,

(*c*) *Flattening Test.*—The tube is to be flattened at the end, or at any point where defective material is suspected, by a few blows (not more than six), till the sides are not more than three times the thickness of the metal apart. The tubes must stand this treatment without cracking.

(*d*) *Crushing Test.*—Samples of the tube selected are to be crushed endwise until the outside diameter is increased in one zone by 25 per cent., or until one complete fold is formed. The samples must stand this treatment without cracking.

AIR BOARD

T. 10. *April*, 1917.

SCHEDULE OF STANDARD SIZES OF STEEL TUBES FOR AIRCRAFT

(THIS SCHEDULE DOES NOT APPLY TO AXLE TUBES)

17 GAUGE. Thickness 0·056 inch (Tolerance — 0, + 0·004 inch).

Nominal size. Outside diameter (Tolerance ± ·004 in.).	Minimum area of section.	Maximum weight per foot.	Moment of inertia I.	Modulus of section Z.	Nominal size. Outside diameter (Tolerance ± ·004 in.).	Minimum area of section.	Maximum weight per foot.	Moment of inertia I.	Modulus of section Z.
inches.	inches.^{2}	lbs.	inches.^{4}	inches.^{3}	inches.	inches.^{2}	lbs.	inches.^{4}	inches.^{3}
¼	·034	·122	·000174	·00139	1⅛	·188	·683	·0269	·0479
5/16	·045	·162	·000384	·00246	1¼	·210	·763	·0375	·0600
⅜	·056	·202	·000736	·00393	1⅜	·232	·843	·0506	·0735
7/16	·067	·242	·00125	·00568	1½	·254	·923	·0663	·0884
½	·078	·282	·00196	·00783	1⅝	·276	1·00	·0851	·105
⅝	·100	·362	·00409	·0131	1¾	·298	1·08	·107	·122
¾	·122	·442	·00740	·0197	1⅞	·320	1·16	·132	·141
⅞	·144	·522	·0121	·0278	2	·342	1·24	·162	·162
1	·166	·602	·0186	·0372	2⅛	·364	1·32	·195	·183

20 GAUGE. (Approx. 1 $^m/_m$.) Thickness ·036 inch (Tolerance — 0, + 0·004 inch).

¼	·0242	·0897	·000142	·00114	1⅛	·123	·464	·0183	·0325
5/16	·0312	·116	·000305	·00195	1¼	·137	·517	·0253	·0405
⅜	·0383	·143	·000557	·00297	1⅜	·151	·570	·0340	·0494
7/16	·0454	·170	·000923	·00422	1½	·166	·624	·0444	·0592
½	·0525	·197	·00142	·00568	1⅝	·180	·677	·0567	·0698
⅝	·0666	·250	·00290	·00928	1¾	·194	·731	·0712	·0814
¾	·0808	·303	·00516	·0138	1⅞	·208	·784	·0880	·0938
⅞	·0949	·357	·00836	·0191	2	·222	·837	·107	·107
1	·109	·410	·0127	·0254	2⅛	·236	·891	·129	·121

Approximately 22 GAUGE. Thickness ·025 inch (Tolerance — 0, + 0·004 inch).

¼	·0177	·0685	·000113	·000905	1⅛	·0864	·339	·0131	·0233
5/16	·0226	·0878	·000234	·00150	1¼	·0962	·378	·0181	·0289
⅜	·0275	·107	·000423	·00226	1⅜	·106	·417	·0242	·0351
7/16	·0324	·127	·000693	·00317	1½	·116	·456	·0315	·0420
½	·0373	·146	·00106	·00422	1⅝	·126	·494	·0403	·0495
⅝	·0471	·185	·00212	·00680	1¾	·135	·533	·0504	·0576
¾	·0569	·223	·00375	·00999	1⅞	·145	·572	·0622	·0663
⅞	·0668	·262	·00605	·0138	2	·155	·610	·0759	·0756
1	·0766	·301	·00911	·0182	2⅛	·165	·649	·0909	·0855

EXTRA TUBES FOR SOCKETS ONLY

Approximately 22 Gauge. Thickness 0·025 inch (Tolerance — 0 + 0·005 inch).

Outside diameter.	To take	Minimum area.	Maximum weight per foot.	Outside diameter.	To take	Minimum area.	Maximum weight per foot.
inches.	inches.	inches.²	lbs.	inches.	inches.	inches.²	lbs.
$\frac{9}{16}$	$\frac{1}{2}$	0·042	0·166	$1\frac{7}{16}$	$1\frac{3}{8}$	0·111	0·437
$\frac{11}{16}$	$\frac{5}{8}$	0·052	0·204	$1\frac{9}{16}$	$1\frac{1}{2}$	0·120	0·460
$\frac{13}{16}$	$\frac{3}{4}$	0·062	0·244	$1\frac{11}{16}$	$1\frac{5}{8}$	0·130	0·515
$\frac{15}{16}$	$\frac{7}{8}$	0·071	0·282	$1\frac{13}{16}$	$1\frac{3}{4}$	0·140	0·554
$1\frac{1}{16}$	1	0·081	0·321	$1\frac{15}{16}$	$1\frac{7}{8}$	0·150	0·592
$1\frac{3}{16}$	$1\frac{1}{8}$	0·091	0·360	$2\frac{1}{16}$	2	0·160	0·631
$1\frac{5}{16}$	$1\frac{1}{4}$	0·102	0·405				

NOTES

1. **The Areas, Moments of Inertia and Moduli (bending)** are calculated for tubes of the minimum thickness, *i.e.* 0·056 inch for 17 gauge tubes, 0·036 inch for 20 gauge tubes and 0·025 inch for 22 gauge tubes.

2. **The Weights per Foot** are calculated for tubes of the maximum thickness, *i.e.* 0·060 inch for 17 gauge tubes, 0·040 inch for 20 gauge tubes, and 0·029 inch for 22 gauge tubes. The weight is taken to be 490 lbs. per cubic foot.

3. **Telescopic Tubes.**—The 17 gauge tubes (omitting the $\frac{5}{16}$ and $\frac{7}{16}$ sizes) form a telescopic series, each fitting over the next size, $\frac{1}{8}$ inch smaller in diameter.

The 22 gauge tubes (with the socket sizes) also form a telescopic series, each fitting over the next size $\frac{1}{16}$ inch smaller in diameter.

4. **Tubes for Struts.**—Strut tubes are carefully straightened and then blued, but to avoid bending during handling they must not be ordered in lengths exceeding 10 to 12 feet.

5. **Tubes for General Purposes.**—These are treated in the same way as strut tubes. Long tubes for boundary edges and similar purposes can be made much straighter if built up of shorter pieces, and should be ordered in lengths not exceeding 10 feet.

6. **Tubes for Sockets.**—The 17 gauge and 22 gauge tubes may be used for sockets, *see* Clause 3, and also tubes of the special standard thicknesses given below. Additional sizes are provided in the table headed *Tubes for Sockets.* Tubes for use as sockets are more readily obtainable than strut tubes, because strut tubes have to be carefully straightened and more accurately heat treated; orders should therefore always state when tubes are intended only for sockets. Such tubes must not be used for carrying loads.

Tubes for use as sockets should not be ordered in lengths exceeding 4 feet; if the lengths they are going to be cut into are specified, the tubes will be supplied in multiples of these lengths; if not they will be supplied in random lengths of 4 feet and under.

7. **Special Standard Thicknesses.**

No. 1. Special $0{\cdot}090\,{}^{-0}_{+\cdot007}$ —fits over a tube $\frac{3}{16}$ inch smaller in O.D.

,, 2. ,, $0{\cdot}120\,{}^{-0}_{+\cdot009}$,, ,, $\frac{1}{4}$,, ,,

,, 3. ,, $0{\cdot}150\,{}^{-0}_{+\cdot011}$,, ,, $\frac{5}{16}$,, ,,

8. **Tolerances and Fits.**—The tolerances specified give an average clearance between tube and socket of about 0·008 inch.

9. **Orders.**—Orders for standard tubes must state if they are for *struts* or *sockets*.

If they are for *struts* the order must give for each tube :—

(*a*) Number of tubes and length of each, *or* Total length and lengths in which it is to be used.

(*b*) Outside diameters and thicknesses or gauges.

(*c*) Specification number, which defines the quality of the steel.

If the tubes are for *sockets* the order must give :—

(*d*) Total length required, and when practicable, the lengths the tubes are going to be cut into, so that exact multiples may be supplied.

(*e*) Outside diameters and sizes they are to take.

(*f*) Specification number.

Example.—Supply 1,000 strut tubes 5 feet 6 inches long, 1 inch diameter, 17 gauge, *or* Supply 6,500 feet of 1 inch diameter, 17 gauge tube for cutting into 5 feet 6 inch lengths. Specification T. 6.

Also supply 100 feet of socket tubing $1\frac{1}{8}$ inch O.D. to take 1 inch, for cutting into 9 inch pieces. Specification T. 26.

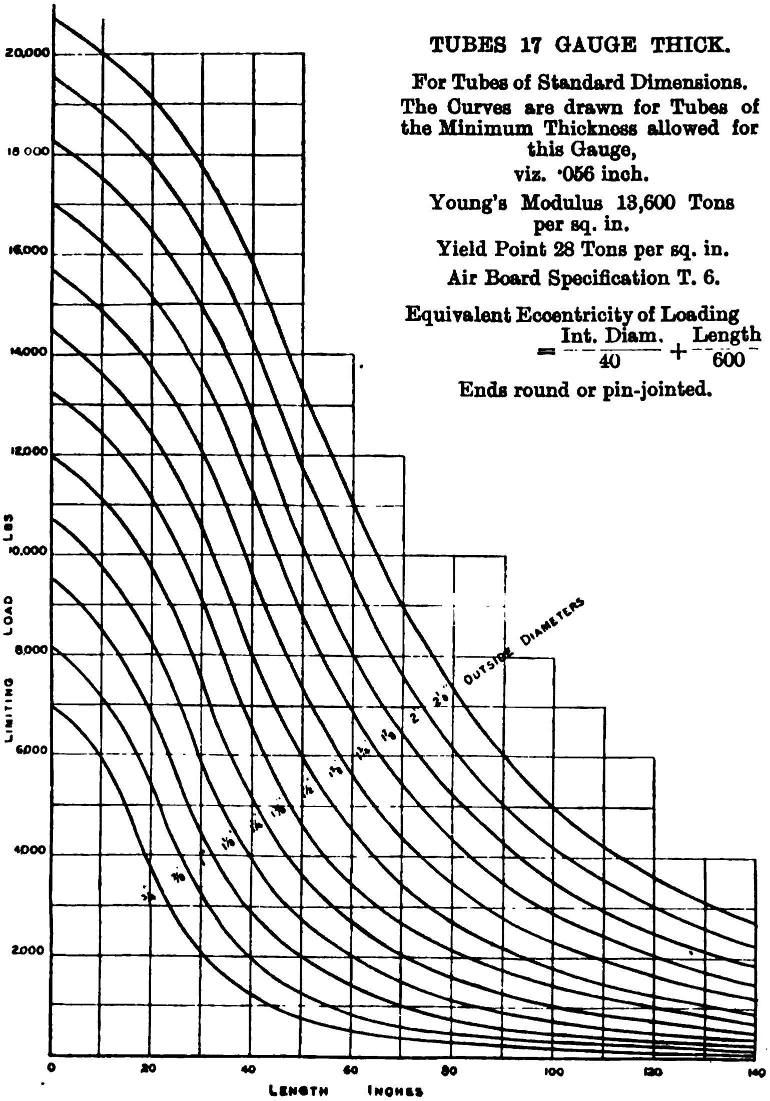
10. **Strength of Struts.**—The curves on pp 305–307 give the limiting loads for struts of any length made of any of the standard sizes of steel tubes.

Allowance is made for the maximum crookedness and eccentricity of bore which is permitted in the Air Board Specifications.

The *Limiting Load* given by the curves is the load which makes the maximum stress in the tube equal to the yield point, *i.e.* to 28 tons per square inch.

The *Collapsing Load* of a strut is either the same as, or a little higher than, the limiting load.

LIMITING LOADS FOR TUBULAR STEEL STRUTS



TUBES 17 GAUGE THICK.

For Tubes of Standard Dimensions.

The Curves are drawn for Tubes of the Minimum Thickness allowed for this Gauge, viz. ·056 inch.

Young's Modulus 13,600 Tons per sq. in.

Yield Point 28 Tons per sq. in.

Air Board Specification T. 6.

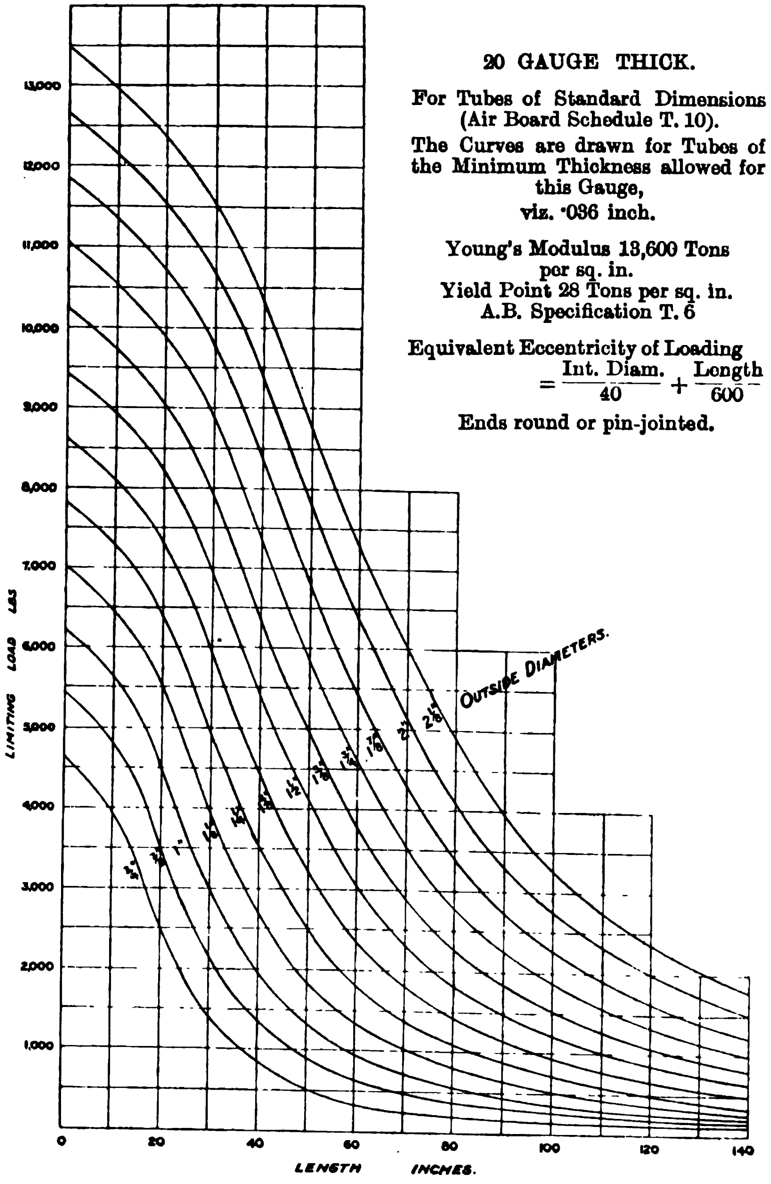
Equivalent Eccentricity of Loading

$$= \frac{\text{Int. Diam.}}{40} + \frac{\text{Length}}{600}$$

Ends round or pin-jointed.

FIG. 260.

LIMITING LOADS FOR TUBULAR STEEL STRUTS.



20 GAUGE THICK.

For Tubes of Standard Dimensions (Air Board Schedule T. 10).

The Curves are drawn for Tubes of the Minimum Thickness allowed for this Gauge, viz. ·036 inch.

Young's Modulus 13,600 Tons per sq. in.
Yield Point 28 Tons per sq. in.
A.B. Specification T. 6

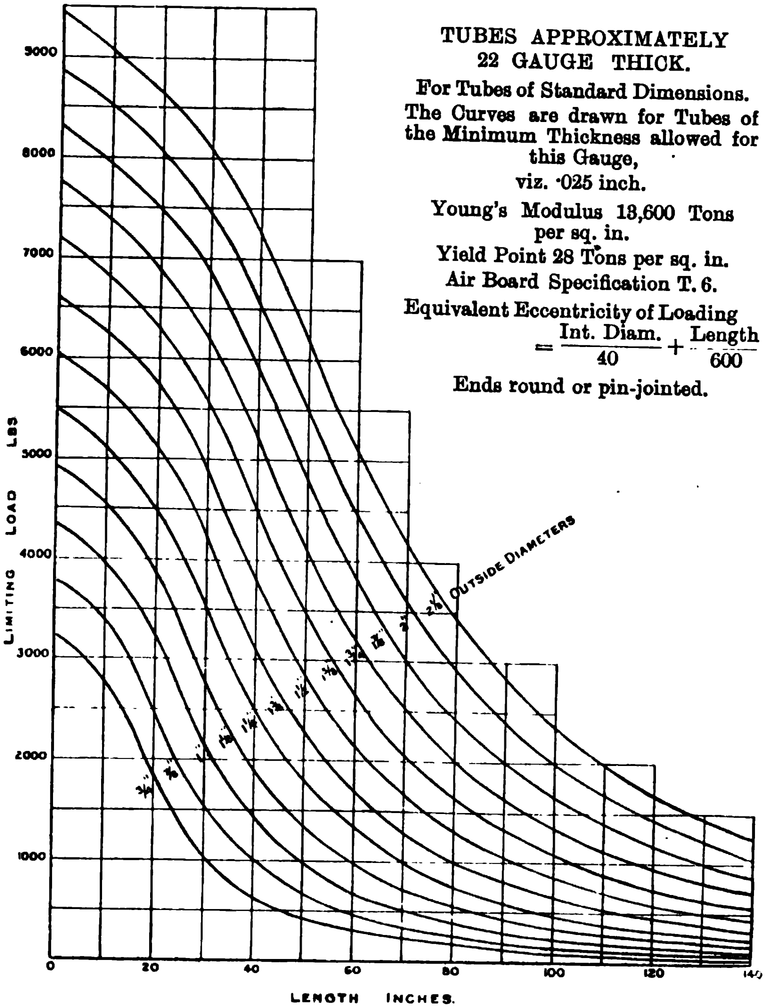
Equivalent Eccentricity of Loading

$$= \frac{\text{Int. Diam.}}{40} + \frac{\text{Length}}{600}$$

Ends round or pin-jointed.

FIG. 261.

LIMITING LOADS FOR TUBULAR STEEL STRUTS



TUBES APPROXIMATELY
22 GAUGE THICK.

For Tubes of Standard Dimensions.
The Curves are drawn for Tubes of the Minimum Thickness allowed for this Gauge,
viz. ·025 inch.

Young's Modulus 13,600 Tons per sq. in.
Yield Point 28 Tons per sq. in.
Air Board Specification T. 6.
Equivalent Eccentricity of Loading

$$= \frac{\text{Int. Diam.}}{40} + \frac{\text{Length}}{600}$$

Ends round or pin-jointed.

FIG. 262.

ADMIRALTY AIR DEPARTMENT

T. 10. *December*, 1916. *Supplement.*

SUPPLEMENTARY SCHEDULE OF LARGER STANDARD SIZES OF ROUND STEEL TUBES FOR AIRCRAFT

TUBES FOR STRUTS, STRAIGHTENED AND NORMALIZED

Outside diameter (Tolerance ± ·004 in.).	To take	Minimum area of section (*see* Note).	Maximum weight per foot (*see* Note).	Radius of gyration (*see* Note).	Outside diameter (Tolerance ± ·004 in.).	To take	Minimum area of section (*see* Note).	Maximum weight per foot (*see* Note).	Radius of gyration (*see* Note).

Approximately 11 Gauge, Thickness ·118 inch min. to ·122 inch max.

inches.	ins.	sq. ins.	lbs.	inches.	inches.	ins.	sq. ins.	lbs.	inches.
2¼	2	·790	2·779	·753	3¾	3½	1·347	4·735	1·283
2½	2¼	·883	3·105	·841	4	3¾	1·439	5·061	1·371
2¾	2½	·976	3·431	·930	4¼	4	1·532	5·387	1·460
3	2¾	1·068	3·757	1·018	4½	4¼	1·625	5·713	1·548
3¼	3	1·161	4·083	1·106	4¾	4½	1·717	6·039	1·636
3½	3¼	1·955	4·409	1·95	5	4¾	1·810	6·365	1·725

Approximately 17 Gauge, Thickness ·056 inch min. to ·060 inch max.

2¼	2⅛	·386	1·407	·774	3¾	3⅝	·650	2·369	1·304
2⅜	2¼	·408	1·487	·818	3⅞	3¾	·672	2·449	1·348
2½	2⅜	·430	1·567	·862	4	3⅞	·694	2·529	1·393
2⅝	2½	·452	1·647	·907	4⅛	4	·716	2·609	1·437
2¾	2⅝	·474	1·727	·951	4¼	4⅛	·738	2·689	1·481
2⅞	2¾	·496	1·808	·995	4⅜	4¼	·760	2·769	1·525
3	2⅞	·518	1·888	1·039	4½	4⅜	·782	2·850	1·569
3⅛	3	·540	1·968	1·083	4⅝	4½	·804	2·930	1·614
3¼	3⅛	·562	2·048	1·128	4¾	4⅝	·826	3·010	1·658
3⅜	3¼	·584	2·128	1·172	4⅞	4¾	·848	3·090	1·702
3½	3⅜	·606	2·208	1·216	5	4⅞	·870	3·170	1·746
3⅝	3½	·628	2·288	1·260					

Approximately 20 Gauge, Thickness about 1 mm., from ·036 inch min. to ·040 inch max.

2¼		·250	·946	·781	3		·335	1·267	1·046
2½		·278	1·053	·869	3¼		·363	1·374	1·135
2¾		·307	1·160	·958	3½		·392	1·481	1·223

Approximately 22 Gauge, Thickness ·025 inch min. to ·029 inch max.

2¼	$2\frac{3}{16}$	·174	·689	·785	2⅝	$2\frac{9}{16}$	·204	·806	·917
2⅜	$2\frac{5}{16}$	·184	·728	·829	2¾	$2\frac{11}{16}$	·214	·844	·962
2½	$2\frac{7}{16}$	·194	·767	·873	2⅞	$2\frac{13}{16}$	·224	·883	1·006
					3	$2\frac{15}{16}$	·233	·922	1·050

EXTRA TUBES FOR SOCKETS ONLY

Approximately 22 Gauge, Thickness ·025 inch min. to ·029 inch max.

inches.	ins.	sq. ins.	lbs.		inches.	ins.	sq. ins.	lbs.
$2\frac{3}{16}$	$2\frac{1}{8}$	·169	·670		$2\frac{11}{16}$	$2\frac{5}{8}$	·209	·825
$2\frac{5}{16}$	$2\frac{1}{4}$	·179	·709		$2\frac{13}{16}$	$2\frac{3}{4}$	·219	·864
$2\frac{7}{16}$	$2\frac{3}{8}$	·189	·747		$2\frac{15}{16}$	$2\frac{7}{8}$	·228	·902
$2\frac{9}{16}$	$2\frac{1}{2}$	·199	·786		$3\frac{1}{16}$	3	·238	·941

NOTE.—Sectional areas are calculated for tubes of minimum thickness, viz., ·118, ·056, ·036, and ·025 inch respectively.

Weights per foot are calculated for maximum thickness, viz., ·122, ·060, ·040, and ·029 inch respectively.

Radii of gyration are calculated for minimum diameter and thickness.

Sockets.—All the tubes except those of 20 gauge may be used as sockets for other standard tubes.

Struts.—Curve A (Fig. 263) gives the limiting stress, and hence the limiting load, for a tubular strut of any size and length, allowing for the crookedness and eccentricity of bore allowed in the Air Board Tube Specifications.

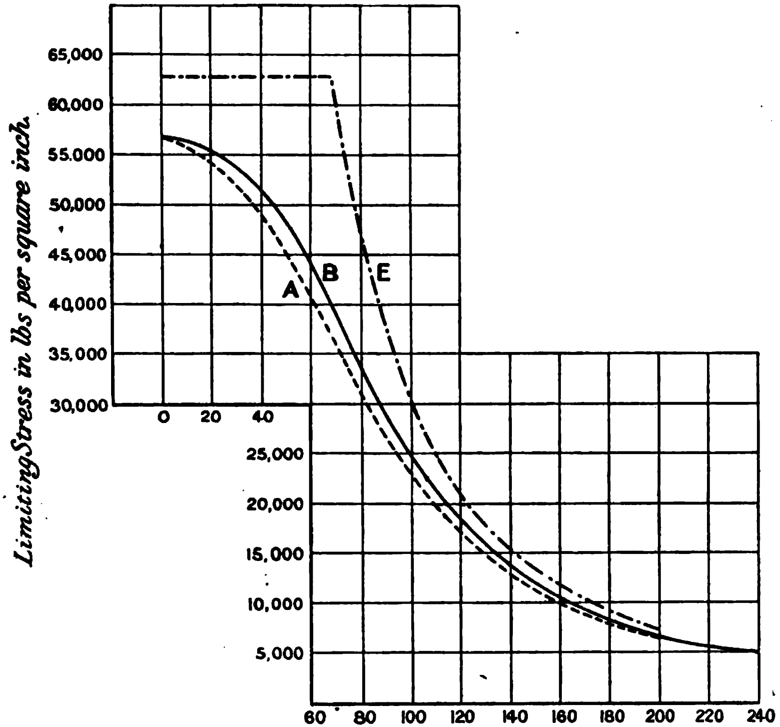


FIG. 263.

Specially Straight Tubes.—The tubes in this schedule can be supplied, when so ordered, with a deviation from straightness not exceeding one-half the amount allowed in the Tube Specifications. Curve B (Fig. 263) gives the limiting stress, and hence the limiting load, for struts of any size and length allowing for this reduced crookedness and the eccentricity of bore allowed in the Air Board Specifications.

AIR BOARD

T. 11. *April*, 1917.

OVAL STEEL TUBES FOR AIRCRAFT

(See Fig. 264.)

SCHEDULES OF STANDARD SIZES AND PROPERTIES OF SECTIONS

Outside diameters.		Minimum area of section. *See* Note 1.	Weight per ft. run of tube of maximum section. *See* Note 2.	Radii of gyration. *See* Note 1.		Moduli of section. *See* Note 1.		Limiting bending loads at 10 ins. overhang. *See* Note 3 and Fig. 265.	
B. Tolerance ± ½ per cent. of B.	D. Tolerance ± 1 per cent. of D.			About axis XX.	About axis YY.	About axis XX.	About axis YY.	W*x*.	W*y*.
17 Gauge. Thickness, ·056 inch to ·060 inch.									
inches.	inches.	sq. ins.	lbs.	inches.	inches.	inches.^{3}	inches.^{3}	lbs.	lbs.
1	·4	·120	·436	·298	·132	·0214	·0105	132	66
1¼	·5	·151	·556	·377	·170	·0344	·0176	216	110
1½	·6	·184	·673	·455	·208	·0513	·0268	321	168
1¾	·7	·217	·791	·534	·246	·0711	·0379	446	238
2	·8	·249	·910	·613	·284	·0943	·0509	592	319
2¼	·9	·282	1·028	·692	·322	·1204	·0659	756	412
2½	1·0	·314	1·147	·771	·361	·1503	·0827	943	518
20 Gauge. Thickness, ·036 inch to ·040 inch.									
1	·4	·079	·299	·304	·139	·0147	·0077	92	48
1¼	·5	·100	·378	·383	·177	·0236	·0127	148	80
1½	·6	·121	·457	·462	·216	·0345	·0189	217	118
1¾	·7	·142	·536	·541	·254	·0476	·0264	299	165
2	·8	·163	·616	·619	·292	·0628	·0350	394	220
2¼	·9	·184	·695	·698	·330	·0800	·0449	502	282
2½	1·0	·205	·774	·777	·368	·0994	·0562	623	352
Approximately 22 Gauge. Thickness, ·025 inch to ·029 inch.									
⅝	·25	·0341	·134	·189	·086	·0039	·0020	24	13
¾	·3	·0413	·163	·228	·105	·0058	·0031	36	19
1	·4	·0559	·220	·307	·143	·0106	·0058	67	36
1¼	·5	·0704	·278	·386	·181	·0169	·0094	106	59
1½	·6	·0849	·335	·465	·220	·0246	·0138	154	86
1¾	·7	·0996	·392	·544	·258	·0338	·0191	212	119
2	·8	·1140	·450	·623	·296	·0444	·0250	278	156

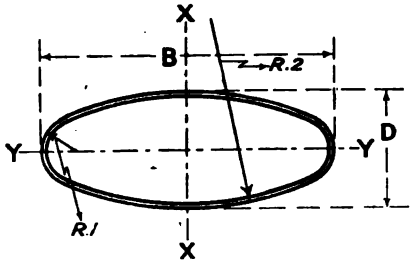


FIG. 264.—2½ : 1 Oval.

$B = 2{\cdot}5\,D$ or $D = {\cdot}4\,B$.
$R_1 = {\cdot}3\,D$ or ${\cdot}12\,B$. $R_2 = 1{\cdot}1\,B$ (approx.).
Area of Cross Section $= 1{\cdot}66(b+d)t$.
Radius of Gyration about $XX = 0{\cdot}317b$.
„ „ „ $YY = 0{\cdot}386d$.
Where $b = B - t$ and $d = D - t$ and $t =$ thickness.

NOTES.

1. **The Areas, Radii of Gyration and Moduli** are calculated for tubes of the minimum thickness, *i.e.* 0·056 inch for 17 gauge tubes, 0·036 inch for 20 gauge tubes, and 0·025 inch for 22 gauge tubes.

2. **The Weights per Foot** are calculated for tubes of the maximum thickness, *i.e.* 0·060 inch for 17 gauge tubes, 0·040 inch for 20 gauge tubes, and 0·029 inch for 22 gauge tubes.

Allowance has been made in each case for the tolerances.

3. **The Limiting Bending Loads** are the loads which, at an overhang of 10 inches (*see* Fig. 265), will produce a maximum fibre stress of 28 tons per square inch, which is the minimum yield stress allowed in Specification T. 6. The limiting loads are calculated for tubes of the minimum section.

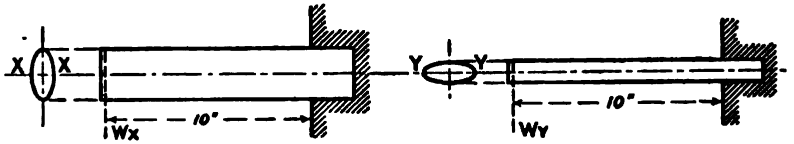


FIG. 265.

4. **Specification.**—Oval tubes should be ordered to Specification T. 1 or T. 6. They are straightened and normalized. They should not be ordered in lengths exceeding 10 feet.

Long tubes for trailing edges and similar purposes can be made straighter if built up of shorter pieces, and should be ordered in lengths not exceeding 10 feet.

5. **Sockets.**—Properly formed socket tubes for all standard ovals can be obtained from the tube makers. These socket tubes are $\frac{1}{10}$ inch larger in each outside diameter than the tubes they are to go over; they will therefore be an easy fit when made 20 gauge thick and a close fit when made 18 gauge thick. Unless specially ordered in stated lengths the socket tubes may be supplied in lengths of about 6 inches.

Satisfactory sockets can be made by pressing round tubes into the proper shape. The inside diameter of the round tube should be three-quarters of the larger diameter B of the oval it has to fit.

6. **Struts.**—Fig. 266 gives the limiting stress, and hence the limiting load, for an oval strut of any size and length, allowing for crookedness and eccentricity of bore permitted in the A.B. Tube Specifications.

LIMITING STRESS FOR OVAL TUBULAR STEEL STRUTS WITH ROUND OR PIN-JOINTED ENDS.

Yield point of steel . . . 28 tons per square inch
Young's Modulus, E . . . 13,600 „ „

L = Length of strut in inches.
K = Radius of gyration of cross section of oval about the axis YY (*see* Table, p. 310).

Limiting load = limiting stress × area of cross section.

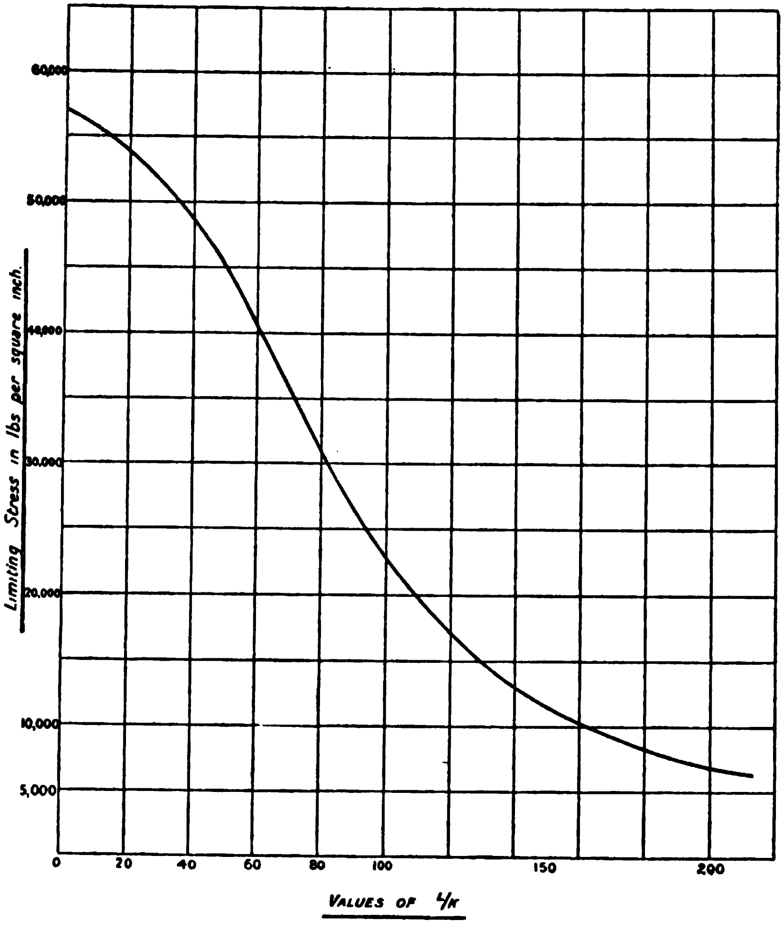


Fig. 266.

AIR BOARD

2 T. 12. *December*, 1917.

SCHEDULE OF STANDARD STEEL TUBES OF SPECIAL SECTIONS

These tubes are not suitable for parts which are highly stressed.

1. **Sizes and Thicknesses.**—The tubes are only made in the sizes and thicknesses shown on the accompanying drawings.

2. **Material and Strength.**—If solid drawn the tubes are to be made to Specification T. 26. If welded, the tubes are to be of equal quality, except exhaust pipes, which may be hot finished.

3. **Streamline Exhaust Pipes.**—Tubes of the form shown in Fig. 270 are only to be used as exhaust pipes; they are to be welded, not solid drawn. Larger sizes of the same form may, if required, be ordered, but large exhaust pipes are preferably made by hand and riveted up.

4. **Weights per Foot** of each tube, of the maximum thickness:—

Fig. 267	0·11 lb.
Fig. 268	0·13 lb.

Fig. 269 . . . .

—	¾-in.	⅞-in.	1-in.
Gauge. 17	lb. 0·520	lb. 0·623	lb. 0·725
20	0·358	0·426	0·494

Fig. 270	0·97 lb.
Fig. 271	0·26 lb.

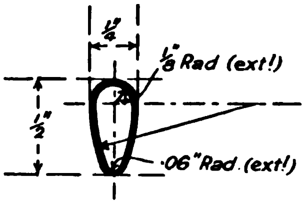


FIG. 267.

½" Aerofoil Edge and Sockets for $\frac{9}{16}$" Edge.

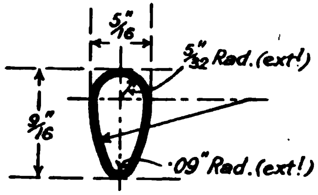


FIG. 268.

$\frac{9}{16}$" Aerofoil Edge and Sockets for ½" Edge.

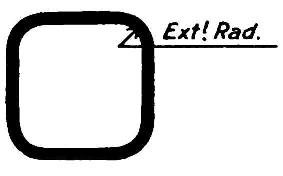


FIG. 269.

Squares. External Dimensions.

¼", ⅜", ½", ⅝". Rad. ·05"
17 and 20 gauge.
¾", ⅞", 1", 1⅛", 1¼", 1⅜", ½"
Rad. ·15"—17 and 20 gauge.
1⅝", 1¾", 1⅞", 2". Rad. ·15"
17 gauge and ·09" thick.

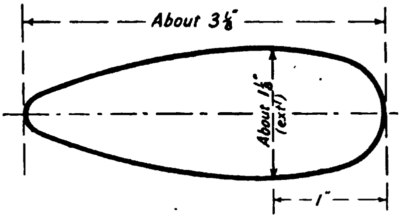


Fig. 270.

Streamline Exhaust Pipe. (20 G.)
Not to be formed from solid drawn tube.

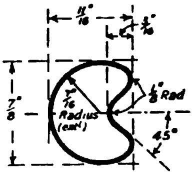


Fig. 271.

Cowl Ring Edge. (22 G.)

Figs. 267, 268, 269 and 271. Limits on dimensions ± ·01 inch.

AIR BOARD

T. 13. *June,* 1918.

SCHEDULE OF STANDARD SIZES OF STREAMLINE STEEL TUBES

(*For Aerofoil Edges and Exhaust Tubes, see* T. 12.)

STREAMLINE STEEL TUBES FOR AIRCRAFT

(See Fig. 272.)

Dimensions of Standard Sizes.

Standard streamline.	B. Maximum external diameter. Tolerance ± ½ %.	D. Minimum external diameter. Tolerance ± 1 %.	C. Centre of gravity below nose.	R 1.	R 2.	R 3.	R 4.	External diameter of equivalent round tube.
				External radii.				
Number.	inches.	inches.	inches.	inches.	inches.	inches.	inches.	inches.
1	3·4	1·175	1·628	7·637	2·35	0·540	0·242	2·463
2	3⅛	1·080	1·495	7·02	2·160	0·497	0·222	2·267
3 and 5	2¾	0·950	1·316	6·175	1·900	0·437	0·196	1·995
4 and 6	2	0·691	0·957	4·485	1·381	0·318	0·142	1·450

Properties of Standard Sizes.

Standard streamline.	Minimum area of section.	Weight per ft. maximum section.	Radii of gyration.		Modulus.		Moment of inertia.	
			About XX.	About YY.	About XX.	About YY.	About XX.	About YY.
Mean Thickness, $\cdot056\,^{+\cdot004}_{-\cdot0}$ inch.								
Number.	inches.^{2}	lbs.	inches.	inches.	inches.^{3}	inches.^{3}	inches.^{4}	inches.^{4}
1	·4235	1·54	1·045	0·426	·261	·131	·4624	·0769
2	·389	1·412	0·960	0·390	·2196	·1094	·3580	·0591
3	·341	1·239	0·842	0·340	·1685	·0829	·2415	·0394
4	·245	0·890	0·608	0·241	·0868	·0414	·0906	·0143
Mean Thickness, $\cdot036\,^{+\cdot004}_{-\cdot0}$ inch.								
5	·2213	0·8335	0·847	0·348	·1108	·0566	·1590	·0268
6	·1599	0·602	0·613	0·249	·0576	·0287	·0601	·0099

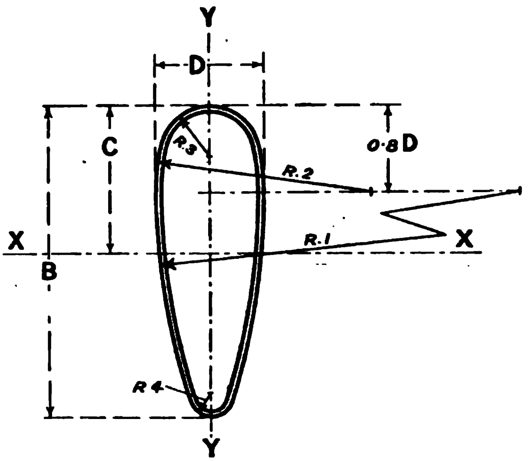


FIG. 272.

Round tubing properly faired weighs less, and has a less wind resistance per foot, than streamline steel tubing of equal strength; the use of streamline tubing is permissible only when greater stiffness is required about one axis than the other, or it effects considerable simplification of connections and details.

NOTES

1. **The Areas, Radii of Gyration, Moments of Inertia and Moduli** are calculated for tubes of the minimum thickness, *i.e.* 0·056 inch and 0·036 inch.

2. **The Weights per Foot** are calculated for tubes of the maximum thickness, *i.e.* 0·060 inch and 0·040 inch.

3. **Specification.**—Tubes should be ordered to Specification T. 1. They will be straightened and blued in accordance with that specification. They should not be ordered in lengths exceeding 10 feet. For special streamline tubes for aerofoil edges and exhausts, *see* Schedule T. 12.

4. **Sockets.**—(*a*) Properly formed socket tubes for all standard streamline tubes may be obtained from the tube makers. Standard sockets are made of ·056 inch tube, and are made an easy fit over the standard tubes. Unless specially ordered in stated lengths, the socket tubes may be supplied in lengths of about 6 inches.

(*b*) Socket tubes will be known as " Streamline socket SS. 1 " to fit streamline tube No. 1; " Streamline socket SS. 2 " to fit streamline tube No. 2, etc. Socket tubes will comply with Specification T. 26 unless specially ordered otherwise.

(*c*) Satisfactory sockets can be made by pressing round tubes into the proper shape. The inside diameter of the round tube should be slightly greater than the " equivalent round " of the streamline tube. (*See* last column in the upper table.)

AIR BOARD

T. 14. *September*, 1917. *Abridged.*

SPECIFICATION FOR TEMPERED CARBON STEEL AXLE TUBES

Note.—*Axle tubes are seriously weakened where drilled.*

Accuracy of Form, Size, and Straightness.—(*a*) The tubes are to be accurately circular.

(*b*) The mean outside diameter (*i.e.* the mean between the maximum and minimum diameter) at any section is not to differ from the size shown in column 2 of the Schedule by more than ±0·005 inch.

(*c*) The ends of the tubes for a distance of 14 inches from each end are to be rounded by pressure, so that no diameter exceeds the nominal diameter (column 1).

(*d*) No axle is to exceed the maximum specified weight (calculated by multiplying its length by the weight per foot given in column 5 of the Schedule.

(*e*) At no point in a tube is the thickness to fall short of the nominal thickness by more than 10 per cent. or exceed it by more than 20 per cent.

(*f*) The tubes are to be as straight as possible, and in no part of the length is the departure from straightness to exceed one three-hundredth of the length of that part.

Mechanical Tests.—(*a*) The tubes are to comply with the following tests.

(*b*) *Proof Load.*—Every axle is to be tested by having a proof bending moment applied to it near one end, and at least one axle in ten is to be tested in this way at both ends. The proof load is given in column 7 and the leverage L is given in column 8 of the Schedule at the end of this specification.[1]

(*c*) *Flattening Test.*—After heat treatment a short piece, one or two inches long, is to be cut from one axle in ten and bent into an oval section by pressure, so that its diameter is reduced when under load to 0·85 times the original diameter. The sample must stand this treatment without cracking. The sample may be nearly severed from the tube before heat treatment while the tube is soft.

(*d*) *Destruction Test.*—At least one sample tube from each batch heat treated together is to be bent to destruction. The sample must be of sufficient length to test, but need not be the full length of an axle.

(*e*) The sample is first to be tested under the proof load to verify that it will pass that test; if it fail, another sample must be selected which will pass it. The sample is then to be further bent in the same apparatus by increasing the lever arm till the gap at B reaches the value given in column 9 of the Schedule. It must stand this without cracking. If it fails, the batch it represents will be rejected, but may be re-heat-treated.

---

[1] See Chapter XX.

SCHEDULE

1. Nominal diameter.	2. True outside diameter (Tolerance ± ·005 in.).	3. Thickness.	4. Minimum area of section.	5. Maximum weight per foot.	6. Modulus of section Z.	7. Proof load.	8. Proof load leverage L.	9. Destruction test, minimum deflection without crack.
inches.	inches.	inches.	inches.^{2}	lbs.	inches.^{3}	lbs.	inches.	inches
2·375	2·362	·120 $^{-\cdot000}_{+\cdot010}$	0·850	3·12	0·460	2,080	34·65	1·5
2·165	2·152	·130 $^{-\cdot000}_{+\cdot010}$	0·831	3·05	0.399	2,080	30·0	1·5
2·165	2·152	·090 $^{-\cdot000}_{+\cdot007}$	0·587	2·15	0·292	1,527	30·0	1·5
1·75	1·743	·120 $^{-\cdot000}_{+\cdot010}$	0·615	2·25	0·235	1,475	25·0	2·0
1·75	1·743	·090 $^{-\cdot000}_{+\cdot007}$	0·469	1·75	0·185	1,160	25·0	2·0
1·75	1·743	·072 $^{-\cdot000}_{+\cdot006}$	0·380	1·40	0·153	960	25·0	2·0
1·75	1·743	·056 $^{-\cdot000}_{+\cdot004}$	0·298	1·08	0·122	766	25·0	2·0
1·50	1·493	·090 $^{-\cdot000}_{+\cdot007}$	0·399	1·46	0·133	960	21·7	2·5
1·50	1·493	·072 $^{-\cdot000}_{+\cdot006}$	0·323	1·18	0·110	795	21·7	2·5
1·50	1·493	·056 $^{-\cdot000}_{+\cdot004}$	0·254	0·932	0·0884	638	21·7	2·5
1·10	1·093	·090 $^{-\cdot000}_{+\cdot007}$	0·286	0·43	0·067	795	13·2	3·0

AIR BOARD

T. 21. *April*, 1917. *Abridged.*

SPECIFICATION FOR ANNEALED CARBON STEEL TUBES

Carbon about 0·3 per Cent.

(*For Blued Tubes, see Specification* T. 1.)

This specification should be used only for tubes which have to be much bent, or cold worked, and are heavily stressed.

Accuracy of Form, Size, and Straightness.—(*a*) Round tubes are to be accurately circular.

(*b*) The mean outside diameter (*i.e.* the mean between the maximum and minimum diameter) at any point is not to differ from the size ordered by more than ± 0·004 inch (or for tubes over 2 inches diameter by more than diameter/500) The mean inside diameter is not to be less than the correct outside diameter *minus* twice the maximum permissible thickness, nor greater than same *minus* twice the minimum permissible thickness.

(*c*) Oval tubes are to be of the correct form and dimensions within the tolerances specified in Schedule T. 11.

(*d*) No tube is to have a mean thickness less than the specified gauge, or exceeding it by more than ·004 inch except of tubes thicker than ·060 inch, for which the tolerance is to be 7½ per cent. of their thickness. (Tubes ordered to be 22-gauge thick are to be ·025 inch thick with a tolerance of +·004 inch, to agree with the dimensions set out in Schedule T. 10.)

(*e*) At no point in a tube is the thickness to fall short of the nominal thickness by more than 10 per cent. or exceed it by more than 15 per cent.

(*f*) The tubes are to be as straight as possible, and in no part of the length is the departure from straightness to exceed one six-hundredth of the length of that part.

Tests.—The tubes are to comply with the following mechanical tests.

(*a*) *Softening Test.*—A test piece consisting of a suitable length cut off the tube is to be heated to a full red heat at one end for at least five minutes while the other end remains cold, and is then to be allowed to cool freely in the air. The sample so treated must give the following results when tested in tension :—

Ultimate strength in tension not less than 28 tons per square inch.
Yield point ,, ,, 18 ,, ,,

(*b*) *Flattening Test.*—The tube is to be flattened at the end or at any point where defective material is suspected by a few blows (not more than six) till the sides are not more than the thickness of the metal apart. The tubes must stand this treatment without cracking.

(*c*) *Crushing Test.*—Samples of the tube selected are to be crushed endwise until the outside diameter is increased in one zone by 25 per cent. or until one complete fold is formed. The samples must stand this treatment without cracking.

AIR BOARD

T. 26. *April*, 1917. *Abridged.*

SPECIFICATION FOR MILD STEEL TUBES

HALF HARD[1] OR FULLY SOFTENED. CARBON ABOUT 0·15 PER CENT.

(*For Blued Tubes, see Specification* T. 6.)

Tubes ordered to this specification will be delivered in a half hard condition, unless ordered to be softened outright. They must not be relied on to have a strength greater than 20 *tons per square inch ultimate, and* 11 *tons per square inch yield point.*

This Specification should be used for :—

Socket Tubes.
Special Tubes. (T. 12).
Induction and Exhaust Pipes. (*Exhaust pipes which are not to be bent should be made of welded tube.*)
Tubes under ½ inch o.d., unless strength is essential. Small tubes are difficult to make to Specification T. 6.

Accuracy of Form, Size, and Straightness.—(*a*) Round tubes are to be accurately circular.

(*b*) The mean outside diameter (*i.e.* the mean between the maximum and minimum diameter) at any point is not to differ from the size ordered by more than ± 0·004 inch (or for tubes over 2 inches diameter by more than diameter/500). The mean inside diameter is not to be less than the correct outside diameter *minus* twice the maximum permissible thickness, nor greater than same *minus* twice the minimum permissible thickness.

(*c*) Oval tubes are to be of the correct form and dimensions within the tolerances specified in Schedule T. 11.

(*d*) Special tubes are to be of the correct form and dimensions within the tolerances specified in Schedule T. 12.

(*e*) No tube is to have a mean thickness less than the specified gauge, or exceeding it by more than ·004 inch except tubes thicker than ·060 inch, for which the tolerance is to be 7½ per cent. of their thickness. (Tubes ordered to be 22-gauge thick are to be between ·025 inch and ·029 inch thick, to agree with the dimensions set out in Schedule T. 10.)

(*f*) At no point in a tube is the thickness to fall short of the nominal thickness by more than 10 per cent. or exceed it by more than 15 per cent.

(*g*) The tubes are to be as straight as possible, and in no part of the length is the departure from straightness to exceed one six-hundredth of the length of that part.

Tests.—The tubes are to comply with the following mechanical tests.

(*a*) *Flattening Test.*—The tube is to be flattened at the end, or at any point where defective material is suspected, by a few blows (not more than six) till the sides are not more than three times the thickness of the metal apart if the tubes are half hard. Annealed tubes are to be flattened close. The tubes must stand this treatment without cracking.

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[1] Bright drawn tubes which will stand the flattening and crushing tests will be accepted as half hard.

(*b*) *Crushing Test.*—Samples of the tube are to be crushed endwise until the outside diameter is increased in one zone by 25 per cent., or until one complete fold is formed. The samples must stand this treatment without cracking.

(*c*) *Bending Test.*—Tubes of less than $\frac{3}{4}$ inch diameter, and all trailing edge tubes must stand being bent through 90°, round a radius not greater than 10 diameters without serious deformation in section or showing signs of failure.

ADMIRALTY, AIR DEPARTMENT

T. 4. *December*, 1915. *Abridged.*

SPECIFICATION FOR DURALUMIN TUBES

1. **Material and Manufacture.**—The tubes are to be solid-drawn seamless tubes made of duralumin alloy of specific gravity not exceeding 2·85.

The tubes are to be straight, smooth, cylindrical, of uniform quality and sectional thickness, and of equal diameter throughout, free from dirt specks, seaming, blisters, lamination, grooving, or other surface defects.

2. **Toleration.**—The diameter of the tubes is not to differ at any point from the size specified by more than :—

For tubes under 2 inches diameter	± ·003 inch.
,, of 2 inches and up to 3 inches	± ·005 inch.

The mean thickness of the tubes is nowhere to be less than the specified gauge or to exceed it by more than ·004 inch for tubes less than ·08 inch thick, or by more than 5 per cent. for thicker tubes. Any variations of thickness due to eccentricity of the bore is not to exceed ± 10 per cent. of the specified thickness.

3. **Tests.**—The tubes are to comply with the following mechanical tests.

Tensile Tests.—A test piece consisting of a short length cut off the tube must, without further heating or other manipulation, show an ultimate strength, yield point, and elongation not less than the following :—

——	Ultimate stress. Tons per sq. in.	Yield point. Tons per sq. in.	Elongation per cent. on 2 in.
Tubes up to ·06 in. thick	25	16	8
,, between ·06 and ·1 in. thick . .	25	16	10
,, over ·1 in. thick	25	16	12½

ADMIRALTY, AIR DEPARTMENT

T. 9. *June*, 1916. *Abridged.*

SPECIFICATION FOR ALUMINIUM TUBES

Material and Manufacture.—The tubes are to be solid-drawn seamless tubes made of aluminium assaying not less than 98 per cent.

The tubes are to be straight, smooth, cylindrical, of uniform quality and sectional thickness, and of equal diameter throughout, free from dirt specks, seaming, blisters, lamination, grooving, or other surface defects.

Toleration.—The diameter of the tubes is not to differ at any point from the size specified by more than 1 per cent.

The mean thickness of the tubes is nowhere to be less than the specified gauge or to exceed it by more than 5 per cent. Any variations of thickness due to eccentricity of the bore is not to exceed ± 10 per cent. of the specified thickness.

Tests.—The tubes are to comply with the following mechanical tests.

Tensile Tests.—A test piece consisting of a short length cut off the tube must, without further heating or other manipulation, show an ultimate strength and elongation not less than the following :—

Ultimate strength.	10 tons per square inch.
Elongation in 2 inches	2½ per cent.

Crushing Test.—A sample of the tube one and a half diameters long is to be crushed endways to half its length. The sample must stand this treatment without splitting or cracking.

AIR BOARD

W. 3. *November*, 1917. *Abridged.*

SPECIFICATION FOR STREAMLINE WIRES

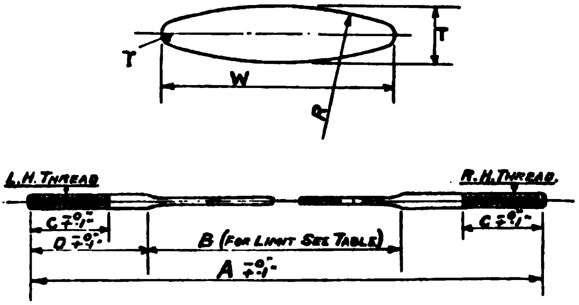

FIG. 273.

Size.	Thread.	W in.	T in.	R in.	r in.	Area sq. in.	Strength lbs.
4 B.A.	4 B.A.	·192	·048	·288	·011	·0071	1,050
2 B.A.	2 B.A.	·256	·064	·384	·014	·0125	1,900
$\frac{1}{4}$ in. B.S.F.	$\frac{1}{4}$ in. B.S.F.	·348	·087	·522	·019	·0234	3,450
$\frac{9}{32}$ in. B.S.F.	$\frac{9}{32}$ in. B.S.F.	·404	·101	·606	·022	·0313	4,650
$\frac{5}{16}$ in. B.S.F.	$\frac{5}{16}$ in. B.S.F.	·440	·110	·660	·024	·0376	5,700
$\frac{11}{32}$ in. B.S.F.	$\frac{11}{32}$ in. B.S.F.	·496	·124	·744	·027	·0475	7,150
$\frac{3}{8}$ in. B.S.F.	$\frac{3}{8}$ in. B.S.F.	·540	·135	·810	·030	·0563	8,500
$\frac{13}{32}$ in B.S.B.	$\frac{13}{32}$ in. B.S.F.	·596	·149	·894	·033	·0682	10,250
$\frac{7}{16}$ in. B.S.F.	$\frac{7}{16}$ in. B.S.F.	·636	·159	·954	·035	·0781	11,800
$\frac{15}{32}$ in. B.S.F.	$\frac{15}{32}$ in. B.S.F.	·692	·173	1·038	·038	·0921	13,800
$\frac{1}{2}$ in. B.S.F.	$\frac{1}{2}$ in. B.S.F.	·732	·183	1·098	·040	·1026	15,500
$\frac{9}{16}$ in. B.S.F.	$\frac{9}{16}$ in. B.S.F.	·836	·209	1·254	·045	·1354	20,200
$\frac{5}{8}$ in. B.S.F.	$\frac{5}{8}$ in. B.S.F.	·924	·231	1·386	·050	·1655	24,700

(*a*) The cross-sectional area of the wire is not to be less than that given in the table and is not to exceed it by more than 7½ per cent. The sectional area may be assumed to be 0·769 W×T where W and T are the diameters of the oval (*see* Fig. 273), or may be ascertained by weighing a length of the wire. The weight per foot run of wire of 0·1 square inch section is to be taken as 0·34 lb.

Tests.—The blanks are to comply with the following mechanical tests:—

(*a*) *Tensile Tests.*—A test piece, cut off one end of the wire selected for testing so as to include the round end, must show an ultimate stress not less than the value given in the table accompanying this specification. The test piece is to be held at one end by the round part so that the shoulder between the round and oval parts is included in the part under stress.

(*b*) *Tensile Tests on Screwed End.*—The round end of the blank selected for testing is to be cut off, and is to be screwed with a pair of screwing dies so that it is a good fit in an approved gauge. It is to be fitted with fork ends or with nuts, by which the load can be applied, and is then to be subjected to a tensile test, which must give an ultimate load not less than that obtained from the oval section of the blank from which it was cut. Where there are difficulties in screwing the pieces, temporary permission may be granted to substitute a test on unscrewed pieces for some of these tests.

(c) *Bending Test.*—Two samples cut from the wire selected for testing are to be subjected to the following bending test. One of the samples is to be cut from the end of the wire, so as to include the shoulder, and the test is to be applied so that the sample is bent at the shoulder, the sample being held by the oval part. The other sample is to be bent in the oval part. The sample is to be fixed in a vice which has the inner edges of the jaws rounded to a radius equal to three times the thickness of the wire given in the table. The projecting end of the wire is then to be bent at right angles to the fixed part, first to one side, then to the other, for a number of times till it breaks. Both samples must stand without breaking the number of bends specified in tests on swaged wires, each through 180°. The first bend through 90° is not counted.

AIR BOARD

W. 8. *November*, 1917. *Abridged.*

SPECIFICATION FOR SWAGED WIRES

Size.	Thread.	Area.	Strength.
		sq. in.	lbs.
4 B.A.	4 B.A.	·0085	1,050
2 B.A.	2 B.A.	·0129	1,900
$\frac{1}{4}$ in. B.S.F.	$\frac{1}{4}$ in. B.S.F.	·0230	3,450
$\frac{9}{32}$ in. B.S.F.	$\frac{9}{32}$ in. B.S.F.	·0337	4,650
$\frac{5}{16}$ in. B.S.F.	$\frac{5}{16}$ in. B.S.F.	·0390	5,700
$\frac{3}{8}$ in. B.S.F.	$\frac{3}{8}$ in. B.S.F.	·0590	8,500
$\frac{7}{16}$ in. B.S.F.	$\frac{7}{16}$ in. B.S.F.	·0835	11,800
$\frac{1}{2}$ in. B.S.F.	$\frac{1}{2}$ in. B.S.F.	·1120	15,500

Mechanical Tests.—The following tests are to be carried out:—

(*a*) *Tensile Test.*—A sample is to be cut from every coil, and when tested in tension must give the following results:—

For wire rods of sizes from 4BA to $\frac{11}{32}$ inch inclusive.	Ult. stress not less than 55 tons per sq. inch, or more than 65 tons per sq. inch.
For wire rods of sizes from $\frac{3}{8}$ inch to $\frac{1}{2}$ inch, inclusive.	Ult. stress not less than 52 tons per sq. inch, or more than 62 tons per sq. inch.

(*b*) *Bending Test.*—A sample is to be cut from every coil and subjected to the following bending test:—

The sample is to be fixed in a vice, or between dies, of which the inner edges are rounded to a radius equal to three times the diameter of the wire. The projecting end is then to be bent at right angles to the fixed part, and is then to be bent backwards and forwards through an angle of 180° till it breaks. The wire must stand without breaking the following number of bends through 180° (the first bend through 90° is not counted):—

For wires of sizes—

4BA and 2BA	Minimum number of bends, 6.
$\frac{1}{4}$ in., $\frac{9}{32}$ in., and $\frac{5}{16}$ in.	,, ,, ,, 5.
$\frac{11}{32}$ in., $\frac{3}{8}$ in., and $\frac{13}{32}$ in.	,, ,, ,, 4.
$\frac{7}{16}$ in. and upwards	,, ,, ,, 3.

AIR BOARD

2 W 2. *November*, 1917. *Abridged.*

SPECIFICATION FOR FLEXIBLE STEEL WIRE ROPE

SCHEDULE

Item.	Minimum breaking strength.	Maximum diameter.	Construction.	Weight of 100 feet.
		EXTRA FLEXIBLE ROPES.		
	cwts.	inch.		lbs.
0	5	·075	4 × 7	1·0
1	10	·115	4 × 19	2·0
2	15	·137	„	3·2
3	20	·150	7 × 19	3·8
51	25	·168	7 × 19	5·0
52	35	·195	„	6·4
53	45	·228	„	9·0
54	60	·262	„	11·7
55	70	·270	„	12·4
56	80	·305	„	15·1
57	100	·349	„	19·6
58	120	·378	7 × 27	22·5
59	140	·388	„	25·5
60	160	·418	7 × 37	28·9
		STANDARD STRAINING CORDS.		
41	10	·085	1 × 19	1·6
42	15	·105	„	2·3
43	20	·125	„	3·4
44	25	·143	„	4·3
45	35	·161	1 × 37	5·5
46	45	·189	„	7·5
47	60	·210	„	9·3
48	75	·238	„	11·9
49	90	·259	„	14·1

Tests on Wires.—The following tests are to be carried out:—

(*a*) *Tensile Strength.*—A sample from each coil of wire is to be tested in tension. The ultimate tensile strength must be not more than 135 tons per square inch, and not less than a value to be specified by the manufacturer of the wire rope.

(*b*) *Torsion Test.*—A sample from each coil of wire is to be tested in torsion. The wire is to be twisted in a torsion machine until it breaks. It must stand before breaking a number of turns not less than the number given by the formula :—

Number of turns=27·5 per length of 100*d* for all wires up to and including ·018 inch diameter.

Number of turns=20 per length of 100*d* for larger wires.

Where *d*=the diameter of the wire in inches.

(*c*) All wires showing a tendency to brittleness are to be rejected.

Tests on Ropes.—The following tests on the rope are to be carried out:—

(*a*) *A Tensile Test* is to be made on a sample cut from each piece of rope and must give at least the specified breaking load. The length of the sample tested is to be not less than six times the circumference of the rope, and in no case less than 15 inches in the clear between the points of security. The load is to be gradually applied till the sample breaks.

(*b*) Should the test pieces fail to reach the specified load, the rope represented by the test piece is to be rejected. Should, however, any wire break before 50 per cent. of the specified load has been applied, a re-test may be taken. No further re-test will be allowed, and should the re-test fail at any load less than that specified, the rope represented by the test piece will be rejected.

(*c*) *A Bending Test* is to be made on a sample cut from each piece of rope. Each sample must be bent once round its own part and straightened again at least 20 times in succession without any of the wires breaking or the rope opening up.

AIR BOARD

W. 1. *July*, 1917. *Abridged.*

SPECIFICATION FOR HIGH TENSILE STEEL WIRE

Tests.—(*a*) *A tensile test* is to be made on a sample cut from each length of wire, and must give a breaking stress between the limits specified in the table at the end of this specification.

(*b*) *A torsion test* is to be made on a sample cut from each length of wire. The wire is to be twisted in a torsion machine till it breaks. It must stand before breaking a number of turns given by the formula :—

Number of turns $= 20$ per length of $100d$,
where $d =$ diameter of the wire in inches.

(*c*) *A bending test* is to be made on a sample cut from each length of wire. The sample is to be fixed in a vice which has the inner edges of the jaws rounded to a radius of 5 mm. The projecting end of the wire is then to be bent at right angles to the fixed part, first to one side, then to the other, for a number of times till it breaks. The wire must stand before breaking the number of such reversals, each through 180°, stated in the table at the end of this specification. The first bend through 90° is not to be counted.

TABLE.

S.W.G.	Breaking Stress.		Number of bends through 180°	S.W.G.	Breaking Stress.		Number of bends through 180°
	Minimum T. per sq. in.	Maximum T. per sq. in.			Minimum T. per sq. in.	Maximum T. per sq. in.	
8	80	90	3	14	95	115	7
9	80	90	3	15	95	115	10
10	85	95	3½	16	100	120	14
11	90	100	4	17	100	120	17
12	90	105	5	18	100	120	20
13	90	110	6	19	100	120	24

BRITISH ENGINEERING STANDARDS ASSOCIATION

V. 4. *November,* 1918. *Abridged.*

BRITISH STANDARD SPECIFICATION FOR AIRCRAFT MATERIAL

ASH

(Issued for and under the authority of the Ministry of Munitions, Department of Aircraft Production.)

1. **Quality.**—The timber shall be butt lengths of Ash (*Fraxinus excelsior*, L.) of European growth. It shall not have more than 16 rings per inch, except in the case of approved small trees.

NOTE.—Wide ringed ash is preferable to narrow ringed material. Timber having more than 16 rings per inch may be accepted provided that its toughness and strength have been found satisfactory by mechanical tests. Small trees, well-grown, having more than 16 rings per inch frequently provide satisfactory material. The timber should preferably be felled between the months of November and March.

2. **Weight per Cubic Foot.**—The weight per cubic foot of the timber shall be not less than 38 lbs. when the moisture content is 15 per cent.

3. **Seasoning.**—(*a*) *For ordinary use.* The timber shall be naturally seasoned if possible. If kiln-dried timber is used the following conditions shall be complied with :—

(i) The timber shall be dried in an approved kiln.
(ii) Autograph records shall be kept showing the temperature and humidity conditions during the process.
(iii) The maximum temperature shall not exceed 125° F.
(iv) The moisture content at the conclusion of the process shall be as follows :

Summer (Mid. April to Mid. Oct.) not more than 20 per cent. and
not less than 15 ,,
Winter (Mid. Oct. to Mid. April) not more than 18 ,, and
not less than 14 ,,

(*b*) *For Steam Bending.*—Timber which is to be subjected to straightening or bending by steam treatment shall preferably not be seasoned, and shall not be kiln dried.

4. **Dryness of Timber.**—(*a*) *Seasoned Timber.* The timber shall not be reduced to its final shape and size until the moisture has been reduced to 16 per cent. or less.

(*b*) *Unseasoned Timber for Bending.*—Timber that is to be subjected to straightening or bending by steam treatment shall contain at least 25 per cent. of moisture, which should preferably be the original sap moisture.

NOTE.—The process of steaming and subsequent drying takes the place of artificial seasoning.

5. **Dryness Tests.**—(*a*) *Selection of Test Pieces.* Samples shall be cut from the timber, whether unseasoned or seasoned, and shall be tested for moisture content.

6. **Conversion.**—Parts cut from planks should, as far as possible, be cut following the grain; and may subsequently be straightened by steam treatment. Planks in which the grain is too curved to allow of straightening should not be rejected. Irregular and curved parts should, as far as possible, be obtained from such timber with naturally curved grain, so that the grain of the wood is continuous throughout the length of the part.

7. **Straightness of Grain.**—The maximum inclination of the grain to the length of the part shall not exceed one in ten.

8. **Freedom from Defects.**—The timber shall contain no deleterious knots or shakes, it shall not contain curls, burrs, rammy figure, caney grain, signs of prolonged weathering, black heart, foxiness or other forms of rot. At least half of each annual ring shall consist of autumn grown wood of close texture.

NOTE.—Material containing abnormally large and open pores, or pores filled with brilliant yellow powdery secretion may be expected to be brittle. As a rule the lighter coloured material is the better quality.

9. **Mechanical Tests.**—(*a*) *Compression Tests.* A test piece 1 inch by 1 inch and 2 inches long shall give an ultimate strength not less than the value shown in the following table, depending on the moisture in the test piece :—

Percentage moisture content of sample.	Minimum ultimate strength.
	lbs. per sq. in.
13	5,600
14	5,300
15	5,000
16	4,700
17	4,400

(*b*) *Notched Bar Tests for Brittleness.*—A test piece of the size and shape shown in Chapter XX. when tested in an impact testing machine shall not absorb less than 10 ft.-lbs. The sides of the test piece shall be cut radially and tangentially and the blow shall be applied in the tangential direction.

NOTE.—Where a suitable machine is not available the test piece shall be broken by a blow with a hammer. The fracture shall show a satisfactory amount of fibrous splinter.

10. **Bending Test.**—*Selection of Test Piece.* A lath of the material, ¼ inch thick, shall withstand the following test :—

The lath shall be bent round a semicircle of 1½ feet in diameter without showing signs of fracture.

11. **Steam Bending.**—(*a*) *Temperature of Steam.* During steam treatment the maximum temperature of the steam shall not exceed 220° F. (2 lbs. per square inch) and the timber shall remain in the steam no longer than is necessary to secure the necessary degree of pliability.

(*b*) *Curvature.*—The timber shall not be bent to a curve of less radius than 12 times its radial width, or straightened from a curve of less radius than 40 times its radial width.

Curved pieces of smaller radius are undesirable, but, if necessary, such pieces shall be produced of laminated timber or by bending the timber in a master strap clamp.

BRITISH ENGINEERING STANDARDS ASSOCIATION

V. 6. 1918. *Abridged.*

BRITISH STANDARD SPECIFICATION FOR AIRCRAFT MATERIAL

MAHOGANY

(West African)

(*Issued for and under the authority of the Ministry of Munitions, Department of Aircraft Production.*)

1. **Quality.**—The timber shall be mahogany (*Khaya sp.*) having the same general characteristics as first quality mahogany imported from Grand Bassam or Benin.

2. **Weight per Cubic Foot.**—The weight per cubic foot of the timber shall not be less than 34 lbs. when the moisture content is 15 per cent.

3. **Seasoning.**—The timber shall be naturally seasoned if possible. If kiln-dried timber is used the following conditions shall be complied with :—

(i) The timber shall be dried in an approved kiln.
(ii) Autographic records shall be kept showing the temperature and humidity conditions during the process.
(iii) The maximum temperature shall not exceed 125° F.
(iv) The moisture content at the conclusion of the process shall be as follows :—

Summer (Mid. April to Mid. Oct.)	not more than	16	per cent.	and
	not less than	12	,,	
Winter (Mid. Oct. to Mid. April)	not more than	14	,,	and
	not less than	10	,,	

4. **Dryness of Timber.**—The timber shall not be reduced to its final shape and size until the moisture has been reduced to 14 per cent. or less, nor shall any parts of the wood be glued together until this degree of dryness is attained.

5. **Dryness Tests.**—*Selection of Test Pieces.* Samples shall be cut by the contractor from the timber, whether unseasoned or seasoned, and shall be tested for moisture content as and when required.

6. **Freedom from Defects.**—The wood in the finished parts shall contain no deleterious wormholes, sapwood, gum veins, spongy heart or other forms of rot, knots, shakes, cross shake, fiddle-back figure, roe figure and other forms of cross and curly grain.

7. **Mechanical Tests.**—(*a*) *Selection of Test Pieces.*—Samples shall be cut by the contractor from the seasoned timber, and a number of test pieces shall be prepared from these samples for the following tests.

(*b*) *Notched Bar Tests for Brittleness.*—A test piece of the size and shape shown in Chapter XX., when tested in an imp [illegible] hine of the type illustrated in the same chapter, shall not absorb [illegible]. The sides of the test piece shall be cut radially and tangentially and the blow shall be applied in the tangential direction.

Note.—Where a suitable machine is not available the test piece shall be broken by a blow with a hammer. The fracture shall show a satisfactory fibrous splinter.

(c) *Bending Test for Modulus of Rupture.*—A test piece cut parallel to the grain having a square section 1 inch by 1 inch, when tested shall give the following result:—

Modulus of Rupture . . .	Not less than 11,000 lbs. per square inch.

BRITISH ENGINEERING STANDARDS ASSOCIATION

V. 6. 1918. *Abridged.*

BRITISH STANDARD SPECIFICATION FOR AIRCRAFT MATERIAL

MAHOGANY

(West African)

(*Issued for and under the authority of the Ministry of Munitions, Department of Aircraft Production.*)

1. **Quality.**—The timber shall be mahogany (*Khaya sp.*) having the same general characteristics as first quality mahogany imported from Grand Bassam or Benin.

2. **Weight per Cubic Foot.**—The weight per cubic foot of the timber shall not be less than 34 lbs. when the moisture content is 15 per cent.

3. **Seasoning.**—The timber shall be naturally seasoned if possible. If kiln-dried timber is used the following conditions shall be complied with :—

(i) The timber shall be dried in an approved kiln.
(ii) Autographic records shall be kept showing the temperature and humidity conditions during the process.
(iii) The maximum temperature shall not exceed 125° F.
(iv) The moisture content at the conclusion of the process shall be as follows :—

Summer (Mid. April to Mid. Oct.) not more than 16 per cent. and
not less than 12 ,,
Winter (Mid. Oct. to Mid. April) not more than 14 ,, and
not less than 10 ,,

4. **Dryness of Timber.**—The timber shall not be reduced to its final shape and size until the moisture has been reduced to 14 per cent. or less, nor shall any parts of the wood be glued together until this degree of dryness is attained.

5. **Dryness Tests.**—*Selection of Test Pieces.* Samples shall be cut by the contractor from the timber, whether unseasoned or seasoned, and shall be tested for moisture content as and when required.

6. **Freedom from Defects.**—The wood in the finished parts shall contain no deleterious wormholes, sapwood, gum veins, spongy heart or other forms of rot, knots, shakes, cross shake, fiddle-back figure, roe figure and other forms of cross and curly grain.

7. **Mechanical Tests.**—(*a*) *Selection of Test Pieces.*—Samples shall be cut by the contractor from the seasoned timber, and a number of test pieces shall be prepared from these samples for the following tests.

(*b*) *Notched Bar Tests for Brittleness.*—A test piece of the size and shape shown in Chapter XX., when tested in an impact testing machine of the type illustrated in the same chapter, shall not absorb less than 6 ft.-lbs. The sides of the test piece shall be cut radially and tangentially and the blow shall be applied in the tangential direction.

Note.—Where a suitable machine is not available the test piece shall be broken by a blow with a hammer. The fracture shall show a satisfactory fibrous splinter.

(c) *Bending Test for Modulus of Rupture.*—A test piece cut parallel to the grain having a square section 1 inch by 1 inch, when tested shall give the following result:—

Modulus of Rupture . . .	Not less than 11,000 lbs. per square inch.

AIR BOARD

2 V. 1. *July,* 1918.

SPECIFICATION FOR SILVER SPRUCE AND APPROVED SUBSTITUTES

1. **Quality.**—(*a*) The timber is to be the first quality of any of the following woods, viz. :—Silver Spruce (*Picea sitchensis*, Carr.), Quebec Spruce (*Picea alba* and *Picea nigra*, Link), White Sea White Deal (*Picea excelsa*, Link), White Sea Red Deal (*Pinus sylvestris*, L.), West Virginia Spruce (*Picea rubeus*, Sargent), and North Carolina Spruce, when this is the same wood as West Virginia Spruce, but grown in North Carolina, Port Orford Cedar (*Chamæcyparis Lawsoniana*, Murr.), New Zealand Kauri (*Agathis* [*Dammara*] *Australis*, Salisb.), Canadian White Pine (*Pinus Strobus*, L.), Oregon Pine (*Pseudotsuga Douglasii*, Carr.).

(*b*) It should be butt lengths, slow grown (not less than six annual rings per inch) and preferably rift-sawn.

(*c*) *Grades.*—All approved timbers complying with Grade A tests are to be classed as Grade A.

All approved timbers complying with Grade B tests are to be classed as Grade B.

2. **Freedom from Defects.**—The timber is to be clean, straight-grained, free from dote, deleterious shakes, knots, and resin pockets. It is to be cut parallel to the grain (as determined by the Splitting Test specified in Clause 5).

3. **Seasoning.**—The timber is to be thoroughly seasoned naturally, if possible ; but, if not, may be conditioned after cutting into overhead sizes. The conditioning is to be carried out in a well-ventilated place at a temperature not exceeding 85° Fahr. The moisture at the end of the process is to be between 14 and 17 per cent., calculated on the weight of the dry wood. An autographic record is to be kept, showing the temperature and humidity during the process.

4. **Weight.**—The weight is not to be less than 25 lbs. per cubic foot when it contains 15 per cent. of moisture.

5. **Mechanical Tests.**—(*a*) The timber is to comply with the following tests :—

(*b*) Suitable methods and apparatus for carrying out the tests are described in Chapter XX.

(*c*) *Compression Test.*—Test pieces turned parallel to the grain and to the form and dimensions shown in Chapter XX. (or alternatively cut 1 inch square and between 2 and 3 inches long), when tested in compression must give an ultimate strength not less than :—

Ultimate strength, Grade A	. . .	5,000 lbs. per square inch.
,, ,, Grade B	. . .	4,000 ,, ,,

The load is to be applied at a rate between 3,000 and 6,000 lbs. per minute.

(*d*) *Dryness Correction.*—These compression tests are for timber containing 15 per cent. of moisture. If the timber when tested contains more moisture the specified strengths are to be reduced by 230 lbs. per square inch for every 1 per cent. increase of moisture above 15 per cent., and if the timber contains less moisture the specified strengths are to be increased at the same rate. The percentage of moisture is calculated on the weight of the dried sample.

(*e*) *Bending Test.*—Test pieces, 40 inches long by 2 inches deep by 1 inch wide, cut from samples parallel to the grain, are to be loaded so as to produce in the middle part a pure bending moment (without shear). The deflection of the part subject to simple bending is to be measured and the value of Young's Modulus calculated from it.

The results must not be less than—

Young's Modulus Grade A . .	1,600,000	lbs. per square inch.
,, ,, Grade B . .	1,200,000	,, ,,

(*f*) *Splitting Test.*—Short samples, say 4 to 6 inches long, are to be split in two planes, one tangential and one radial. The split faces will show the true direction of the grain, which must not be inclined to the length of the plank by more than 1 in 20 for Grades A and B.

(*g*) *Brittleness Test.*—A notched test piece of the dimensions shown in Chapter XX., when broken in a notched bar testing machine must not absorb less than :—

For Grade A timber	8 ft.-lbs.
,, B ,, ...	4 ,,

When a suitable testing machine is not available the test piece is to be broken by a blow with a hammer, and the fracture must show a satisfactory amount of fibrous splinter.

BRITISH ENGINEERING STANDARDS ASSOCIATION

V. 5. *November*, 1918. *Abridged.*

BRITISH STANDARD SPECIFICATION FOR AIRCRAFT MATERIAL

WALNUT

(*Issued for and under the authority of the Ministry of Munitions, Department of Aircraft Production.*)

1. **Quality.**—The timber shall be butt lengths of walnut (*Juglans*) of any of the following species, viz. : American Black Walnut (*Juglans nigra*, L.), European or Asiatic Walnut (*Juglans regia*, L.) or Kurumi Walnut (*Juglans sieboldiana*, Maxim) from Japan.

African walnut (*Khaya-sp*) and " Satin walnut " (properly called *Red Gum*) are not included in this specification.

Note.—The timber shall preferably be cut from trees that have been felled during the leafless season.

2. **Weight per Cubic Foot.**—The weight per cubic foot of the timber shall be not less than 35 lbs. when the moisture content is 15 per cent.

3. **Seasoning.**—The timber shall be naturally seasoned if possible. If kiln-dried timber is used the following conditions shall be complied with :—

(i) The timber shall be dried in an approved kiln.
(ii) Autographic records shall be kept showing the temperature and humidity conditions during the process.
(iii) The maximum temperature shall not exceed 125° F.
(iv) The moisture content at the conclusion of the process shall be as follows :—

Summer (Mid. April to Mid. Oct.)	not more than	18	per cent.	and
	not less than	12	,,	
Winter (Mid. Oct to Mid April)	not more than	16	,,	and
	not less than	10	,,	

4. **Dryness of Timber.**—The timber shall not be reduced to its final shape and size until the moisture has been reduced to 13 per cent. or less, nor shall any parts of the wood be glued together until this degree of dryness is attained.

5. **Dryness Tests.**—*Selection of Test Pieces.* Samples shall be cut from the timber, whether unseasoned or seasoned, and shall be tested for moisture content.

6. **Conversion.**—Logs containing a simple bend shall be planked so that planks are produced parallel to the plane of the bend. Irregular and curved parts should, as far as possible, be obtained from timber having naturally curved grain, so that the grain of the wood is continuous throughout the length of the part. Walnut, however, bends readily when steamed, and parts may be bent when necessary.

7. **Straightness of Grain.**—The maximum inclination of the grain to the length of the part shall not exceed one in twelve.

8. **Freedom from Defects.**—The timber shall contain no deleterious knots or shakes, curls, burrs, rammy figure, caney grain, nor show signs of prolonged weathering, mildew pock, dead streak, or other forms of rot. Sapwood may be permitted in any part if it is sound, bright, tough and strong. Pinholes, if not too large or numerous, may be permitted.

NOTE.—Wide-ringed material is usually heavier and is the tougher and better material.

9. **Mechanical Tests.**—*Notched Bar Tests for Brittleness.* A test piece of the size and shape shown in Chapter XX. when tested in an impact testing machine shall not absorb less than 9 ft.-lbs. The sides of the test piece shall be cut radially and tangentially and the blow shall be applied in the tangential direction.

NOTE.—Where a suitable machine is not available the test piece shall be broken by a blow with a hammer. The fracture shall show a satisfactory amount of fibrous splinter.

ADMIRALTY, AIR DEPARTMENT

L. 3. *December*, 1915. *Abridged.*

PROVISIONAL SPECIFICATION FOR DURALUMIN SHEETS

1. **Material.**—The sheets are to be made of Duralumin alloy of specific gravity not exceeding 2·85; they are to be sound, homogeneous, free from blisters, laminations, and surface defects of any kind, and are to be quite fair and free from buckle.

2. **Accuracy of Dimensions.**—The sheets are to be of the dimensions specified. A variation in thickness to the extent of the following limits will be tolerated:—

	inch.
For I.W.G. of 23 and over	± ·001
,, ,, 19 to 22 (inclusive)	± ·002
,, ,, 14 to 18 ,, ...	± ·004
,, ,, 6 to 13 ,, ...	± ·006

3. **Tests.**—The sheets are to comply with the following test:—

Tensile Test.—Strips cut from the sheets lengthwise or crosswise are to show when tested in tension not less than the following results:—

	Tensile strength. Tons per sq. inch.	Elongation per cent. on 2 inches.
Sheets ·05 inch thick and above .	25	15
Sheets below ·05 inch thick . . .	25	10

AIR BOARD

L. 4. *January,* 1918. *Abridged.*

SPECIFICATION FOR ALUMINIUM SHEETS

Specific Gravity 2·75

1. **Quality and Manufacture.**—The sheets are to be made of aluminium assaying not less than 98 per cent.

They are to have bright, clean, and smooth surfaces, free from discoloration, blisters, lamination, or surface defects of any kind, and are to be quite fair and free from buckle.

2. **Accuracy of Dimensions.**—The sheets are to be made of the sizes and gauges stated on the order. The thickness is not to differ from the thickness specified by more than the following tolerances :—

For sheet of 23 S.W.G. and thinner	± ·001
,, ,, 19 to 22 S.W.G.	± ·002
,, ,, 14 to 18 ,,	± ·004
,, ,, 6 to 13 ,,	± ·006

3. **Mechanical Tests.**—The sheets are to comply with the following tests :—

Tensile Tests.—Strips 12 inches long × 1¾ inches wide cut longitudinally from any portion of the sheet are to give results not less than the following. The effective part of the test piece is to be 4 inches long × 1¼ inches wide :—

Ultimate strength	10 tons per square inch.
Elongation in 4 inches	3 per cent.

Bending Tests.—Strips cut longitudinally or transversely are to stand bending double over a radius equal to the thickness of the strip without cracking.

AIR BOARD

L. 16. *July*, 1918. *Abridged.*

SPECIFICATION FOR HALF-HARD ALUMINIUM SHEETS

Specific Gravity 2·75

1. **Quality and Manufacture.**—The sheets are to be made of aluminium assaying not less than 98 per cent.

They are to have bright, clean, and smooth surfaces, free from discoloration, blisters, lamination, or surface defects of any kind, and are to be quite fair and free from buckle.

2. **Accuracy of Dimensions.**—The sheets are to be made of the sizes and gauges stated on the order. The thickness is not to differ from the thickness specified by more than the following tolerances :—

Thickness of sheets.		Tolerances.
Gauge.	Inch.	Inch.
10	·128 and thicker	± ·008
14	·080 ,,	± ·006
19	·040 ,,	± ·004
23	·024 ,,	± ·003
Thinner		± ·002

3. **Mechanical Tests.**—The sheets are to comply with the following tests :—

Tensile Tests.—Strips 12 inches long × 1¾ inches wide cut longitudinally from any portion of the sheet are to give results not less than the following. The effective part of the test piece is to be 4 inches long × 1½ inches wide :—

Ultimate strength	7·5 tons per square inch.
Elongation in 4 inches	10·0 per cent.

Bending Tests.—Strips cut longitudinally or transversely are to stand bending double over a radius equal to the thickness of the strip without cracking.

AIR BOARD

L. 17. *July,* 1918. *Abridged.*

SPECIFICATION FOR SOFT ALUMINIUM SHEETS

Specific Gravity 2·75

1. **Quality and Manufacture.**—The sheets are to be made of aluminium assaying not less than 98 per cent.

They are to have bright, clean, and smooth surfaces, free from discoloration, blisters, lamination, or surface defects of any kind, and are to be quite fair and free from buckle.

2. **Accuracy of Dimensions.**—The sheets are to be made of the sizes and gauges stated on the order. The thickness is not to differ from the thickness specified by more than the following tolerances :—

Thickness of sheets.		Tolerances.
Gauge.	Inch.	Inch.
10	·128 and thicker	±·008
14	·080 ,,	±·006
19	·040 ,,	±·004
23	·024 ,,	±·003
Thinner		±·002

3. **Mechanical Tests.**—The sheets are to comply with the following tests :—

Tensile Tests.—Strips 12 inches long × 1¾ inches wide cut longitudinally from any portion of the sheet are to give results not less than the following. The effective part of the test piece is to be 4 inches long × 1¼ inches wide :—

Ultimate strength	5½ tons per square inch.
Elongation in 4 inches	15 per cent.

Bending Tests.—Strips cut longitudinally or transversely are to stand bending double and closing tight down without cracking.

AIR BOARD

S. 3. *April,* 1917. *Abridged.*

SPECIFICATION FOR MILD STEEL SHEETS (FOR WELDING)

Freedom from Defects.—The sheets must be sound, homogeneous, and free from blisters, lamination, or surface defects of any kind, and be quite fair and free from buckle.

Accuracy of Dimensions.—(*a*) The sheets are to be of the dimensions specified. The thickness is not to differ from the thickness specified by more than the following tolerances:—

For sheets of	6 to 13 S.W.G.	±·006 inch.
,, ,,	14 to 18 ,,	±·004 ,,
,, ,,	19 to 22 ,,	+·002 or −·003 inch.
,, ,,	23 S.W.G. or thinner	+·001 or −·002 ,,

(*b*) The above tolerances are for sheets used for small parts such as wiring plates. For large sheets over 3 feet wide to be used for pressings, the tolerances may be increased to the following:—

For sheets of	6 to 13 S.W.G.	+·008 or −·006 inch.
,, ,,	14 to 18 ,,	+·006 or −·004 ,,
,, ,,	19 to 22 ,,	±·003 inch.
,, ,,	23 S.W.G. or thinner	±·002 ,,

Mechanical Tests.—The sheets are to comply with the following tests:—

(*a*) *Tensile Test.*—Strips cut from the samples selected lengthwise or crosswise are to be machined for testing to a width of 1¼ inches. When tested in tension they must show not less than—

Ultimate strength	26 tons per square inch.
Yield point	18 ,, ,, ,,

(*b*) *Bending Tests.—Close Bend.* The corner of every sheet is to be bent over and closed down on to a sheet of the same thickness. The sheets must stand this treatment without showing any signs of cracking. All sheets failing in this test will be rejected.

(*c*) *Reverse Bends.*—A strip 1¼ inches wide cut from the sheet lengthwise or crosswise, and filed or machined so as to have smooth edges and rounded corners, is to be bent backwards and forwards in the following way:—The strip is to be fixed in a vice which has the inner edges of the jaws rounded to a radius equal to three times the thickness of the sheet. The projecting end of the strip is then to be bent at right angles to the fixed part, first to one side, then to the other, for a number of times till it breaks. The strip must stand without cracking at least three such reversals. The first bend through 90° is not counted.

AIR BOARD

S. 4. *March,* 1918. *Abridged.*

SPECIFICATION FOR NICKEL STEEL SHEETS

(NOT SUITABLE FOR WELDING)

Parts made of this steel are always to be hardened and tempered.

Freedom from Defects.—The sheets are to be sound, homogeneous, and free from blisters, lamination, or surface defects of any kind, and are to be quite fair and free from buckle.

Accuracy of Dimensions.—(*a*) The sheets are to be of the dimensions specified. The thickness is not to differ from the thickness specified by more than the following tolerances :—

For sheets of 6 to 13 S.W.G.	±·006 inch.
,, ,, 14 to 18 ,, ...	±·004 ,,
,, ,, 19 to 22 ,, ...	+·002 or −·003 inch.
,, ,, 23 S.W.G. or thinner	+·001 or −·002 ,,

Mechanical Tests.—(*a*) The sheets are to comply with the following tests :—

(*b*) *Tensile Test on Hardened and Tempered Samples.*—Strips are to be machined to a width of 1¼ inches, with a length of 4 inches between gauge points, and must give the following results :—

Ultimate stress not less than 48 tons per square inch.
Yield point ,, ,, 34 ,, ,, ,,

(*c*) *Bending Test on Hardened and Tempered Samples.*—A strip 1¼ inches wide, filed or machined so as to have smooth edges and rounded corners, is to be bent over a radius equal to 1½ times the thickness of the sheet until the sides are parallel. The sheets must stand this treatment without showing any signs of cracking.

AIR BOARD

L. 1. *May,* 1917. *Abridged.*

SPECIFICATION FOR DURALUMIN BAR

1. **Material.**—The bars are to be made of duralumin alloy of specific gravity not exceeding 2·85; they are to be sound, straight, free from twists, seams and damaged ends, and are to have a workmanlike finish. They are to be uniform in quality, within the stipulated margins of manufacture, capable of being turned and screwed satisfactorily and taking a good finish.

2. **Margins of Manufacture.**—(*a*) The margins of manufacture for bars up to 2⅝ inches diameter (extruded) are those given in Tables I. and II. of the British Standard Specification No. 35.

(*b*) The margin of manufacture for bars above 2⅝ inch diameter (hammered) is ±⅛ inch.

3. **Tests.**—The bars are to comply with the following test:—

Tensile Test.—Test pieces turned from the bars to the dimensions of the British Standard test piece, or, if the bars are too small, turned to a similar form having the same geometrical proportions, must give the following results:—

—	Bars above 2⅝ inches diameter (hammered).	Bars up to 2⅝ inches diameter (extruded).
Ultimate stress not less than	17 tons per square inch	25 tons per square inch
Yield point ,, ,,	10 ,, ,, ,,	16 ,, ,, ,,
Elongation ,, ,,	15 per cent. . . .	15 per cent.
Reduction of area ,, ,,	—	20 ,,

BRITISH ENGINEERING STANDARDS ASSOCIATION

F. 16. *May*, 1918. *Abridged.*

BRITISH STANDARD SPECIFICATION FOR RUBBER SHOCK ABSORBER CORD FOR AIRCRAFT

(*Issued for and under the authority of the Ministry of Munitions, Department of Aircraft Production.*)

NOTE.—The Association desires to call attention to the fact that this Specification is intended to include the technical provisions necessary for the supply of the material herein referred to, but does not purport to comprise all the necessary provisions of a contract.

1. **Construction.**—The cord shall be made of multiple strands of cut rubber thread tightly encased in two coverings of cotton braid. The cord shall be as smooth and as uniform in diameter as possible.

2. **Quality of the Rubber Thread.**—The thread shall be of hard fine Para rubber vulcanized with the admixture of sulphur or other approved agent. All the cut strands shall be square in section.

3. **Sizes of Cut Rubber Thread.**—The count of the cut rubber thread shall be not larger in section than $\frac{1}{18}$ inch square nor smaller than $\frac{1}{30}$ inch square. Only one count of cut rubber thread shall be used in any one cord. A tolerance of plus or minus 5 per cent. will be permitted in the respective counts.

4. **Test of Bare Rubber.**—The degree of vulcanization shall be such that a strand 6 inches long shall extend to a length between 27 and 37 inches 60 seconds after the application of the load given in the following table. The extension of any of the threads in any one warp shall not differ by more than 5 inches.

Counts.	Inch. $\frac{1}{18}$	Inch. $\frac{1}{20}$	Inch. $\frac{1}{22}$	Inch. $\frac{1}{24}$	Inch. $\frac{1}{26}$	Inch. $\frac{1}{28}$	Inch. $\frac{1}{30}$
Load applied in grammes .	315	255	210	180	150	130	115

Sixty seconds after removal of the load a strand originally 6 inches long shall return to a length not exceeding $6\frac{3}{8}$ inches.

5. **Method of Testing Cut Rubber Thread.**—The cut rubber thread shall be held in suitable grips.[1] The top grip should be fixed vertically, and to the bottom grip, which should be a floating one, weights should be added to bring the combined weight to the required value as given in the above table. The extension of each rubber thread shall be measured by means of two marks upon the thread, each mark at least $\frac{1}{2}$ inch distant from the jaws of the grips and 6 inches distant from each other when the rubber thread is in its normal condition. These marks may be indicated conveniently by tying white cotton on to the cut rubber thread. The load shall be applied gradually by supporting the weight

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[1] Strong spring wooden clothes pegs faced with fine glass paper or rubber upon the gripping surfaces are suitable.

in the hand and allowing it to sink slowly to the point where the rubber thread carries the entire weight. The temperature of the test room shall be between 60° F. and 70° F., at which temperature the rubber shall have been kept for at least three hours prior to the test.

6. **Braiding.**—Both the inner and the outer coverings shall be made of best American two-fold cotton yarn in the grey; the inside cover shall be of soft American cotton, and the outer cover of hard polished glacé American cotton.

7. **Sizes of Finished Cord.**—The overall diameter, including the braid, shall be one of the following three sizes :—

$\frac{3}{8}$ inch, $\frac{1}{2}$ inch or $\frac{5}{8}$ inch.

A tolerance of ± 10 per cent. shall be permitted on the overall diameter.

8. **Number of Rubber Strands.**—The standard sizes of cord shall be obtained by using a minimum number of rubber strands in accordance with the following table :—

Size of cord.	Minimum number of strands of rubber.						
	Inch. $\frac{1}{18}$	Inch. $\frac{1}{20}$	Inch. $\frac{1}{22}$	Inch. $\frac{1}{24}$	Inch. $\frac{1}{26}$	Inch. $\frac{1}{28}$	Inch. $\frac{1}{30}$
Inch.							
$\frac{3}{8}$	77	95	114	136	160	186	213
$\frac{1}{2}$	139	172	208	247	290	336	386
$\frac{5}{8}$	215	266	322	383	450	522	600

9. **Finished Cord.**—The cord shall be made up whilst the bare rubber is under an initial strain calculated to give the following loads in the finished cord :—

Size of cord.	Load in lbs. to give 10 % extension.	Load in lbs. to give 100 % extension.		Load in lbs. to give an extension of 5 % beyond 100 %.
Inch.	Min.	Min.	Max.	Max.
$\frac{3}{8}$	20	70	90	15
$\frac{1}{2}$	35	130	155	25
$\frac{5}{8}$	55	200	240	40

10. **Tests upon Finished Cord.**—Any coil of finished cord may be tested at four regions, selected indiscriminately throughout its length, and the average test results shall be in accordance with the loads and extensions set out in the table in clause 9.

11. **Method of Testing Finished Cord.**—The temperature of the test room shall be between 60° F. and 70° F., at which temperature the cord shall have been kept for at least three hours prior to the test. The portion of cord to be tested shall be stretched three times to 100 per cent. extension in order that any "shrinkage" or "creeping" may take place before determining the test loads. Five-inch lengths of cord shall be tested in a spring balance machine designed to give a uniform rate of stretch, and which is capable of testing the cord without cutting the coil. The complete extension—not including the hysteresis measurement—shall be completed in ninety seconds.

APPENDIX III

CHARACTERISTICS OF VARIOUS AEROFOILS

NOTE.—In the following tables, in addition to the standard symbols explained in Chapter II., the following have been used:—

k_N = coefficient of force normal to the chord.
k_T = „ „ along the chord.

Aerofoil R.A.E. 3.

Angle of incidence.	k_L	k_D	L/D	k_N	k_T	C.P. coeff.	k_M
—6°	—0·123	·0400	—3·08	—·127	·0270	·228	·0288
—4	—0·059	·0298	—1·98	—·061	·0256	—0·101	·0062
—3	—0·0017	·0248	—0·07	—·003	·0248	—18·300	—0·033
—2	·056	·0207	2·70	·055	·0227	·958	—0·053
—1	·112	·0185	6·03	·112	·0204	·659	—0·073
0	·165	·0175	9·44	·165	·0175	·530	—0·087
1	·202	·0167	12·1	·202	·0132	·476	—0·096
2	·241	·0164	14·6	·242	·0080	·437	—0·105
3	·284	·0172	16·4	·285	·0023	·406	—0·115
4	·318	·0193	16·4	·318	—·0030	·387	—0·123
5	·359	·0224	16·0	·359	—·0090	·369	—0·132
6	·398	·0256	15·5	·399	—·0161	·358	—0·142
8	·466	·0333	14·0	·466	—·0318	·347	—0·161
10	·534	·0414	12·9	·533	—·0520	·332	—0·177
12	·590	·0517	11·4	·588	—·0720	·321	—0·189
14	·632	·0616	10·2	·628	—·0932	·311	—0·190
16	·615	·0772	7·96	·612	—·0950	·308	—0·189
18	·601	·113	5·32	·606	—·0785	·343	—0·208
20	·568	·160	3·55	·589	—·0438	·385	—0·226

Aerofoil R.A.E. 6c.

Angle of incidence.	k_L	k_D	L/D	k_N	k_T
—6°	—·147	·0367	—4·0	—·150	·021
—4	—·072	·0256	—3·0	—·074	·021
—2	·009	·0186	0·5	·008	·019
0	·105	·0154	6·8	·105	·015
2	·207	·0149	13·9	·207	·008
4	·288	·0180	16·0	·289	—·002
6	·357	·0240	14·9	·358	—·013
8	·432	·0324	13·5	·432	—·028
10	·499	·0411	12·2	·499	—·046
12	·567	·0513	11·0	·565	—·068
14	·609	·0617	9·9	·606	—·087
16	·608	·7059	8·0	·605	—·095
18	·559	·1302	4·3	·572	—·049
20	·502	·1637	3·1	·528	—·018

Aerofoil R.A.E. 9.

Angle of incidence.	k_L	k_D	L/D	k_N	k_T	C.P. coeff.	k_M
−6°	−·138	·0325	−4·3	−·140	·0179		·0201
−4	−·064	·0221	−2·9	−·0665	·0176		−·0048
−2	·011	·0165	·6	·0104	·0169	2·257	−·0236
0	·112	·0141	7·9	·112	·0141	·467	−·0523
2	·203	·0138	14·7	·2035	·0067	·383	−·0778
3	·241	·0150	16·08	·242	·0024		
4	·277	·0172	16·0	·277	−·0021	·340	−·0944
6	·351	·0237	14·9	·352	−·0131	·317	−·1114
8	·422	0318	13·3	·422	−·0273	·303	−·1280
10	·494	·0413	12·0	·494	−·0451	·297	−·1466
12	·559	·0508	11·0	·558	−·0664	·290	−·1616
14	·604	·0618	9·8	·601	−·0861	·283	−·1700
16	·561	·1065	5·3	·569	−·0522	·320	−·1819
18	·485	·1433	3·4	·505	−·014	·370	−·1873
20	·459	·1650	2·8	·489	−·002	·383	−·1867

Aerofoil R.A.E. 14.

Angle of incidence.	k_L	k_D	L/D	k_N	k_T	C.P. coeff.	k_M
−6°	−·163	·0354	−4·6	−·166	·0181	·178	·0296
−4	−·0682	·0225	−3·03	−·0696	·0176	−·144	−·0100
−2	·0388	·0125	3·1	·0382	·0138	1·35	−·0516
0	·134	·0094	14·25	·134	·0094	·520	−·0700
2	·215	·0102	21·1	·215	·0027	·413	−·0887
4	·284	·0143	19·9	·284	−·0046	·370	−·105
6	·356	·0199	17·9	·356	−·0176	·330	−·117
8	·419	·0270	15·5	·419	−·0315	·316	−·132
10	·474	·0360	13·2	·473	−·0469	·297	−·140
12	·510	·0484	10·5	·509	−·0586	·288	−·143
14	·536	·0753	7·12	·538	−·0567	·290	−·156
16	·544	·1140	4·77	·554	−·0403	·324	−·179

Aerofoil R.A.E. 15.

Angle of incidence.	k_L	k_D	L/D	k_N	k_T	C.P. coeff.	k_M
−6°	−·1615	·0285	−5·67	−·163	·101	·161	·0262
−4	−·0901	·0147	−6·12	−·091	·008	·101	·0091
−3	−·0556	·0109	−5·12	−·056	·008	·027	·0015
−2	−·0236	·0088	−2·68	−·024	·008	−·241	−·0057
−1	·0112	·0081	1·38	·011	·008	1·256	−·0138
0	·0491	·0074	6·66	·049	·007	·467	−·0229
1	·0930	·0080	11·6	·093	·006	·381	−·0354
2	·1430	·0092	15·6	·143	·004	·364	−·0520
3	·1916	·0111	17·3	·192	·001	·338	−·0649
4	·2260	·0132	17·1	·226	−·003	·319	−·0721
5	·2603	·0156	16·7	·261	−·007	·307	−·0801
6	·2958	·0184	16·1	·296	−·013	·299	−·0884
8	·3691	·0259	14·3	·369	−·026	·290	−·1104
10	·434	·0337	12·9	·433	−·042	·282	−·122
12	·485	·0440	11·0	·483	−·058	·276	−·133
14	·513	·0686	7·47	·514	−·058	·288	−·148
16	·511	·1160	4·41	·523	−·029	·339	−·177
18	·484	·1460	3·32	·505	−·011	·384	−·194
20	·458	·1640	2·79	·487	−·003	·391	−·190

Aerofoil R.A.E. 16.

Angle of incidence.	k_L	k_D	L/D	k_N	k_T	C.P. coeff.	k_M
—6°	—·1231	·0267	—4·64	—·1252	·0137	·080	·0100
—4	—·0507	·0149	—3·40	—·0515	·0113	—·175	—·0090
—3	—·0125	·0117	—1·07	—·0131	·0111	—1·37	—·0177
—2	·0222	·0101	2·19	·0218	·0109	1·16	—·0252
—1	·0597	·0091	6·55	·0595	·0101	·575	—·0343
0	·1025	·0089	11·5	·1025	·0089	·441	—·0450
1	·1457	·0101	14·6	·1459	·0076	·399	—·0581
2	·2001	·0121	16·5	·200	·0051	·366	—·0735
3	·2370	·0144	16·5	·238	·0020	·343	—·0812
4	·2730	·0170	16·1	·273	—·0021	·326	—·0890
5	·312	·0200	15·5	·313	—·0073	·316	—·0986
6	·347	·0235	14·7	·348	—·0129	·305	—·106
8	·418	·0319	13·1	·418	—·0265	·294	—·123
10	·482	·0418	11·5	·482	—·0425	·257	—·138
12	·542	·0538	10·1	·541	—·0601	·281	—·153
14	·568	·0983	4·77	·575	—·0421	·289	—·167
16	·445	·1236	3·30	·462	—·0038	·367	—·170
18	·446	·1461	3·05	·469	—·0013	·382	—·179
20	·446	·1652	2·66	·476	—·0029	·388	—·185

Aerofoil R.A.E. 17.

Angle of incidence.	k_L	k_D	L/D	k_N	k_T	C.P. coeff.	k_M
—6°	—·177	·0298	—5·96	—·179	·0111	·176	·0312
—4	—·104	·0175	—5·92	—·105	·0102	·121	·0127
—3	—·068	·0137	—5·00	—·069	·0101	·083	·0057
—2	—·035	·0112	—3·14	—·035	·0100	—·046	—·0016
—1	—·0023	·0094	—0·25	—·0025	·0094	—·725	—·0067
0	·031	·0080	3·92	·031	·0080	·511	—·0159
1	·073	·0077	9·32	·073	·0064	·374	—·0273
2	·120	·0088	13·6	·120	·0046	·349	—·0418
3	·167	·0105	15·8	·168	·0018	·320	—·0534
4	·203	·0127	15·9	·203	—·0015	·301	—·0611
5	·236	·0151	15·7	·236	—·0055	·295	—·0695
6	·271	·0174	15·6	·271	—·0110	·286	—·0775
8	·340	·0239	14·2	·340	—·0236	·277	—·0942
10	·414	·0329	12·5	·413	—·0395	·271	—·112
12	·475	·0421	11·3	·474	—·0576	·267	—·126
14	·508	·0628	8·1	·508	—·0621	·269	—·136
16	·471	·1110	4·24	·483	—·0230	·320	—·154
18	·447	·1368	3·28	·467	—·008	·368	—·172
20	·434	·1560	2·78	·461	—·0018	·376	—·173

Avro Wing Section.

Angle of incidence.	k_L	k_D	L/D	k_N	k_T	C.P.	k_M
—6°	—·1345	·0381	—3·53	—·138	·0238	·140	·0193
—4	—·0391	·0253	—1·54	—·041	·0225	—·524	—·0215
—3	·0131	·0201	0·65	·012	·0208	3·4	—·0408
—2	·0637	·0155	4·11	·063	·0177	·912	—·0575
—1	·110	·0135	8·21	·110	·0154	·648	—·0713
0	·155	·0131	11·8	·155	·0131	·545	—·0845
1	·196	·0129	15·1	·196	·0095	·487	—·0955
2	·235	·0142	16·6	·236	·0060	·445	—·105
3	·269	·0155	17·3	·270	·0014	·419	—·113
4	·303	·0179	16·9	·303	·0033	·395	—·1196
5	·338	·0205	16·5	·338	—·0091	·377	—·1275
6	·368	·0231	16·0	·368	—·0155	·362	—·133
8	·426	·0294	14·5	·426	—·0301	·339	—·1445
10	·479	·0381	12·6	·479	—·0457	·323	—·1556
12	·520	·0492	10·5	·519	—·0600	·311	—·161
14	·547	·0640	8·55	·547	—·0702	·303	—·166
16	·554	·0964	5·75	·559	—·0599	·326	—·182
18	·543	·1368	3·98	·558	—·0377	·359	—·200
20	·507	·1638	3·10	·532	—·0195	·383	—·204

Bristol Wing Section.

Angle of incidence.	k_L	k_D	L/D	k_N	k_T	C.P. coeff.	k_M
—6°	—·1383	·0203	—6·81	—·140	·0057	·094	·0132
—4	—·0806	·0124	—6·50	—·0813	·0067	·032	·0026
—3	—·0504	·0102	—4·94	—·0509	·0076	—·050	—·0025
—2	—·0164	·0089	—1·84	—·0167	·0083	—·609	—·0102
—1	·0154	·0087	1·77	·0152	·009	1·146	—·0174
0	·0504	·0086	5·86	·0504	·0086	·534	—·0269
1	·1017	·0098	10·4	·102	·008	·437	—·0446
2	·1575	·0114	13·8	·158	·006	·404	—·0638
3	·2135	·0131	16·3	·214	·0019	·376	—·0805
4	·2480	·0150	16·5	·248	—·0023	·345	—·0856
5	·2775	·0174	15·9	·278	—·0069	·330	—·0917
6	·306	·0204	15·0	·306	—·0117	·315	—·0964
8	·362	·0303	12·0	·363	—·0204	·293	—·106
10	·409	·0436	9·39	·411	—·0281	·281	—·1155
12	·450	·0698	6·45	·455	—·0253	·296	—·135
14	·455	·1030	4·41	·466	—·01	·342	—·159
16	·438	·1260	3·48	·456	·0004	·370	—·169
18	·425	·144	2·95	·449	·0057	·380	—·172
20	·418	·161	2·60	·448	·0083	·388	—·174

Handley Page Aerofoil.

Angle of incidence.	k_L	k_D	L/D	k_N	k_T	C.P. coeff.	k_M
—4°	—·066	·0254	—2·6	—·068	·0207	—·145	—·0098
—2	·023	·0172	1·3	·022	·018	1·62	—·036
—1	·071	·0150	4·75	·071	·0162	·724	—·051
0	·126	·0140	9·0	·126	·0140	·538	—·0678
2	·220	·0136	16·2	·220	·0059	·433	—·095
3	·258	·0153	16·8	·259	·0018	·399	—·104
4	·296	·0178	16·7	·296	—·0029	·375	—·111
5	·332	·0206	16·1	·332	—·0085	·362	—·120
6	·369	·0241	15·3	·370	—·0146	·352	—·130
8	·440	·0322	13·7	·440	—·0293	·337	—·148
10	·510	·0411	12·4	·509	—·048	·319	—·162
12	·575	·0506	11·3	·573	—·07	·311	—·178
14	·615	·0615	10·0	·612	—·089	·304	—·186
16	·605	·0748	8·1	·602	—·095	·297	—·179
18	·598	·100	6·0	·600	—·09	·313	—·188
20	·561	·159	3·52	·581	—·043	·359	—·208

Aerofoil B.I.R. 33a.

Angle of Incidence.	k_L	k_D	L/D	k_N	k_T	C.P. coeff.	k_M
—6°	—0·112	·0406	—2·76	—·116	·0287	·138	·016
—4	·002	·0277	0·08	·000	·0277	99·0	—·0064
—3	·059	·0234	2·54	·058	·0265	1·05	—·0607
—2	·110	·0206	5·36	·109	·0244	·697	—·0767
—1	·154	·0189	8·13	·154	·0216	·570	—·0878
0	·193	·0182	10·6	·193	·0182	·497	—·096
1	·226	·0182	12·4	·226	·0143	·458	—·1036
2	·265	·0190	13·9	·266	·0098	·423	—·112
3	·301	·0201	14·9	·302	·0044	·397	—·1195
4	·339	·0225	15·1	·340	—·0014	·378	—·1281
5	·374	·0248	15·1	·375	—·0079	·364	—·1361
6	·404	·0278	14·5	·405	—·0146	·356	—·144
8	·481	·0366	13·1	·481	—·0306	·338	—·1626
10	·546	·0460	11·8	·546	—·0495	·327	—·1782
12	·612	·0555	11·0	·610	—·0730	·322	—·1964
14	·669	·0672	9·95	·665	—·0956	·316	—·21
16	·705	·0790	8·92	·699	—·1183	·308	—·2155
18	·686	·0958	7·17	·682	—·121	·318	—·218
20	·646	·115	5·63	·646	—·113	·313	—·202

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